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Note On An Inverse Transient Thermoelastic Behavior of Circular Plate

Badrinath E Ghonge
Department of Mathematics,
(S.R. Arts,Com. & Sci. College Ghansawangi, Dist. Jalna, MS, India)

Abstract: *This work deals with determination of unknown temperature and thermal deflection of thin circular plate with the stated conditions. The inverse heat conduction equation is solved by using the Marchi-Fasulo integral transform and Laplace transform simultaneously and the results for unknown temperature and thermal deflection are obtained in terms of infinite series and it is solved for special case by using Math-cad software and illustrated graphically by using Microsoft office excel 2010.*

KEYWORDS –*Inverse transient, Laplace Transform, Marchi-Fasulo Transform, Thin circular plate.*

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I. INTRODUCTION

The inverse problems of thermoelasticity consist of determination of temperature distribution and thermal deflection of solids when the conditions of temperature and deflection are known at the some points of the solid under consideration. Grysa and Cialkowski [1], Grysa and Koalowski [2] studied one-dimensional transient thermoelastic problems and derived the heating temperature and heat flux on the surface of an isotropic infinite slab. In [3] Dai et. al studied Based on the thermoviscoelastic theory and the classic plate theory, thermoviscoelastic behavior of a circular plate made from high strength low alloy (HSLA) steel material is investigated. The entire problem is solved by utilizing the finite difference method, Newmark method and iterative method. Khobragade et al.[4] and [5] discuss an inverse steady state and transient thermoelastic problem of thin circular plate and annular disc in Marchi-Fasulo transform domain. Deshmukh et al. [6] investigated inverse heat conduction problem of semi-infinite, clamped thin circular plate and their thermal deflection by quasi-static approach. Ghonge and Ghadle [7]-[10] investigated an inverse problems of thermoelastic behavior in different solids by integral transform methods. Further Ghonge and Ghadle [11]-[14] derived the analytical solution to deflection of thermoelastic circular plates for different conditions by using Marchi-Fasulo, Marchi-Zgrablich and Laplace integral transform.

In this work, the temperature, unknown temperature on curved surface and quasi-static thermal deflection due to unknown temperature $g(z,t)$ are discuss. The inverse heat conduction equation is solved by using finite Marchi-Fasulo transform and Laplace transform simultaneously and the results for unknown temperature and thermal deflection are obtained in terms of infinite series of Bessel's function and it is solved for special case by using Math-cad software and illustrated graphically by using Microsoft office excel 2010.

II. CONSTRUCTION OF THE PROBLEM

Consider a thin circular plate of thickness 2h occupying the space $D:\left\{(x,y,z)\,|\,0\le r=\sqrt{x^2+y^2}\le a,-h\le z\le h\right\}$. Suppose the plate is subjected to arbitrary known interior temperature $f(z,t)$ within the $0\le r\le a$ region with third kind condition which assume to be zero at upper surface $z=h$ and lower surface $z=-h.$ Under this more realistic prescribed conditions, the unknown temperature on lower surface and quasi-static thermal deflection due to unknown temperature $g(z,t)$ are required to determine. The differential equation satisfying the deflection function as in [15,16] is given as

$$\nabla^4 w=\frac{-1}{(1-\nu)D}\nabla^2 M_T \tag{0.1}$$

Where, the operator ∇^2 is defined by

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} \tag{0.2}$$

M_T is the thermal moment of the plate defined as

$$M_T = \alpha E \int_{-h}^{h} zT(r,z,t)dz \tag{0.3}$$

and D is the flexural rigidity of the plate denoted as

$$D = \frac{Eh^3}{12(1-\nu^2)} \tag{0.4}$$

α, E and ν are the coefficients of the linear thermal expansion, the Young's modulus and the Poisson's ration of the plate material respectively.

Since the edge of the circular plate is fixed and clamped;

$$w = \frac{\partial w}{\partial r} = 0 \qquad at \;\; r = a \tag{0.5}$$

The temperature of the circular plate satisfying the heat conduction equation as in [15, 16] is as

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{k}\frac{\partial T}{\partial t} \qquad in \; 0 \le r \le a, -h \le z \le h \tag{0.6}$$

with initial condition

$$T(r,z,t) = 0 \qquad 0 \le r \le a, -h \le z \le h \tag{0.7}$$

the boundary condition's

$$\frac{\partial T}{\partial r} = g(z,t) \qquad (unknown)\, r = a, -h \le z \le h \tag{0.8}$$

$$\left[T(r,z,t) + k_1 \frac{\partial T(r,z,t)}{\partial z} \right]_{z=h} = 0 \tag{0.9}$$

$$\left[T(r,z,t) + k_2 \frac{\partial T(r,z,t)}{\partial z} \right]_{z=-h} = 0 \tag{0.10}$$

and interior condition

$$\left[T(r,z,t) + c \frac{\partial T(r,z,t)}{\partial r} \right]_{r=\xi} = f(z,t) \;\; (known) \tag{0.11}$$

where k_1 and k_2 are the radiation constants on the two plane surfaces, k is the thermal diffusivity of the material of the circular plate. The equations (0.1) to (0.11) constitute the mathematical formulation of the inverse transient thermoelastic deflection problem of circular plate.

