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Fuzzy strongly c -semi irresolute and fuzzy strongly c -semi continuous functions in fuzzy topological spaces

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ABSTRACT: *In this paper we introduce the concept of fuzzy strongly C – semi interior and fuzzy strongly C – semi closure operators by using arbitrary complement function **C** of a fuzzy topological space where **C** :[0, 1]→ [0, 1] is a function and investigate some of their basic properties of a fuzzy topological space.*
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I. INTRODUCTION

The concept of complement function is used to define a fuzzy closed subset of a fuzzy topological space. That is a fuzzy subset λ is fuzzy closed if the standard complement 1–λ = λ′ is fuzzy open. Here the standard complement is obtained by using the function **C** : [0, 1]→ [0, 1] defined by **C** (x) = 1–x, for all x ∈[0, 1]. Several fuzzy topologists used this type of complement while extending the concepts in general topological spaces to fuzzy topological spaces. But there are other complements in the fuzzy literature [10]. This motivated the author to introduce the concepts of fuzzy **C** -closed sets and fuzzy **C** - semi closed sets in fuzzy topological spaces, where **C** : [0, 1]→ [0, 1] is an arbitrary complement function.

Bin [6] defined the notion of fuzzy strongly semi interior and fuzzy strongly semi closure operators in fuzzy topological spaces and studied their properties. In this paper, we generalize the concept of fuzzy strongly semi interior and fuzzy strongly semi closure operators by using the arbitrary complement function **C**, instead of the usual fuzzy complement function, fuzzy strongly C – semi interior instead of fuzzy strongly semi interior and by using fuzzy strongly C – semi closure instead of fuzzy strongly semi closure.

For the basic concepts and notations, one can refer Chang [7]. The concepts that are needed in this paper are discussed in the second section. The concepts of fuzzy strongly C – semi interior and strongly C – semi closure operators in fuzzy topological spaces and studied their properties in the third section. The fourth section is devoted to the applications of fuzzy strongly C – semi open and fuzzy strongly C – semi closed sets to fuzzy continuous functions and fuzzy irresolute functions.

II. PRELIMINARIES

Throughout this paper (X,τ) denotes a fuzzy topological space in the sense of Chang. Let C : [0, 1]→[0, 1] be a complement function. If λ is a fuzzy subset of (X,τ) then the complement C λ of a fuzzy subset λ is defined by C λ(x) = C (λ(x)) for all x∈X. A complement function C is said to satisfy

(i) the boundary condition if C (0) = 1 and C (1) = 0,
(ii) monotonic condition if x ≤ y ⇒ C (x) ≥ C (y), for all x, y ∈ [0, 1],
(iii) involutive condition if C (C (x)) = x, for all x ∈[0, 1].

The properties of fuzzy complement function C and C λ are given in George Klir **[8]** and Bageerathi et al **[2]**. The following lemma will be useful in sequel.

Definition 2.1 [Definition 3.1, [2]]

Let (X,τ) be a fuzzy topological space and **C** be a complement function. Then a fuzzy subset λ of X is fuzzy **C** -closed in (X,τ) if **C** λ is fuzzy open in (X,τ).

Lemma 2.2 [Proposition 3.2, [2]]

Let (X, τ) be a fuzzy topological space and **C** be a complement function that satisfies the involutive condition. Then a fuzzy subset λ of X is fuzzy open in (X,τ) if **C** λ is a fuzzy **C** -closed subset of (X,τ).

Definition 2.3 [Definition 4.1, [2]]

Let (X, τ) be a fuzzy topological space. Then for a fuzzy subset λ of X, the fuzzy **C** -closure of λ is defined as the intersection of all fuzzy **C** -closed sets μ containing λ. The fuzzy **C** -closure of λ is denoted by $Cl_{\mathbf{C}}\,\lambda$ that is equal to $\wedge\{\mu: \mu \geq \lambda, \mathbf{C}\,\mu \in \tau\}$.

Lemma 2.4 [Lemma 4.2, [2]]

If the complement function **C** satisfies the monotonic and involutive conditions, then for any fuzzy subset λ of X, (i) $\mathbf{C}\,(Int\,\lambda) = Cl_{\mathbf{C}}\,(\mathbf{C}\,\lambda)$ and (ii) $\mathbf{C}\,(Cl_{\mathbf{C}}\,\lambda) = Int\,(\mathbf{C}\,\lambda)$.

