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Comparison of Various Methods of Analysis of Grid Floor Frame

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ABSTRACT: *An assembly of intersecting beams placed at regular interval and interconnected to a slab of nominal thickness is known as Grid floor or Waffle floor. These slabs are used to cover a large column free area and therefore are good choice for public assembly halls. The structure is monolithic in nature and has more stiffness. It gives pleasing appearance. The maintenance cost of these floors is less. However, construction of the grid slabs is cost prohibitive. By investigating various parameters the cost effective solution can be found for the grid slabs, for which proper method of analysis need to be used. There are various approaches available for analyzing the grid slab system. In present study some of these approaches are studied and compared with each other. The comparison is done on the basis of flexural parameters such as bending moments and shear forces obtained from various methods. For carrying out study, halls having constant width 10.00m and varying ratio of hall dimensions (L/B) from 1 to 1.5 are considered.*

KEYWORDS: *Grid Slabs, Orthotropic Plate, Plate theory, Rankine-Grashoff method, Stiffness method*

I. INTRODUCTION

The slab which is resting on the beams running in two directions is known as grid slab. In these types of slab, a mesh or grid of beams running in both the directions is the main structure, and the slab is of nominal thickness. It is used to cover a large area without obstruction of internal columns. They are generally employed for architectural reasons for large rooms such as auditoriums, vestibules, theatre halls, show rooms of shops where column free space is often the main requirement.

1.1 Typical Plan of Grid System and Notations Used

Fig. No.1 shows a plan of typical grid system configuration for present study.

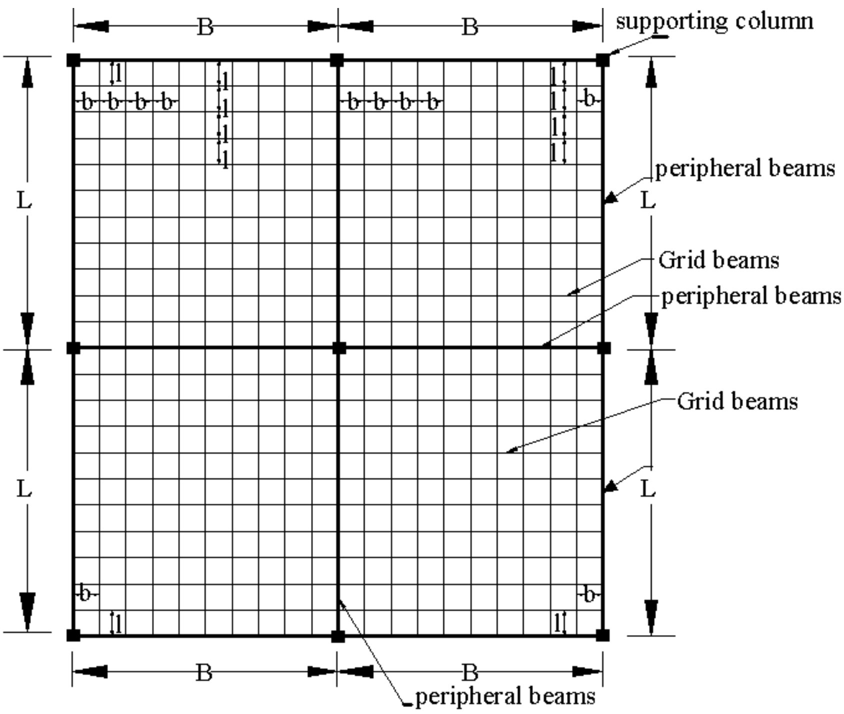

Figure No. 1 plan of grid system

Notations used

1) L = Length of Hall (Longer side of hall)
2) B = Width of Hall (Shorter side of hall)
3) l = Spacing of grid beams in the direction of the length of the hall
4) b = Spacing of grid beams in the direction of the width of the hall
5) Mx= Bending moment in the beams running in x-direction
6) My= Bending moment in the beams running in y-direction
7) Qx= Shear force in the beams running in x-direction
8) Qy= Shear force in the beams running in y-direction

1.2 Dimensions Considered

For the comparison purpose, the width of the hall is kept constant as 10.00 m and length is increased by an interval of 1.00 m, so that L/B ratio varies from 1 to 1.5 at the interval of 0.1.

