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Analysis of Cantilever beams in Liquid Media: A case study of a microcantilever

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ABSTRACT: *In modern era of atomic force microscope, it can be used in micro sensing applications in aerospace and fluid-flow engineering. The micro-sensor in such applications encounters various types of fluid media. The study of conventional micro-cantilevers is not applicable in liquids. The behavior of the AFM cantilever in liquid media has been studied by many researchers during the past five years. Hydrodynamic forces in the system are often modeled as nonlinear functions of the tip displacement. On the other hand micro-cantilevers sensors can also be used for measurement of micro scale viscosity, density, and temperature in avionic applications by analyzing frequency response of the cantilever. In this paper, a micro-cantilever with its tip operating in tapping mode is considered with liquid environment and modeled using a continuous system dynamics. The hydrodynamic forces and additional mass from the liquid are accounted in the equation of motion. Also the first mode dynamics is considered for solving equations of motion using Galerkin's method. It is also shown a methodology to measure fluid density and viscosity using microcantilever probes.*

Keywords*: Flow physics, Flexural vibration, Hydrodynamic force, Modal approximation.*

I. INTRODUCTION

The study of flexural vibrations of beams and plates submerged in a viscous fluid is drawing an increased attention in many research fields such as atomic force microscopy, micromechanical oscillators for sensing and actuations, micro scale energy harvesters and biomimetic propulsions. In all these applications the estimation of forces exerted by the fluid on the structure is of primary importance. Such forces include distributed lift and thrust produced by momentum transferred to the fluid. These forces are related to complex flow field generated by solid body motion which is influenced by inertial and viscous phenomenon. A first estimate of distributed lift of thin beam with rectangular cross section is given by Sader [1]. In this work, length to width ratio was selected very large and is subjected to low frequency excitation, so that beam is locally considered as infinitely long cylinder and fluid loading is analyzed using numerical findings based on unsteady Stokes flow. Brenetto et al. [2] explored the possibilities of extracting energy from mechanical vibration using ionic polymer metal composites in which the hydrodynamic function-expressions were proposed over some range of Renault's numbers. Aureli et al. [3] proposed an extension to take in to account finite amplitude oscillations for a two dimentional numerical simulations of flow physics induced by rigid lamina oscillating in viscous fluid. It is demonstrated in other papers [4] that as the amplitude increases, the relevant nonlinear hydrodynamic damping would always exists.

In the present work, we consider the flow induced by vibration of cantilever beam submerged in viscous fluid to determine the influence of parameters, such as frequency and amplitude of oscillation, aspect ratio on the forces exerted by fluid on the structure. An in-plane flexural vibration of the beam modeled using classical linear beam theory and is assumed to be vibrating along its fundamental mode shape. The fluid is assumed to be Newtonian and flow is incompressible.

II. PROBLEM STATEMENT

2.1 Beam vibration in liquids

We considered flexural vibration of cantilever beam under harmonic base excitation. Let x be the co-ordinate along beam axis with y and z are the co-ordinates along width and thickness. Beam is slender and composed of homogeneous and isotropic material. The classical linear Euler-Bernoulli beam theory gives the equation of motion as:

$$\frac{\partial^2}{\partial x^2}\left[K \times \frac{\partial^2 w(x,t)}{\partial x^2}\right] + (\rho bh) \times \frac{\partial^2 w(x,t)}{\partial t^2} = F_{hyd}(x,t) + S(x,t) + F(t) \qquad (1)$$

where, $K = \dfrac{Ebh^3}{12}$,

b and h are width and thickness, $\rho =$ Mass density of cantilever,

$w(x,t) =$ Beam deflection, $F(t) = F_0 \sin(\omega t) =$ Harmonic base excitation,