Definition 2.5 [Definition 2.15, 3]

A fuzzy topological space (X, τ) is **C** -product related to another fuzzy topological space (Y, σ) if for any fuzzy subset ν of X and ζ of Y, whenever $\mathbf{C}\,\lambda \ngeq \nu$ and $\mathbf{C}\,\mu \ngeq \zeta$ imply $\mathbf{C}\,\lambda \times 1 \vee 1 \times \mathbf{C}\,\mu \geq \nu \times \zeta$, where $\lambda \in \tau$ and $\mu \in \sigma$, there exist $\lambda_1 \in \tau$ and $\mu_1 \in \sigma$ such that $\mathbf{C}\,\lambda_1 \geq \nu$ or $\mathbf{C}\,\mu_1 \geq \zeta$ and $\mathbf{C}\,\lambda_1 \times 1 \vee 1 \times \mathbf{C}\,\mu_1 = \mathbf{C}\,\lambda \times 1 \vee 1 \times \mathbf{C}\,\mu$.

Definition 2.6 [Definition 3.1, [7 & 8]]

A function f: (X, τ) $\rightarrow$ (Y, σ) is called fuzzy **C** -semi continuous if $f^{-1}(\mu)$ is a fuzzy **C** -semi open set of X for each fuzzy open subset μ of Y.

Lemma 2.7 [1]

Let $\{\lambda_\alpha\}$ be the family of fuzzy subsets of a fuzzy topological space X. Then
$\vee\, Cl_{\mathbf{C}}\,(\lambda_\alpha) \leq Cl_{\mathbf{C}}\,(\vee\lambda_\alpha)$.

Lemma 2.8 [Lemma 5.1, [2]]

Suppose f is a function from X to Y. Then $f^{-1}(\mathbf{C}\,\mu)=\mathbf{C}\,(f^{-1}(\mu))$for any fuzzy subset μ of Y.

III. FUZZY STRONGLY C – SEMI INTERIOR AND FUZZY STRONGLY C – SEMI CLOSURE

In this section, we define the concept of fuzzy strongly C – semi interior and fuzzy strongly C – semi closure and investigate some of their basic properties.

Definition 3.1

Let (X,τ) be a fuzzy topological space and **C** be a complement function. Then for a fuzzy subset λ of X, the fuzzy strongly C – semi interior of λ (briefly $SSInt_{\mathbf{C}}\,\lambda$), is the union of all fuzzy strongly C – semi open sets of X contained in λ.

That is, $SSInt_{\mathbf{C}}\,(\lambda) = \vee\,\{\mu: \mu \leq \lambda, \mu$ is fuzzy strongly C – semi open$\}$.

Proposition 3.2

Let (X, τ) be a fuzzy topological space and let **C** be a complement function that satisfies the monotonic and involvtive conditions. Then for any fuzzy subsets λ and μ of a fuzzy topological space X, we have

(i) $SSInt_{\mathbf{C}}\,\lambda \leq \lambda$,

(ii) λ is fuzzy strongly C – semi open $\Leftrightarrow SSInt_{\mathbf{C}}\,\lambda = \lambda$,

(iii) $SSInt_{\mathbf{C}}\,(SSInt_{\mathbf{C}}\,\lambda) = SSInt_{\mathbf{C}}\,\lambda$,

(iv) If $\lambda \leq \mu$ then $SSInt_{\mathbf{C}}\,\lambda \leq SSInt_{\mathbf{C}}\,\mu$.

Proof.

(i) follows from Definition 3.1.

Let λ be fuzzy strongly C – semi open. Since $\lambda \leq \lambda$, by using Definition 3.1, $\lambda \leq SSInt_{\mathbf{C}}\,\lambda$. By using (i), we get $SSInt_{\mathbf{C}}\,\lambda = \lambda$. Conversely we assume that $SSInt_{\mathbf{C}}\,\lambda = \lambda$. By using Definition 3.1, λ is fuzzy strongly C – semi open. Thus (ii) is proved.

By using (ii), we get $SSInt_{\mathbf{C}}\,(SSInt_{\mathbf{C}}\,\lambda) = SSInt_{\mathbf{C}}\,\lambda$. This proves (iii).