For all hall sizes the thickness of grid slab is assumed as 100 mm. The size of the beams (0.23 m x 0.60 m) is kept same during entire study.

II. METHODS OF ANALYSIS

Grid is highly redundant structural system and therefore statically indeterminate. Various approaches available for the analysis of grid floor frame, are as listed below.

1) Analysis of grid by Rankine – Grashoff method.
2) Analysis by plate theory.
3) Stiffness method.

2.1 Rankine - Grashoff Method

This is an approximate method. It is based on equating deflections in either direction at the junctions of ribs. This method is suitable for small span grids with the spacing of ribs not exceeding 1.50 m. In this method the slab is considered as simply supported on edges. (Refer Figure. No.2) This method computes moments and shear force per unit width of slab strip.

2.2 Plate Analogy Method

This is a rigorous method of analysis. This is based on Timoshenko's analysis of orthotropic plate theory considering plane stress analysis. As in Rankine-Grashoff method, in this method also the analysis is done by considering the grid simply supported on edges (Refer Fig. No.2). Bending & torsion moments and shears are obtained per unit width of slab strip.

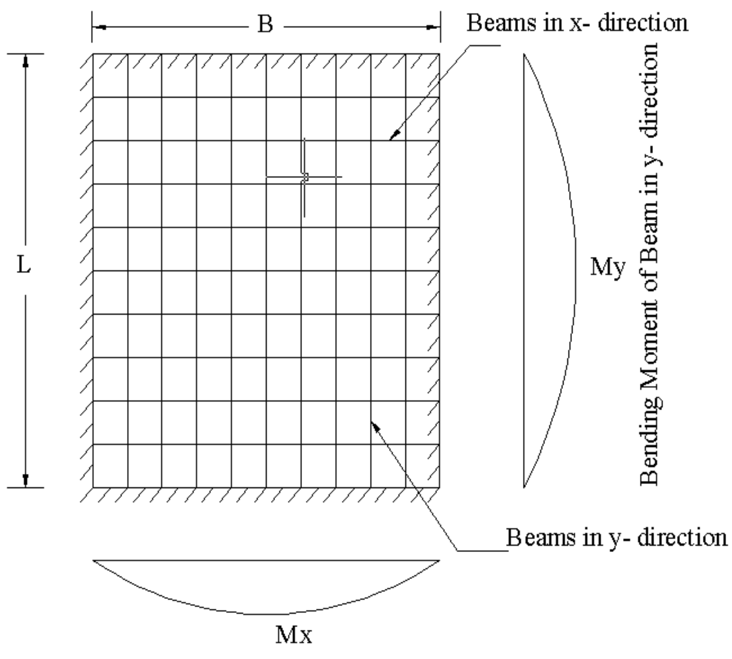

Figure No. 2 Typical grid considered in Rankine–Grashoff and Plate theory (Below grid)

2.3 Stiffness Method

This method is based on matrix formulation of the stiffness of the structure and gives closed form solution. By using this method the analysis can be done by considering rigid supports as well. Various application software's are available to carry out analysis by this method. In the present work while analyzing grid floor frame by stiffness method, the simple supports are considered at closer distance so as to simulate the support conditions similar to Rankine-Grashoff method and Plate theory. (Refer Fig. No.3).

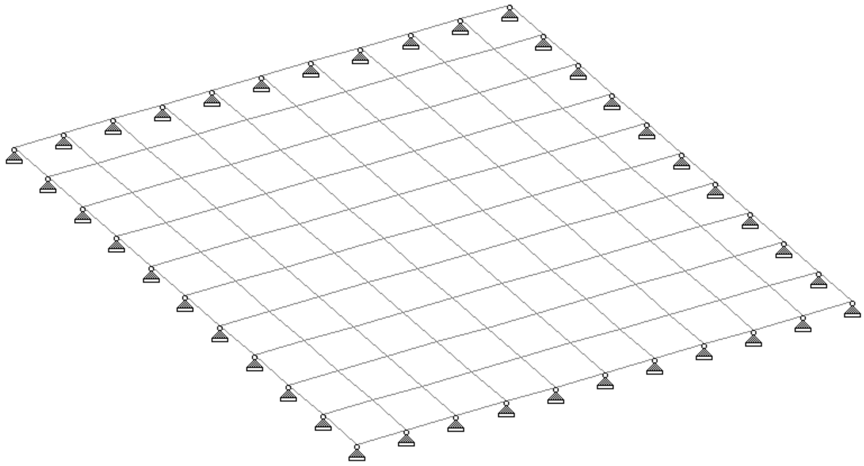

Figure No. 3 Typical Grid floor considered in stiffness method

III. THEORETICAL FORMULATION

3.1 Typical Geometrical Data for L/B=1.0

Width of Hall (a) = 10.00 m, Length of Hall (b) = 10.00 m
Spacing of grids in x-direction (a1) and y-direction (b1) = 1.00 m
Thickness of slab (Df) = 0.1 m
Width of ribs (bw) = 0.23 m