S(x,t)=-B $\dfrac{\partial w(x,t)}{\partial t}$ is the damping force, $L =$ Length of beam, $F_{hyd}(x,t)$ describes hydrodynamic action exerted on the beam by the encompassing fluid. The effect of liquid viscosity can be taken care by a simple model. Researchers have approximated the hydrodynamic forces to be in proportion to the cantilever acceleration and velocity as:

$$F_{hyd}(x,t) = -c_a \frac{dw}{dt} - \rho_a \frac{d^2 w}{dt^2} \qquad (2)$$

Where, $c_a =$ additional hydrodynamic damping coefficient= $\left(3\pi\eta + \frac{3}{4}\pi b\sqrt{2\eta\rho_{liq}\omega}\right)$ and additional mass density $\rho_a = \left(\frac{1}{12}\pi\rho_{liq}b^2 + \frac{3}{4}\pi b\sqrt{\frac{2\rho_{liq}\eta}{\omega}}\right)$. Here, ω is vibrating frequency of the cantilever, η is kinematic viscosity of liquid, ρ_{liq} is density of the liquid.

2.2 Solution methodology

In order to solve the dynamic equations in continuous form, the Galerkin's approximation method is employed. Here we considered w(x,t)= $\sum_{i=1}^{M}\phi_i(x)q_i(t)$ where M is number of modes used, $\phi_i(x)$ is ith normalized modal function. As first mode dominates, often w(x,t) is approximated as $\phi_1(x)q_1(t)$. Here, $\phi_1=\phi_1(x)$ is obtained from the boundary conditions of the beam. The mode shape function $\phi_1(x)$ is multiplied on both sides of the differential eq.(1) and the resultant equation is integrated along the cantilever length. i.e.

$$\int_0^L \phi_1 q_1 K \frac{\partial^4 \phi_1}{\partial x^4}dx + (\rho bh + \rho_a)\ddot{q}_1\int_0^L \phi_1^{\,2}dx + (B + c_a)\dot{q}_1\int_0^L \phi_1^{\,2}dx = F_0 q_1 \sin\omega t\int_0^L \phi_1 dx \qquad (3)$$

III. NUMERICAL EXAMPLE

In order to illustrate the methodology, a microcantilever beam with nano-tip used in AFM sensing [5] subjected to harmonic base excitation is considered as shown in Fig.1. Several earlier works demonstrated the operation of such beams in liquid media. Song and Bhushan [6] used finite element model to know frequency and transient response analysis of cantilevers in tapping mode operating in air as well as liquid. Korayem et al. [7] showed that the frequency response behavior of micro cantilever in liquid is completely different from that in air and studied the influence of mechanical properties of the liquid like viscosity and density on frequency response analysis. Vancura et al.[8] analyzed characteristics of resonant cantilever in viscous liquids using rectangular cantilevers geometries in pure water, glycerol and ethanol solution with different concentration. His study results can be used in resonant cantilevers as biochemical sensors in liquid environments.

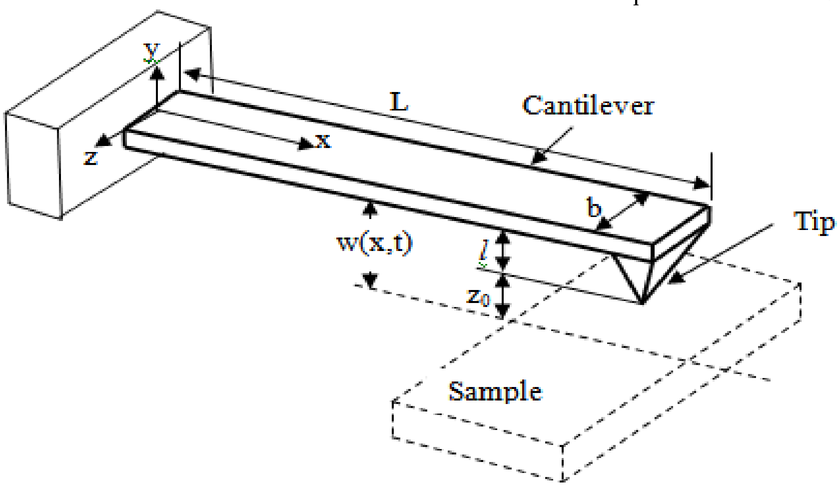

Fig.1 Micro-cantilever beam under consideration

In addition to the hydrodynamic and harmonic forces, the system is subjected to an atomic interaction force $f_{ID}(t)$ in microscopic level. The general mode shape function is obtained from the following boundary conditions:

At x = 0, w(0,t) = 0, and $\frac{\partial w(0,t)}{\partial x} = 0$ (4)

At x = L, $K\frac{\partial^2 w(L,t)}{\partial x^2} = 0$, and $K\frac{\partial^3 w(L,t)}{\partial x^3} = m_e\frac{\partial^2 w(L,t)}{\partial x^2} - f_{ID}(t)$ (5)

Here, $f_{ID}(t)=-k_{ts}w(L,t)$ is linearized tip-sample interaction force, with contact stiffness

$$k_{ts} = -\frac{\partial f_{ID}(t)}{\partial w(L,t)} = \begin{cases} -\frac{HR}{3z_0^3}, if\,(z_0 - w(L,t)) \geq a_0 \\ 2E^*\sqrt{R(a_0 + z_0)}, if\,(z_0 - w(L,t)) < a_0 \end{cases} \quad (6)$$

where, H is Hamaker constant, z_0 is equilibrium distance between cantilever and sample, R is equivalent tip-radius, $E^*=[(1-\nu_t^2)/E_t+(1-\nu_s^2)/E_s]^{-1}$ is effective elastic modulus, a_0 is interatomic distance and m_e is equivalent tip mass added. The frequency equation and eigenfunction can be obtained from above four boundary conditions as follows [9]

$$2\left[k_{ts} - m_e\frac{EI}{\rho A}\beta^4\right]\left[\sin\beta L\cdot\cosh\beta L - \cos\beta L\cdot\sinh\beta L\right] + 2\beta^3 EI\left[1 + \cos\beta L\cdot\cosh\beta L\right] = 0 \quad (7)$$

where $\beta^4 = \frac{\rho A}{EI}\omega^2$. The normalized mode shape is

$$\phi(x) = \frac{1}{N}\left[(\cos\beta L + \cosh\beta L)(\sin\beta x - \sinh\beta x) - (\sin\beta L + \sinh\beta L)(\cos\beta x - \cosh\beta x)\right] \quad (8)$$

where $N = 2(\sin\beta L\cosh\beta L - \cos\beta L\sinh\beta L)$ (9)

The computations are performed with a MATLAB symbolic logic program, which can resolve the equations into ordinary differential form in terms of q_1.

IV. RESULT AND DISCUSSION

Table 1 shows the data considered for analysis.

Table 1. Parameters of simulation for the AFM cantilever [6]

Cantilever length (L)	**200 µm**
Cantilever width (b)	140 µm
Cantilever thickness (t)	7.7 µm
Cantilever mass density (ρ)	2730 Kg/m^3
Cantilever Young's Modulus (E)	130 GPa
Quality factor in air (Q)	900
Liquid density(ρ_{liq})	1030 Kg/m^3
Liquid viscosity(η)	13.2×10^{-4} Kg/m^3
Cantilever angle(α)	15°
Number of elements(n)	20
Tip length(l)	10 µm
Tip radiud(R)	10 nm
Hamarker constant (H)	2.96×10^{-19} J
Intermolecular distance (a_0)	0.38 nm
Effective elastic modulus (E^*)	10.2 GPa
Effective elastic modulus (G^*)	4.2 GPa

The effect of equivalent linear interaction stiffness $\hat{k}_{ts} = k_{ts}/k$, where $k=\rho A l\omega_n^2$ on natural frequencies is as shown in Fig. 2.