Since $\lambda \leq \mu$, by using (i), $SSInt_{\mathbf{C}}\,\lambda \leq \lambda \leq \mu$. By using Definition 3.1, $\mu \leq SSInt_{\mathbf{C}}\,\mu$. This implies that $SSInt_{\mathbf{C}}\,\lambda \leq SSInt_{\mathbf{C}}\,\mu$. This proves (iv).

Proposition 3.3

Let (X, τ) be a fuzzy topological space and let **C** be a complement function that satisfies the monotonic and involutive conditions. Then for any two fuzzy subsets λ and μ of a fuzzy topological space, we have (i) $SSInt_{\mathbf{C}}\,(\lambda\vee\mu) \geq SSInt_{\mathbf{C}}\,\lambda \vee SSInt_{\mathbf{C}}\,\mu$ and (ii) $SSInt_{\mathbf{C}}\,(\lambda\wedge\mu) \leq SSInt_{\mathbf{C}}\,\lambda \wedge SSInt_{\mathbf{C}}\,\mu$.

Proof.

Since $\lambda \leq \lambda \vee \mu$ and $\mu \leq \lambda\vee\mu$, by using Proposition 3.2(iv), we get $SSInt_C\lambda \leq SSInt_C(\lambda\vee\mu)$ and $SSInt_C\,\mu \leq SSInt_C(\lambda\vee\mu)$. This implies that $SSInt_C\,\lambda\vee SSInt_C\,\mu \leq SSInt_C(\lambda\vee\mu)$.

Since $\lambda \wedge \mu \leq \lambda$ and $\lambda \wedge \mu \leq \mu$, by using Proposition 3.2(iv), we get $SSInt_C(\lambda\wedge\mu) \leq SSInt_C\,\lambda$ and $SSInt_C(\lambda\wedge\mu) \leq SSInt_C\,\mu$. This implies that $SSInt_C(\lambda\wedge\mu) \leq SSInt_C\,\lambda \wedge SSInt_C\,\mu$.

Definition 3.4

Let (X,τ) be a fuzzy topological space. Then for a fuzzy subset λ of X, the fuzzy strongly C – semi closure of λ (briefly $SSCl_C\,\lambda$), is the intersection of all fuzzy strongly C – semi closed sets containing λ. That is $SSCl_C\,\lambda = \wedge\{\mu: \mu \geq \lambda, \mu$ is fuzzy strongly C – semi closed$\}$.

The concepts of "fuzzy strongly C – semi closure" and "fuzzy strongly semi closure" are identical if **C** is the standard complement function.

Proposition 3.5

If the complement functions **C** satisfies the monotonic and involutive conditions. Then for any fuzzy subset λ of X,(i)$\mathbf{C}(SSInt_C\,\lambda)=SSCl_C(\mathbf{C}\,\lambda)$ and (ii)$\mathbf{C}(SSCl_C\,\lambda)=SSInt_C(\mathbf{C}\,\lambda)$, where $SSInt_C\,\lambda$ is the union of all fuzzy strongly C – semi open sets contained in λ.

Proof.

By using Definition 3.1, $SSInt_C\,\lambda = \vee\{\mu: \mu \leq \lambda, \mu$ is fuzzy strongly C – semi open$\}$. Taking complement on both sides, we get $\mathbf{C}(SSInt_C(\lambda)(x)) = \mathbf{C}(\sup\{\mu(x): \mu(x) \leq \lambda(x), \mu$ is fuzzy strongly C – semi open$\})$. Since **C** satisfies the monotonic and involutive conditions, by using Proposition 2.9 in [2], $\mathbf{C}(SSInt_C(\lambda)(x)) = \inf\{\mathbf{C}(\mu(x)): \mu(x) \leq \lambda(x), \mu$ is fuzzy strongly C – semi open$\}$. This implies that $\mathbf{C}(SSInt_C(\lambda)(x)) = \inf\{\mathbf{C}\,\mu(x)): \mathbf{C}\,\mu(x) \geq \mathbf{C}\,\lambda(x), \mu$ is fuzzy strongly C – semi open$\}$. By using Proposition 5.3 in [5], $\mathbf{C}\,\mu$ is fuzzy strongly C – semi closed, by replacing $\mathbf{C}\,\mu$ by η, we see that $\mathbf{C}(SSInt_C(\lambda)(x)) =\inf\{\eta(x): \eta(x)\geq \mathbf{C}\lambda(x), \mathbf{C}\,\eta$ is fuzzy strongly C – semi open$\}$. By using Definition 3.4, $\mathbf{C}(SSInt_C(\lambda)(x)) = SSCl_C(\mathbf{C}\,\lambda)(x)$. This proves that $\mathbf{C}(SSInt_C\,\lambda) = SSCl_C(\mathbf{C}\,\lambda)$.