3.2 Load Calculations

The loads on floor slab are calculated on the basis of
Density of reinforced concrete and floor finish considered as 25kN/m2.
Live load intensity=5 kN/m2
Total dead load of floor area (10.00 m x 10.00 m) =730.180 kN
Total live load on floor area (10.00 m x 10.00 m) =500 kN

$$1.5(DL + LL) = 1.5(730.180 + 500) = 1845.27\ kN$$

$$\text{load per unit area } (q) = \frac{1845.27}{10 \times 10} = 18.4527\ kN/m^2 \tag{1}$$

3.3 Rankine-Grashoff Method

Using load intensity given in (1), the design bending moments and shears are calculated as follows:

$$\text{load intensity in x direction } (q_1) = q \times \frac{b^4}{a^4 + b^4} = 9.2263\ kN/m \tag{2}$$

$$\text{load intensity in y direction } (q_2) = q \times \frac{a^4}{a^4 + b^4} = 9.2263\ kN/m \tag{3}$$

3.3.1 Moment calculations

$$\text{Moment in beams running in x direction } (M_x) = \frac{q_1 \times b_1 \times a^2}{8} = 115.329\ kN.m \tag{4}$$

$$\text{Moment in beams running in y direction } (M_y) = \frac{q_1 \times a_1 \times b^2}{8} = 115.329\ kN.m \tag{5}$$

3.3.2 Shear Force calculations

$$\text{Shear force in beams running in x direction } (Q_x) = \frac{q_1 \times a \times b_1}{2} = 46.132\ kN \tag{6}$$

$$\text{Shear force in beams running in y direction } (Q_y) = \frac{q_1 \times b \times a_1}{2} = 46.132\ kN \tag{7}$$

3.4 Plate Theory

Using the load intensity as mentioned in (1) design bending and torsion moments, and shear forces are calculated and presented below:

3.4.1 Section Properties of Ribs

$$\frac{D_f}{D} = \frac{0.1}{0.6} = 0.167 \text{ and } \frac{b_w}{b_f} = \frac{0.23}{1.00} = 0.23 \quad (8)$$

Second moment of inertia of beam in x-direction (I_1) and in y-direction (I_2) $= k.b_w.D^3$ (9)

Where k= constant taken from Reynolds designers handbook [7] = 0.1455

$$I_1 = I_2 = 0.1455 \times 0.23 \times 0.6^3 = 7.22844 \times 10^9 mm^4$$

Flexural rigidity per unit length of plate along x-direction = $D_x = \frac{E I_1}{b_1} = 0.00722.E$ (10)

Flexural rigidity per unit length of plate along y-direction = $D_y = \frac{E I_2}{a_1} = 0.00722.E$ (11)

Torsional rigidity per unit length of plate along x direction $= C_1 = k_1 G (2a^3) 2b$ (12)

Torsional rigidity per unit length of plate along y direction = $C_2 = k_1 G (2b^3) 2a$ (13)

Where,

$$G = \frac{E}{2(1+\mu)} = \frac{E}{2(1+0.15)} = 0.43478 E \quad (14)$$

$k_1 =$ Constant of Torsional rigidity given by Timoshenko [6]

Here in this case, $C_1 = C_2 = 0.252 \times 0.43478 E \times (2 \times 10)^3 \times (2b) = 0.0007998 E$ (15)

$$E = 5700\sqrt{f_{ck}} = 5700\sqrt{20} = 24.4911 \times 10^6 kN/m^2 \quad (16)$$