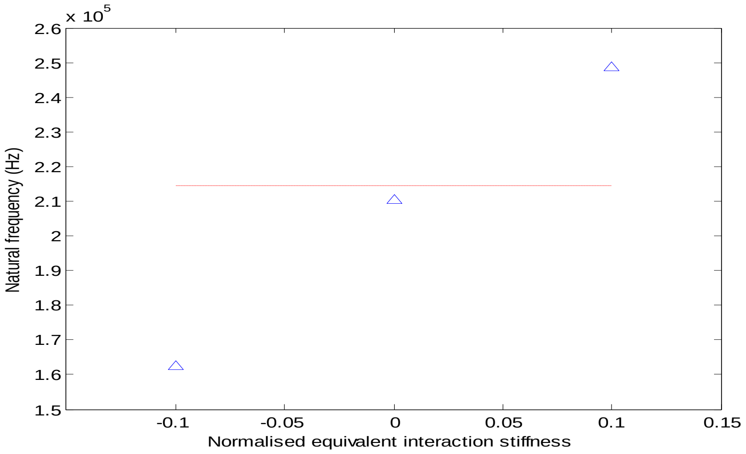

Fig.2 Graph of Normalised equivalent stiffness vs. natural frequency

Here the dotted line indicates the natural frequency of normal cantilever in air without tip mass. It is seen that even if interaction stiffness is zero, the natural frequency mismatch with dashed line is due to the tip-mass boundary condition. Using the modal function available after solving frequency equation, the partial differential is reduced into second order differential equation in terms of variable q1 as per Eq.(3). This is solved with Runge-Kutta's fourth order method, to study the effect equivalent stiffness on time response. The viscous damping ratio considered in present work is 0.001. Fig.3 shows the time history and phase diagram for the system with $\hat{k}_{ts}$ =0.1.

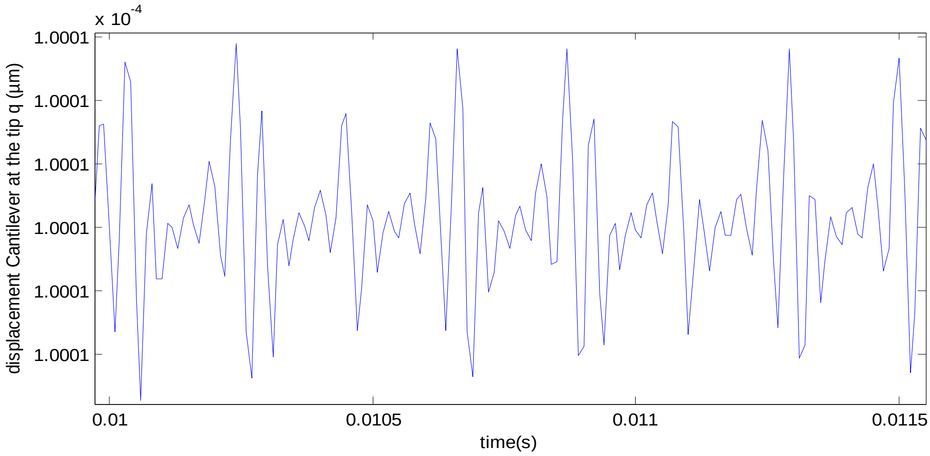

Fig.3 Variation of the displacement(µm) of system with respect to time (s)

Fig.4 shows the graph of the displacement of the cantilever vs. velocity of the cantilever for the system.