By using Definition 3.4, $SSCl_C\,\lambda = \wedge\{\mu: \lambda\leq\mu, \mu$ is fuzzy strongly C – semi closed$\}$. Taking complement on both sides, we get $\mathbf{C}(SSCl_C\,\lambda(x)) = \mathbf{C}(\inf\{\mu(x): \mu(x) \geq \lambda(x), \mu$ is fuzzy strongly C – semi closed$\})$. Since **C** satisfies the monotonic and involutive conditions, by using Proposition 2.9 in [2], $\mathbf{C}(SSCl_C\,\lambda(x)) = \sup\{\mathbf{C}(\mu(x)): \mu(x) \geq \lambda(x), \mu$ is fuzzy strongly C – semi closed$\}$. This implies that $\mathbf{C}(SSCl_C\,\lambda(x)) = \sup\{\mathbf{C}\,\mu(x): \mathbf{C}\,\mu(x)) \leq \mathbf{C}\,\lambda(x), \mu$ is fuzzy strongly C – semi closed$\}$. By using Proposition 5.3 in [5], $\mathbf{C}\,\mu$ is fuzzy strongly C – semi open, by replacing $\mathbf{C}\,\mu$ by η, we see that $\mathbf{C}(SSCl_C(x)) = \sup\{\eta(x): \eta(x)\leq\mathbf{C}\,\lambda(x), \eta$ is fuzzy strongly C – semi open$\}$. By using Definition 3.1, $(SSCl_C\,\lambda(x)) = SSInt_C(\mathbf{C}\,\lambda)(X)$. This proves $\mathbf{C}(SSCl_C(\lambda)) = SSInt_C(\mathbf{C}\,\lambda)$.

Proposition 3.6

Let (X, τ) be a fuzzy topological space and let **C** be a complement function that satisfies the monotonic and involvtive conditions. Then for the fuzzy subsets λ and μ of a fuzzy topological space X, we have

(i) $\lambda \leq SSCl_C\,\lambda$,

(ii) λ is fuzzy strongly C – semi closed $\Leftrightarrow SSCl_C\,\lambda = \lambda$,

(iii) $SSCl_C(SSCl_C\,\lambda) = SSCl_C\,\lambda$,

(iv) If $\lambda \leq \mu$ then $SSCl_C\,\lambda \leq SSCl_C\,\mu$.

Proof.

The proof for (i) follows from $SSCl_C\,\lambda= \inf\{\mu: \mu \geq \lambda, \mu$ is fuzzy strongly C – semi closed$\}$.

Let λ be fuzzy strongly C – semi closed. Since **C** satisfies the monotonic and involvtive conditions. Then by using Proposition 5.3 in [5], $\mathbf{C}\,\lambda$ is fuzzy strongly C – semi open. By using Proposition 3.2(ii) in [5], $SSInt_C(\mathbf{C}\,\lambda) = \mathbf{C}\,\lambda$. By using Proposition 3.5, $\mathbf{C}(SSCl_C\,\lambda) = \mathbf{C}\,\lambda$. Taking complement on both sides, we get $\mathbf{C}(\mathbf{C}(SSCl_C\,\lambda)) = \mathbf{C}(\mathbf{C}\,\lambda)$. Since the complement function **C** satisfies the involutive condition, $SSCl_C\,\lambda = \lambda$.

Conversely, we assume that $SSCl_C\,\lambda=\lambda$. Taking complement on both sides, we get $\mathbf{C}(SSCl_C\,\lambda) =\mathbf{C}\lambda$. By using Proposition 3.5, $SSInt_C\,\mathbf{C}\,\lambda =\mathbf{C}\,\lambda$. By using Proposition 3.2(ii) in [5], $\mathbf{C}\,\lambda$ is fuzzy strongly C – semi open. Again by using Proposition 5.3 in [5], λ is fuzzy strongly C – semi closed. Thus (ii) proved.