3.4.2 Deflection at centre of span

$$2H = \frac{C_1}{b_1} + \frac{C_2}{a_1} = 0.0016 E \quad (17)$$

$$\frac{D_x}{{a_x}^4} = 18.426 \text{ and } \frac{D_y}{{b_y}^4} = 18.426 \quad (18)$$

$$deflectiona = \frac{16 q}{\pi^6}\left[\frac{\sin\left(\frac{\pi x}{a_x}\right)\sin\left(\frac{\pi y}{b_y}\right)}{\frac{D_x}{a_x^4}+\frac{2H}{a_x^2 b_y^2}+\frac{D_y}{b_y^4}}\right] = 0.07526\ m \quad (19)$$

3.4.3 Design moments and Shear force using Plate Theory

$$M_x = -D_x\left(\frac{\partial^2 a}{\partial x^2}\right) = 136.726 \sin\left(\frac{\pi x}{a_x}\right) \sin\left(\frac{\pi y}{b_y}\right) \quad (20)$$

$$M_y = -D_y\left(\frac{\partial^2 a}{\partial y^2}\right) = -136.726 \sin\left(\frac{\pi x}{a_x}\right) \sin\left(\frac{\pi y}{b_y}\right) \quad (21)$$

$$M_{xy} = -\left(\frac{C_1}{b_1}\right)\left(\frac{\partial^2 a}{\partial x \partial y}\right) = -15.1292 \cos\left(\frac{\pi x}{a_x}\right) \cos\left(\frac{\pi y}{b_y}\right) \quad (22)$$

$$M_{yx} = -\left(\frac{C_2}{a_1}\right)\left(\frac{\partial^2 a}{\partial x \partial y}\right) = -15.1292 \cos\left(\frac{\pi x}{a_x}\right) \cos\left(\frac{\pi y}{b_y}\right) \quad (23)$$

$$Q_x = -\frac{\partial}{\partial x}\left[D_x\left(\frac{\partial^2 a}{\partial x^2}\right) + \frac{C_2}{a_1}\left(\frac{\partial^2 a}{\partial_y{}^2}\right)\right] = -47.628 \cos\left(\frac{\pi x}{a_x}\right) \sin\left(\frac{\pi y}{b_y}\right) \quad (24)$$

$$Q_y = -\frac{\partial}{\partial y}\left[D_y\left(\frac{\partial^2 a}{\partial y^2}\right) + \frac{C_1}{b_1}\left(\frac{\partial^2 a}{\partial_x{}^2}\right)\right] = -47.628 \sin\left(\frac{\pi x}{a_x}\right) \cos\left(\frac{\pi y}{b_y}\right) \quad (25)$$

Using these above mentioned equations, moments and shear force are computed. The values of bending moment, twisting moment and shear forces computed at the various salient points of the grid are tabulated below.

Table 1 Moments and Shear Forces per Unit Width of Slab

Position		M_x kN.m	M_y kN.m	M_{xy} kN.m	M_{yx} kN.m	Q_x kN	Q_y kN
X	**Y**						
5	**5**	136.726	136.726	0.000	0.000	-0.038	-0.038
0	**10**	0.000	0.000	15.129	15.129	-0.076	0.000
0	**5**	0.000	0.000	-0.012	-0.012	-47.683	0.000
5	**0**	0.000	0.000	-0.012	-0.012	0.000	-47.683
0	**0**	0.000	0.000	-15.129	-15.129	0.000	0.000

IV. RESULT DISCUSSION

The results of the analysis carried by Rankine-Grashoff method, Plate theory, and Stiffness method are presented below. To carry out analysis of different hall sizes automation in analysis procedure is required which is done by using excel worksheets. These worksheets are used for Rankine-Grashoff method and Plate theory. The analysis by Stiffness method is carried out using STAAD.pro, application software. After analyzing such grids by above discussed three methods, the results are presented here.