III. ANALYSIS OF THE PROBLEM

3.1 Determination of Temperature Function

First applying finite Marchi-Fasulo transform as defined in [4] to the equations (0.6), (0.7), (0.11) and using (0.8), (0.9), then applying Laplace transform as defined in Sneddon [17] to the equations in Marchi-Fasulo transform domain and then using inversion of Laplace transform as defined in Sneddon [17] and Marchi-Fasulo transform as defined in [4] respectively, one obtain the expression for temperature function $T(r,z,t)$ as

$$T(r,z,t) = \frac{2k}{\xi} \sum_{n=1}^{\infty} \frac{P_n(z)}{\lambda_n} \sum_{m=1}^{\infty} \frac{\lambda_m J_0(\lambda_m r)}{J_1(\lambda_m \xi) + c\lambda_m J_1^{'}(\lambda_m \xi)} \int_0^t \overline{f}(n,t') e^{-\left(\lambda_m^2 + a_n^2\right)(t-t')} dt' \tag{0.12}$$

where m, n are positive integers, λ_m are the positive roots of the transcendental equation

$$J_0(\lambda_m a) = 0 \tag{0.13}$$

$$\lambda_n = \int_{-h}^{h} P_n^2(z)dz \;\; \text{and} \;\; \overline{f}(n,t) = \int_{-h}^{h} f(z,t)P_n(z)dz$$

$$P_n(z) = Q_n \cos(a_n z) - W_n \sin(a_n z)$$

$$Q_n = a_n\left(\alpha_1+\alpha_2\right)\cos(a_n h)+\left(\beta_1-\beta_2\right)\sin(a_n h)$$
$$W_n = \left(\beta_1+\beta_2\right)\cos(a_n h)+a_n\left(\alpha_2-\alpha_1\right)\sin(a_n h)$$

equation (0.12) is the desired solution of equation (0.6) with $\beta_1=\beta_2=1, \alpha_1=k_1$ and $\alpha_2=k_2$.

3.2 Determination of Unknown Temperature Function

Using (0.12)in (0.8) once obtain the unknown temperature function $g(z,t)$ as

$$g(z,t)=\frac{2k}{\xi}\sum_{n=1}^{\infty}\frac{P_n(z)}{\lambda_n}\sum_{m=1}^{\infty}\frac{\lambda_m^2 J_0^{'}(\lambda_m r)}{J_1(\lambda_m\xi)+c\lambda_m J_1^{'}(\lambda_m\xi)}\int_0^t \overline{f}\left(n,t'\right)e^{-\left(\lambda_m^2+a_n^2\right)(t-t')}dt' \tag{0.14}$$

equation (0.14) is the desired solution of equation (0.8) with $\beta_1=\beta_2=1, \alpha_1=k_1$ and $\alpha_2=k_2$.

3.3 Determination of Quasi-Static Thermal Deflection Function

Using (0.12) in equation (0.3) , we obtain

$$M_T=\frac{4\alpha Ek}{\xi}\sum_{n=1}^{\infty}\frac{W_n}{a_n\lambda_n}\left(h\cos(a_n h)-\frac{\sin(a_n h)}{a_n}\right)\sum_{m=1}^{\infty}\frac{\lambda_m J_0(\lambda_m r)}{J_1(\lambda_m\xi)+c\lambda_m J_1^{'}(\lambda_m\xi)}\int_0^t \overline{f}\left(n,t'\right)e^{-\left(\lambda_m^2+a_n^2\right)(t-t')}dt' \tag{0.15}$$

Assume the solution of (0.1) satisfying the (0.5) as

$$w(r,t)=\sum_{n=1}^{\infty}C_n(t)\left[J_0(\lambda_m r)-J_0(\lambda_m a)\right] \tag{0.16}$$

Using the (0.15), (0.16) and the result

$$\left[\frac{d^2}{dr^2}+\frac{1}{r}\frac{d}{dr}\right]J_0(\lambda_m r)=-\lambda_m^2 J_0(\lambda_m r),$$

in (0.1), once we obtain the expression for $C_n(t)$ as

$$\begin{aligned}C_n(t)=&\frac{4\alpha Ek}{(1-\nu)D\xi}\sum_{n=1}^{\infty}\frac{W_n}{a_n\lambda_n}\left(h\cos(a_n h)-\frac{\sin(a_n h)}{a_n}\right)\\&\times\sum_{m=1}^{\infty}\frac{1}{\lambda_m\left[J_1(\lambda_m\xi)+c\lambda_m J_1^{'}(\lambda_m\xi)\right]}\int_0^t \overline{f}\left(n,t'\right)e^{-\left(\lambda_m^2+a_n^2\right)(t-t')}dt'\end{aligned} \tag{0.17}$$