By using Proposition 3.5, $\mathbf{C}(SSCl_C\,\lambda) = SSInt_C(\mathbf{C}\,\lambda)$. This implies that $\mathbf{C}(SSCl_C\,\lambda)$ is fuzzy strongly C – semi open. By using Proposition 5.3 in [5], $SSCl_C\,\lambda$ is fuzzy strongly C – semi closed. By applying (ii), we have $SSCl_C(SSCl_C\,\lambda) = SSCl_C\,\lambda$. This proves (iii).

Suppose $\lambda \leq \mu$. Since **C** satisfies the monotonic condition, $\mathbf{C}\,\lambda \geq \mathbf{C}\,\mu$, that implies $SSInt_C\,\mathbf{C}\,\lambda \geq SSInt_C\,\mathbf{C}\mu$.Taking complement on both sides, $\mathbf{C}(SSInt_C\,\mathbf{C}\,\lambda)\leq\mathbf{C}(SSInt_C\,\mathbf{C}\,\mu)$. Then by using Proposition 3.5, $SSCl_C\,\lambda \leq SSCl_C\,\mu$. This proves (iv).

Proposition 3.7

Let (X, τ) be a fuzzy topological space and **C** be a complement function that satisfies the monotonic and involutive conditions. Then for any two fuzzy subsets λ and μ of a fuzzy topological space, we have (i) $SSCl_{\mathbf{C}}\,(\lambda\vee\mu) = SSCl_{\mathbf{C}}\,\lambda \vee SSCl_{\mathbf{C}}\,\mu$ and
(ii) $SSCl_{\mathbf{C}}\,(\lambda\wedge\mu) \leq SSCl_{\mathbf{C}}\,\lambda \wedge SSCl_{\mathbf{C}}\,\mu$.

Proof.

Since **C** satisfies the involutive condition, $SSCl_{\mathbf{C}}\,(\lambda\vee\mu) = SSCl_{\mathbf{C}}\,(\mathbf{C}\,(\mathbf{C}\,(\lambda\vee\mu)))$. Since **C** satisfies the monotonic and involutive conditions, by using Proposition 3.5, $SSCl_{\mathbf{C}}\,(\lambda\vee\mu) = \mathbf{C}\,(SSInt_{\mathbf{C}}\,(\mathbf{C}\,(\lambda\vee\mu)))$. Using Lemma 2.10 in [2], $SSCl_{\mathbf{C}}\,(\lambda\vee\mu) = \mathbf{C}\,(SSInt_{\mathbf{C}}\,(\mathbf{C}\,\lambda\wedge\mathbf{C}\mu))$. Again by Proposition 3.3, $SSCl_{\mathbf{C}}(\lambda\vee\mu)\leq\mathbf{C}\,((SSInt_{\mathbf{C}}\mathbf{C}\,\lambda)\wedge(SSInt_{\mathbf{C}}\mathbf{C}\mu))=\mathbf{C}\,(SSInt_{\mathbf{C}}\mathbf{C}\,\lambda)\vee\mathbf{C}\,(SSInt_{\mathbf{C}}\mathbf{C}\mu)$. By using Proposition 3.5, $SSCl_{\mathbf{C}}\,(\lambda\vee\mu) \leq SSCl_{\mathbf{C}}\,\lambda\vee SSCl_{\mathbf{C}}\,\mu$. Also $SSCl_{\mathbf{C}}\,(\lambda) \leq SSCl_{\mathbf{C}}\,(\lambda\vee\mu)$ and $SSCl_{\mathbf{C}}\,(\mu) \leq SSCl_{\mathbf{C}}\,(\lambda\vee\mu)$ that implies $SSCl_{\mathbf{C}}\,(\lambda)\vee SSCl_{\mathbf{C}}\,(\mu) \leq SSCl_{\mathbf{C}}\,(\lambda\vee\mu)$. It follows that $SSCl_{\mathbf{C}}\,(\lambda\vee\mu)=SSCl_{\mathbf{C}}\,\lambda\vee SSCl_{\mathbf{C}}\,\mu$.

Since $SSCl_{\mathbf{C}}\,(\lambda\wedge\mu) \leq SSCl_{\mathbf{C}}\,\lambda$ and $SSCl_{\mathbf{C}}\,(\lambda\wedge\mu) \leq SSCl_{\mathbf{C}}\,\mu$, it follows that $SSCl_{\mathbf{C}}\,(\lambda\wedge\mu) \leq SSCl_{\mathbf{C}}\,\lambda \wedge SSCl_{\mathbf{C}}\,\mu$.