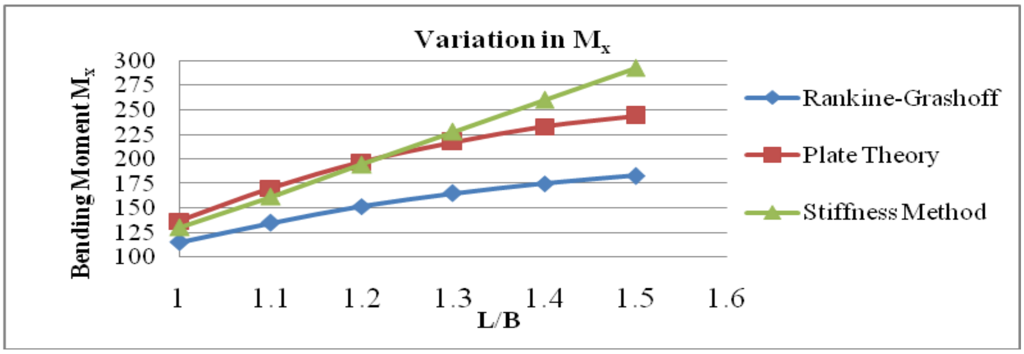

Figure no. 4 comparison of maximum bending moments for beams in x-direction (M_x)

The Figure No.4 shows the variations in maximum bending moment (M_x) for the beams which are running in x- direction for various (L/B) ratios by various methods of analysis.

The graph shows that the bending moment (M_x) is increasing with increasing L/B ratio. The nature of bending moment variation is non-linear for Rankine-Grashoff method and Plate theory approach. However, using stiffness method bending moment (M_x) is increasing almost linearly with increasing L/B ratio.

Up to L/B = 1.3, the bending moment (M_x) is in close proximity for Plate Theory and Stiffness method. With increase in L/B beyond 1.3, the bending moment (M_x) given by Plate theory is lower than those given by the Stiffness method in the range of 5% to 17%.

Rankine-Grashoff method estimates lowest values of M_x, amongst all above three methods.

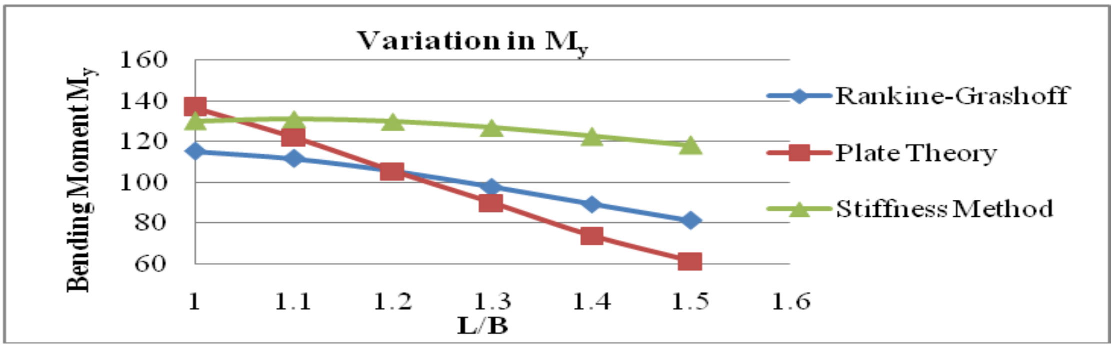

Figure no. 5 comparison of maximum bending moments for beams in y-direction (M_y)

The Figure No.5 shows the variations in maximum bending moment (M_y) for the beams which are running in y- direction for various (L/B) ratios by various methods of analysis.

The bending moment (M_y) is decreasing as L/B goes on increasing for all three methods.
As the L/B ratio increases, the variation is observed to be nonlinear for Rankine-Grashoff method and Stiffness method. However, for Plate theory approach the variation is almost linear. The graph also shows that, the bending moment (M_y) is in close proximity for Plate theory and Stiffness method up to L/B= 1.1. With increase in L/B beyond 1.1, the bending moments (M_y) given by Plate theory is lower than that given by the Stiffness method in the range of 7% to 48%.With increasing L/B ratio these values become lower for plate theory.

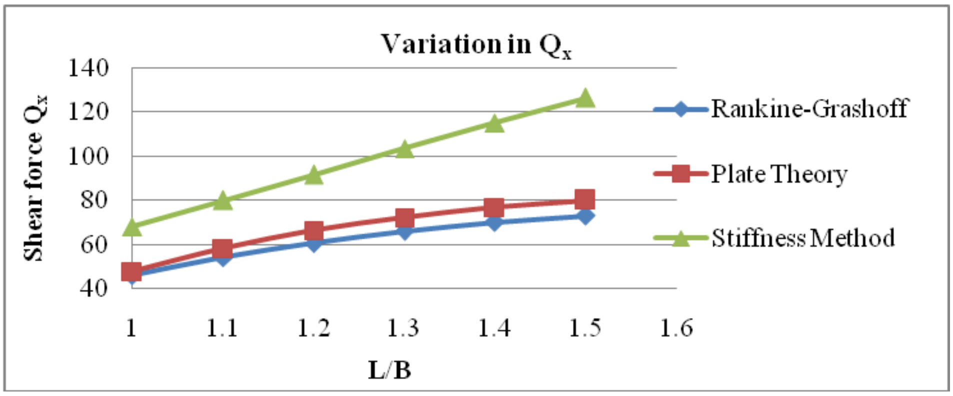

Figure no. 6 comparison of maximum shear force for beams in x-direction (Q_x)