Substituting the equation (0.17) in the equation (0.16), once obtain the expression for thermal deflection function as

$$w(r,t)=\frac{2\alpha E}{D(1-\nu)}\sum_{n=1}^{\infty}\frac{\overline{f}(n)W_n\left[a_n h\cos(a_n h)-\sin(a_n h)\right]\left[I_0(a_n r)-I_0(a_n a)\right]}{\lambda_n a_n^4\left[I_0(a_n\xi)+ca_n I_0^{'}(a_n\xi)\right]} \tag{0.18}$$

IV. PARTICULAR CASE AND NUMERICAL OUTCOMES

For particular case of a circular plate.

Assume that

$$f(z,t)=\left(1-e^{-t}\right)\left(z-h\right)^2\left(z+h\right)^2\xi \tag{0.19}$$

Applying finite Marchi-Fasulo transform as define in [3] to the equations (0.19), one obtain,

$$\overline{f}(n,t)=4\left(k_1+k_2\right)\left(1-e^{-t}\right)\xi\left[\frac{a_n h\cos^2(a_n h)-\cos(a_n h)\sin(a_n h)}{a_n^2}\right] \tag{0.20}$$

Now using (0.20), we obtain the necessary integral bellow

$$\begin{aligned}\int_0^t \overline{f}\left(n,t'\right)e^{-\left(\lambda_m^2+a_n^2\right)(t-t')}dt'=&4\left(k_1+k_2\right)\left(1-e^{-t}\right)\xi\left[\frac{a_n h\cos^2(a_n h)-\cos(a_n h)\sin(a_n h)}{a_n^2}\right]\\&\times\left[\frac{1-e^{-\left(\lambda_m^2+a_n^2\right)t}}{\lambda_m^2+a_n^2}+\frac{e^{-t}-e^{-\left(\lambda_m^2+a_n^2\right)t}}{1-\lambda_m^2-a_n^2}\right]\end{aligned} \tag{0.21}$$

For numerical result we set $k_1 = k_2 = 1,\ a = 1m,\ \xi = 0.5m,\ c = 1, h = 0.1m$ in equations (0.12), (0.18) and once making use of integral in (0.21) and obtain the numerical results for temperature and thermal deflection using MathCAD 2007 and depicted graphically using Microsoft office Excel 2010 as in Fig.1- Fig.2

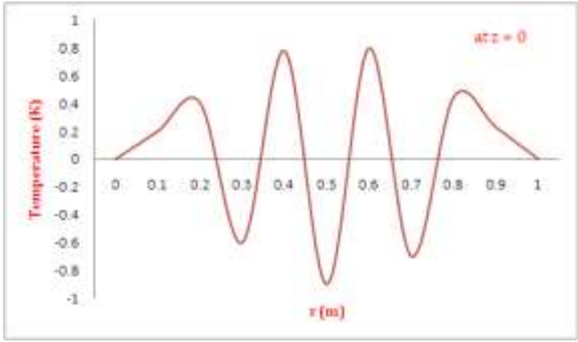

Fig.1 Temperature field

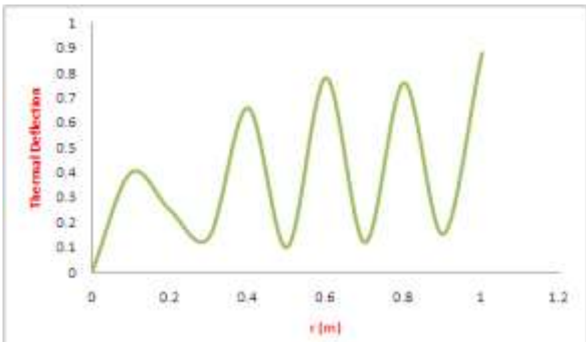

Fig. 2 Quasi-static thermal deflection

V. CONCLUSIONS

This article investigates the temperature, unknown temperature at lower surface and quasi-static thermal deflection due to unknown temperature. First, the mathematical model is constructed, and then the series solutions are obtained in terms of Bessel's function in the form of Marchi-Fasulo transform series by using integral transform methods. . The series solution converges provided we take sufficient number of terms in the series. Since the thickness of the plate is very small, the series solution given here will be definitely convergent.

As a particular case and numerical outcomes the functions and parameters are consider and the temperature, unknown temperature and quasi-static thermal deflection determine by using MathCAD software and illustrated graphically by using Microsoft office excel 2010. This type of inverse problems has the many applications in engineering such as main shaft of a lathe machine and aircraft structure. The results obtained here are mainly useful in the determination of state of strain in a circular plate.

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