Proposition 3.8

Let (X, τ) be a fuzzy topological space and **C** be a complement function that satisfies the monotonic and involutive conditions. Then for any family $\{\lambda_\alpha\}$ of fuzzy subsets of a fuzzy topological space, we have (i) $\vee(SSCl_{\mathbf{C}}\,\lambda_\alpha) \leq SSCl_{\mathbf{C}}\,(\vee\lambda_\alpha)$ and (ii) $SSCl_{\mathbf{C}}\,(\wedge\lambda_\alpha) \leq \wedge(SSCl_{\mathbf{C}}\,\lambda_\alpha)$

Proof.

For every β, $\lambda_\beta \leq \vee\lambda_\alpha \leq SSCl_{\mathbf{C}}\,(\vee\lambda_\alpha)$. By using Proposition 3.6(iv), $SSCl_{\mathbf{C}}\,\lambda_\beta \leq SSCl_{\mathbf{C}}\,(\vee\lambda_\alpha)$ for every β. This implies that $\vee SSCl_{\mathbf{C}}\,\lambda_\beta \leq SSCl_{\mathbf{C}}\,(\vee\lambda_\alpha)$. This proves (i). Now $\wedge\lambda_\alpha \leq \lambda_\beta$ for every β. Again using Proposition 3.6(iv), we get $SSCl_{\mathbf{C}}\,(\wedge\lambda_\alpha) \leq SSCl_{\mathbf{C}}\,\lambda_\beta$. This implies that $SSCl_{\mathbf{C}}\,(\wedge\lambda_\alpha) \leq \wedge SSCl_{\mathbf{C}}\,\lambda_\alpha$. This proves (ii).

Proposition 3.9

Let (X, τ) be a fuzzy topological space and C be a complement function that satisfies monotonic and involutive properties. Let λ be a fuzzy subset of a fuzzy topological space X.
Then (i) $Cl_C\,(SSCl_C\,\lambda) = Cl_C\,\lambda$ and (ii) $Cl_C\,Int\,Cl_C\,\lambda \leq SSCl_C\,\lambda$

Proof

By using Proposition 3.6, we have $\lambda \leq SSCl_C\,\lambda \leq Cl_C\,\lambda$. Since **C** satisfies the monotonic and involutive conditions, by using Lemma 2.6 in [2], $Cl_C\,\lambda\leq Cl_C\,(SSCl_C\,\lambda) \leq Cl_C\,\lambda$. Then $Cl_C\,(SSCl_C\,\lambda) = Cl_C\,\lambda$. This proves (i).
Since $SSCl_C\,\lambda$ is fuzzy strongly C – semi closed, by using Proposition 5.2 in [5], $Cl_C\;Int\,Cl_C\,SSCl_C\,\lambda \leq SSCl_C\,\lambda$. By using (i), $Cl_C\,(SSCl_C\,\lambda)= Cl_C\,\lambda$. This implies that $Cl_C\,Int\,Cl_C\,\lambda\leq SSCl_C\,\lambda$.

IV. APPLICATIONS

This section is devoted to the application of fuzzy strongly C – semi open and fuzzy strongly C – semi closed sets to fuzzy continuous and fuzzy irresolute functions.

Definition 4.1

Let f: (X, τ) → (Y, σ) be a function. Then f is said to be
(i) fuzzy strongly C – semi irresolute if $f^{-1}(\delta)$ is fuzzy strongly C – semi open in X for each fuzzy strongly C – semi open set δ in Y.
(ii) fuzzy strongly C – semi continuous if $f^{-1}(\delta)$ is fuzzy strongly C – semi open in X for each fuzzy open set δ in Y.

Remark 4.2

It is clear that every fuzzy strongly C – semi irresolute mapping is fuzzy strongly C – semi continuous and every fuzzy strongly C – semi continuous mapping is fuzzy **C** -semi continuous. But the converses need not be as shown by the following example.