The Figure No.6 shows the variations in maximum Shear Force (Q_x) for the beams which are running in x- direction for various (L/B) ratios by various methods of analysis.

The variation of shear force (Q_x) is observed to be nonlinear for Rankine-Grashoff method and Plate theory. However, for stiffness method the variation of shear force (Q_x) is almost linear.

Rankine-Grashoff method estimates lowest values of shear force (Q_x) amongst above three methods. For a given L/B ratio, the Stiffness method shows highest value of shear force (Q_x), than that is shown by Plate theory and Rankine-Grashoff method. Plate theory shows less value of shear force (Q_x) by 30% to 37% than the Stiffness method for L/B = 1 to 1.5

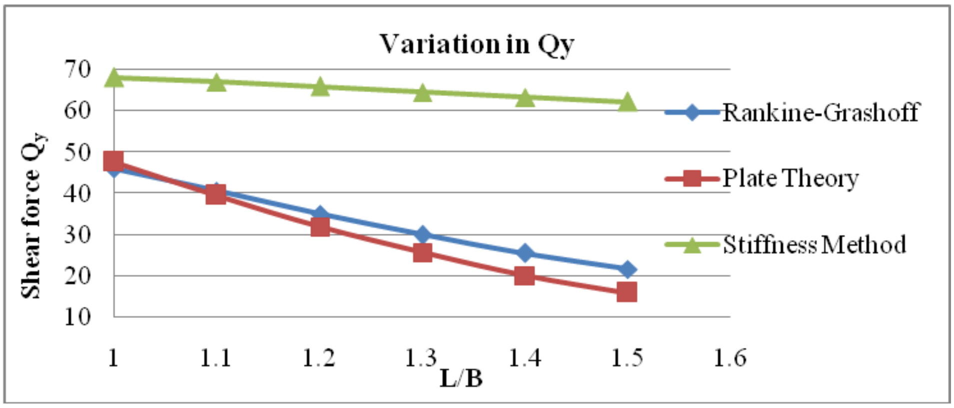

Figure no. 7comparison of maximum shear force for beams in y-direction (q_y)

The Figure No.7 shows the variations in maximum Shear Force (Q_y) for the beams which are running in y- direction for various (L/B) ratios by various methods of analysis.

The graph shows that shear force (Q_y) is decreasing almost linearly with increasing L/B ratio, for all the three methods.

The stiffness method shows highest value of shear force (Q_y) for the given L/B ratio. Rankine-Grashoff method shows lower values in the range of 32% to 60% for L/B= 1 to 1.5 than that of stiffness method. Plate theory shows values lower by 30% to 68% for L/B =1 to 1.5.

V. CONCLUSION

1) Rankine-Grashoff method is an approximate method. Rankine-Grashoff method does not give the values of torsion moments. Rankine-Grashoff method underestimates critical bending moment (M_x) and shear force (Q_x).
2) Plate theory and Rankine-Grashoff method are used for simple support conditions. On the contrary the stiffness method can be used for rigid supports as well.
3) In Plate theory and Rankine-Grashoff method, design moments and shear force in Peripheral beams cannot be obtained. In fact in monolithic framed construction, design moments and shears in peripheral beams will be the maximum.
4) Initially, up to L/B=1.2, Plate theory shows higher value of bending moment (M_x) with respect to stiffness method. With increasing L/B, beyond L/B=1.3 Plate theory shows lower value of bending moment (M_x) as compared to stiffness method.
5) Stiffness method shows higher value of shear force (Q_x) as compared to other methods discussed. Plate theory shows less values of Q_x than that of stiffness method.
6) Stiffness method is accurate & more suitable to arrive at design moments and shear force. Also Stiffness method takes less time for analysis.

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