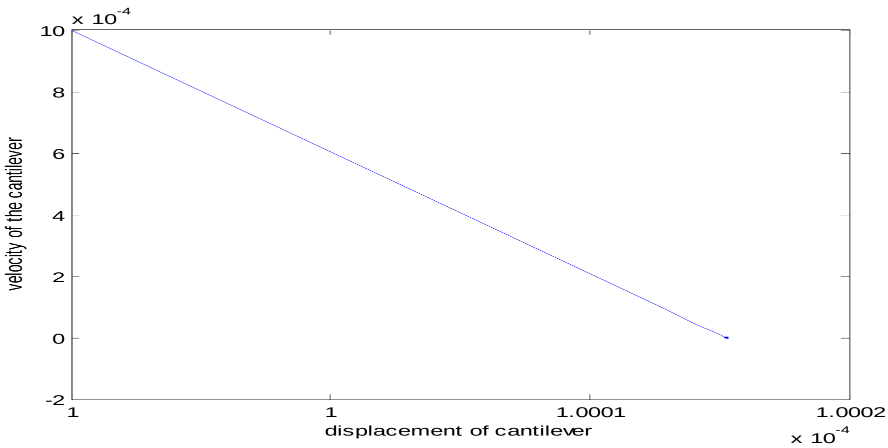

Fig.4 Graph of displacement vs. velocity of the cantilever.

IV. CONCLUSIONS

In this paper cantilever beam dynamics using first mode mechanics in liquids was considered. The effect of hydrodynamic force exerted by encompassing fluid was studied. Galerkin's approximation method was used to get the normalized modal function. Runge-Kutta solver is used to solve this second order ordinary differential equation in time variable. The effect of normalized equivalent interaction stiffness on natural frequency is studied. Further work is going on. It can be concluded that there is a tremendous effect of hydrodynamic forces on the modal characteristics of cantilevers.

APPENDIX

Modal function is approximated in terms of frequency parameter β as:

$$\phi(x) = C_1 \cos\beta x + C_2 \sin\beta x + C_3 \cosh\beta x + C_4 \sinh\beta x$$

The constants C_1 to C_4 are obtained from following boundary conditions:

At $x = 0,$ $\quad w(0,t) = 0 \quad \phi(0) = 0 \Rightarrow C_1 = -C_3$

At $x = 0$ $\quad w'(0,t) = 0 \quad \phi'(0) = 0 \Rightarrow C_2 = -C_4$

$$\therefore\ \phi(x) = C_1(\cos\beta x - \cosh\beta x) + C_2(\sin\beta x - \sinh\beta x)$$

Further at $x = L\ Kw''(L,t) = 0$

$$\Rightarrow C_1(-\cos\beta L - \cosh\beta L) + C_2(-\sin\beta L - \sinh\beta L) = 0 \qquad \text{(A1)}$$

At $x = L$ $\quad Kw'''(L,t) = m_e \dfrac{d^2 w}{dt^2} - k_{ts} w(L,t)$

$$\Rightarrow \begin{aligned} &K\beta^3\left[C_1(\sin\beta\mathrm{L} - \sinh\beta\mathrm{L}) + C_2(-\cos\beta\mathrm{L} - \cosh\beta\mathrm{L})\right]e^{j\omega t} \\ &= m_e\phi(L)(-\omega^2 \times e^{j\omega t}) - k_{ts}\phi(L)e^{j\omega t} \end{aligned}$$

$$\Rightarrow \begin{aligned} &K\beta^3\left[C_1(\sin\beta\mathrm{L} - \sinh\beta\mathrm{L}) + C_2(-\cos\beta\mathrm{L} - \cosh\beta\mathrm{L})\right]e^{j\omega t} \\ &= -(m_e + k_{ts})\left[(C_1(\cos\beta L - \cosh\beta L) + C_2(\sin\beta L - \sinh\beta L)\right] \end{aligned}$$

$$\Rightarrow \begin{aligned} &\left[K\beta^3(\sin\beta\mathrm{L} - \sinh\beta\mathrm{L}) + (m_e + k_{ts})(\cos\beta L - \cosh\beta L)\right]C_1 \\ &-\left[K\beta^3(-\cos\beta\mathrm{L} - \cosh\beta\mathrm{L}) - (m_e + k_{ts})(\sin\beta L - \sinh\beta L)\right]C_2 = 0 \end{aligned} \qquad \text{(A2)}$$

Eliminating C_1 and C_2 frin eqs.(A1) and (A2), we get the frequency equation (7):

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