Example 4.3

Consider the identity map f: (X, τ) → (Y, σ). Let X = Y= {a, b, c}, τ = {0, 1, λ, μ, λ ∧ μ, λ ∨ μ}, where λ = $\{a_{.8}, b_{.4}, c_{.2}\}$ and μ = $\{a_{.2}, b_{.4}, c_{.6}\}$. Now σ = {0, 1, η} , where η = $\{a_{.2}, b_{.5}, c_{.7}\}$. Let **C** (x) $= \dfrac{2x}{1+x}$, 0 ≤ x ≤ 1, be the complement function. Here $f^{-1}(\eta)$ is fuzzy **C** -semi open but not fuzzy strongly C – semi open. Hence f is fuzzy **C** -semi continuous but not fuzzy strongly C – semi continuous.

Example 4.4

Consider the identity map f : (X, τ) → (Y, σ), where X =Y = {a, b, c}, τ = **{**0, {$a_{.8}$, $b_{.7}$, $c_{.3}$}, {$a_{.5}$, $b_{.2}$, $c_{.6}$}, {$a_{.8}$, $b_{.7}$, $c_{.6}$}, {$a_{.5}$, $b_{.2}$, $c_{.3}$}, 1**}** and σ = **{** 0, η = {$a_{.5}$, $b_{.3}$, $c_{.4}$}, 1**}**. The family of all fuzzy **C** -closed sets **C** (τ) = **{**0, {$a_{.6}$, $b_{.7}$, $c_{.95}$}, {$a_{.866}$, $b_{.98}$, $c_{.8}$}, {$a_{.6}$, $b_{.7}$, $c_{.8}$}, {$a_{.866}$, $b_{.98}$, $c_{.95}$}, 1**}**. Here f^{-1}(η) is fuzzy strongly C – semi open in X, where η is fuzzy open in Y. But η is not fuzzy strongly C – semi open in Y. That shows f is fuzzy strongly C – semi continuous but not fuzzy strongly C – semi irresolute.

Theorem 4.5

Let f : (X, τ) → (Y, σ) and **C** be a complement function that satisfies the involutive condition. Then the following conditions are equivalent.

(a) f is fuzzy strongly C – semi irresolute.

(b) f^{-1}(γ) is fuzzy strongly C – semi closed in X for each fuzzy strongly C – semi closed set γ in y.

(c) for each fuzzy set λ in X, f(*SS Cl*$_C$ λ) ≤ *SSCl*$_C$ (f(λ))

(d) for each fuzzy set μ in γ, *SSCl*$_C$ (f^{-1}(μ)) ≤ f^{-1}(*SSCl*$_C$ (μ))

Proof

(a) ⇔ (b)Let γ be a fuzzy strongly C – semi closed in Y. Since the complement function **C** satisfies the monotonic and involutive conditions, by using Proposition 5.3 in [5], C γ is fuzzy strongly C – semi open in Y. By using Definition 4.1, f^{-1}(C γ) is fuzzy strongly C – semi open in X. This implies that C (f^{-1}(γ)) is fuzzy strongly C – semi open in X. Again by using Proposition 5.3 in [5], f^{-1}(γ) is fuzzy strongly C – semi closed in X. That proves (a) ⇒ (b).

Conversely, let δ be a fuzzy strongly C – semi open in Y. Since the complement function **C** satisfies the monotonic and involutive conditions, by using Proposition 5.3 in [5], C δ is fuzzy strongly C – semi closed in Y. By using our assumption and by using Lemma 2.7 in [2], f^{-1}(C δ) = C (f^{-1}(δ)) is fuzzy strongly C – semi closed in X. Again by using Proposition 5.3 in [5], f^{-1}(δ) is fuzzy strongly C – semi open. By using **Definition 4.1**, (b) ⇒ (a).

(b) ⇒ (c). Let λ be a fuzzy set in X. Then *SSCl*$_C$ (f(λ)) is fuzzy strongly C – semi closed in Y. By using (ii), f^{-1}(*SSCl*$_C$ f(λ)) is fuzzy strongly C – semi closed in X. Since the complement function **C** satisfies the monotonic and involutive conditions, by using Proposition 3.5, *SSCl*$_C$ λ ≤ *SSCl*$_C$ (f^{-1}(*SSCl*$_C$ f(λ)) = f^{-1}(*SSCl*$_C$ (f(λ)). This implies that f(*SSCl*$_C$ λ) ≤ f (f^{-1}(*SSCl*$_C$ (f(λ)) ≤ *SSCl*$_C$ (f(λ)).

(c) ⇒ (d) Let μ be a fuzzy set in Y, By using (c), f (*SSCl*$_C$ (f^{-1}(μ) ≤ *SSCl*$_C$ (f(f^{-1}(μ)) ≤ *SSCl*$_C$ μ. This implies f^{-1}(f(*SSCl*$_C$ (f^{-1}(μ)) ≤ f^{-1}(*SSCl*$_C$ μ). From the above, we get *SSCl*$_C$ f^{-1}(μ) ≤ f^{-1} (*SSCl*$_C$ μ).

(d) ⇒ (b) Let γ be a fuzzy strongly C – semi closed in Y. Then by using our assumption, *SSCl*$_C$ f^{-1}(γ) ≤ f^{-1}(*SSCl*$_C$ γ) = f^{-1}(γ) this gives *SSCl*$_C$ f^{-1}(γ) = f^{-1}(γ). Therefore f^{-1}(γ) is fuzzy strongly C – semi closed in X.

Theorem 4.6

Let f : X →Y be a mapping. Let C be a complement function that satisfies monotonic and involutive conditions. Then the following are equivalent.

(a) f is fuzzy strongly C – semi continuous

(b) f^{-1}(γ) is fuzzy strongly C – semi closed set in X for each fuzzy C - closed set γ in Y.

(c) f(*SSCl*$_C$ λ) ≤ *Cl*$_C$ (f(λ) for each fuzzy set λ in X

(d) *SSCl*$_C$ (f^{-1}(μ)) ≤ f^{-1}(*Cl*$_C$ (μ) for each fuzzy set μ in Y.

Proof

Let γ be a fuzzy C -closed set in Y. By using Definition 2.1, C γ is fuzzy open. By using Definition 4.1, f^{-1}(C γ) is fuzzy strongly C – semi open. Since f^{-1}(C γ) = C (f^{-1}(γ)), that implies C (f^{-1}(γ)) is fuzzy strongly C – semi open. Since the complement function **C** satisfies the monotonic and involutive conditions, by using Proposition 5.3 in [5], f^{-1}(γ) is fuzzy strongly C – semi closed. Thus (a) ⇒ (b).

Conversely, Let δ be a fuzzy open set in Y. Since the complement function **C** satisfies the monotonic and involutive conditions, by using Lemma 2.2, then C δ is fuzzy C -closed. By using (b), f^{-1}(C δ) is fuzzy strongly C – semi closed in X. This follows that C (f^{-1}(δ)) is fuzzy strongly C – semi closed in X. by using Proposition 5.3 in [5], f^{-1}(δ) is fuzzy strongly C – semi open in X. Hence (b) ⇒ (a).

(b) ⇒ (c) Let λ be a fuzzy set in X. Then *Cl*$_C$ (f(λ) is fuzzy C -closed in Y. By using (b), f^{-1}(*Cl*$_C$ (f(λ)) is fuzzy strongly C – semi closed in X. Since λ ≤ f^{-1}(f(λ) ≤ f^{-1}(*SSCl*$_C$ (f(λ))), it follows that *SSCl*$_C$ λ ≤ *SSCl*$_C$ (f^{-1}(*Cl*$_C$ f(λ))) = f^{-1}(*Cl*$_C$ (f(λ)). Thus f(*SSCl*$_C$ λ) ≤ f (f^{-1} *Cl*$_C$ f(λ))) ≤ *Cl*$_C$ (f(λ)).

(c) ⇒ (d) Let μ be a fuzzy set in Y. By using (c) , f[*SSCl*$_C$ (f^{-1}(μ))] ≤ *Cl*$_C$ (f(f^{-1}(μ))) ≤ *Cl*$_C$ μ. This gives f^{-1}(f[*SSCl*$_C$ (f^{-1}(μ))] ≤ f^{-1} (*Cl*$_C$ μ),it follows that *SSCl*$_C$ (f^{-1}(μ) ≤ f^{-1}(*SSCl*$_C$ μ).

(d) ⇒ (b) Let γ be a fuzzy C -closed in Y. By using our assumption, *SSCl*$_C$ f^{-1}(γ) ≤ f^{-1}(γ) and also f^{-1}(γ) ≤ *SSCl*$_C$ (f^{-1}(γ). That implies *SSCl*$_C$ f^{-1}(γ) = f^{-1}(γ), this shows that f^{-1}(γ) is fuzzy strongly C – semi closed in X.

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