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Sinusoidal Model Development for the Study of Diurnal Variation of Surface Air Temperature and Surface Dissolved Oxygen for the Lake Rudrasagar in Pre-Monsoon Period

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ABSTRACT : *Fourier analysis could be a very beneficial mathematical tool to develop a mathematical model. Prediction of the diurnal variation of surface temperature and dissolved oxygen(DO) of any lake can also be possible due to the development of useful models. Such models must put a signature in the future aspects of lake water bodies also. Here an attempt is made to develop a mathematical model useful for following the diurnal variation of the surface air temperature and surface DO in the pre-monsoon period of the lake Rudrasagar, a natural lake of western Tripura. Such attempt must generate an important tool in water resource and aquatic habitat management. The proposed model is based on the data collected in the pre-monsoon period for the year 2013 and 2014 over the lake Rudrasagar. During this period of study it is also observed that the diurnal variation of DO and air temperature over the lake almost follow the similar trends of variation; both of them supporting a sinusoidal variation. The results have significant implications for the predictability of diurnal variation of temperature and DO at the lake surface controlling the effect on aquatic habitat. Moreover the observed diurnal and seasonal variation of surface DO can give information about the regional health condition of the natural lake Rudrasagar.*

KEYWORDS: *Fourier analysis, Sinusoidal model, dissolved oxygen, Rudrasagar Lake*

I. INTRODUCTION

Most of the physical properties of lake water are characterized by temperature. Aquatic resources are very much controlled by spatial as well as temporal variation in water temperature (Vannote et al .,1980). The exchange of heat energy with atmosphere is very much active for controlling the lake hydrodynamics. The spatial distribution of temperature and heat exchange has already been studied for various lake using 3-D ELCOM model and also their linkage with regional climate model has been established. Dissolved oxygen is another important parameter which also controls the spatial and temporal distribution of aquatic organisms (Wetzel,1983;Samal and Mazumdar,2005a;2005b).Diurnal variation of incoming solar radiation can able to put it's signature on various physical quantities near the earth surface. Out of these lake's surface air temperature and surface DO are two useful quantities .The magnitude of diurnal variation of surface air temperature might be affected by the nature of underlying surface and its coupling to the atmosphere above. Our atmosphere is almost transparent to the short wave solar radiation but according to Lambert-Beer law it is exponentially decaying while penetrating into the water body. Mathematical model is also available to predict annual cyclic trend in water temperature from recorded air temperature**.(** V. Kothandaraman et.al.,1972**)**

Interests in model development among the scientists and Engineers have been increasing day by day, as the models are only scientific means of predicting the future. The generated model can predict the value of the variables interest at any future time. Models may be used to select an environmental strategy best suited as a solution to specific problems. In order to solve complex environmental problems, numerical modeling is very helpful for Engineers and scientists with the aid of powerful computers and computational methods. Such models plays a very vital role to understand the hydraulic features in natural lakes like Rudrasagar and also to enhance the feasibility study of remedial actions in the shoreline area of the lake. A mathematical model can be formulated to represent a natural phenomenon or system. Lake and watershed models may be able to incorporate the region's unique physical, chemical, biological, social, and economic environment. The model for estimation of inflow temperature and the determination of surface heat transfer co-efficient increase the reliability of hydrothermal model prediction. (Owens.et.al.1998). Modeling of lake Hydrodynamics also plays a significant role for better understanding the algal blooms affecting the water quality (Prakash et.al 2007, Leon et.al 2005).

A developed mathematical lake water temperature model suggested that water temperature profiles changes due to higher air temperature and leads to generation of earlier stratification and consequently the lake water quality.(Hassan et.al.1998a).Though diurnal variation in the gaseous constituents of river water have been studied earlier(Butcher et. al .1927-28), no such study as well as modeling approach for the lake Rudrasagar has been carried out till now. There exists a strong correlation in the temperature and surface DO for the lake Rudrasagar (mihir et. al .2014) and hence the observed sinusoidal diurnal variation in air temperature over the lake should have a direct signature of the almost similar types of variation in Lake Surface DO. Organisms with in the lake therefore may relocate themselves vertically or otherwise in response to diurnal variation of surface air temperature and surface DO.

II. OBJECTIVES AND SCOPES

The primary objective of this study was to investigate the diurnal variation of surface air temperature over the lake Rudrasagar and dissolved oxygen concentration in its surface. In the preliminary level we have developed models based on the recorded data over a 24 hours window of a fair pre-monsoon day only. For more reliability of the model diurnal variation data over a large range of time window is to be considered. Keeping in this view, the study has been carried out for the entire pre-monsoon period. Almost all the days of pre-monsoon period remained fair and the effect of wind was relatively less as it's average value was recoded not more than 0.67km/hour in any day of both the pre-monsoon period of the year 2013 and 2014. Such models may be very much helpful for the future prediction of the two corresponding quantities in the lake surface in pre-monsoon period.

III. METHODOLOGY

Study Area : Geographically the Rudrasagar Lake (23°29' N and 90°01' E) is located in the Melaghar Block under Sonamura Sub-Division and is about 50 km from the state capital of Tripura. Hydromorphologically, Rudrasagar lake is a natural sedimentation water reservoir, that has three evergreen types of influx namely, Noacherra, Durlavnaraya cherra and Kemtali cherra. The actual sediment particles transferred along with the flow and takes up residence within the tank in the occasion as well as the apparent clear water discharges into the river Gomati through a connective route that is Kachigang just as one outflow. Consequently no rock enhancement can be found having 50m is actually silt (Clay loam) and also down below enhancement is actually sandy. Encompassing hillocks are generally involving delicate sedimentary enhancement. Average water basin area of the lake has been identified 1.5 sq kilometer when observation is performed.

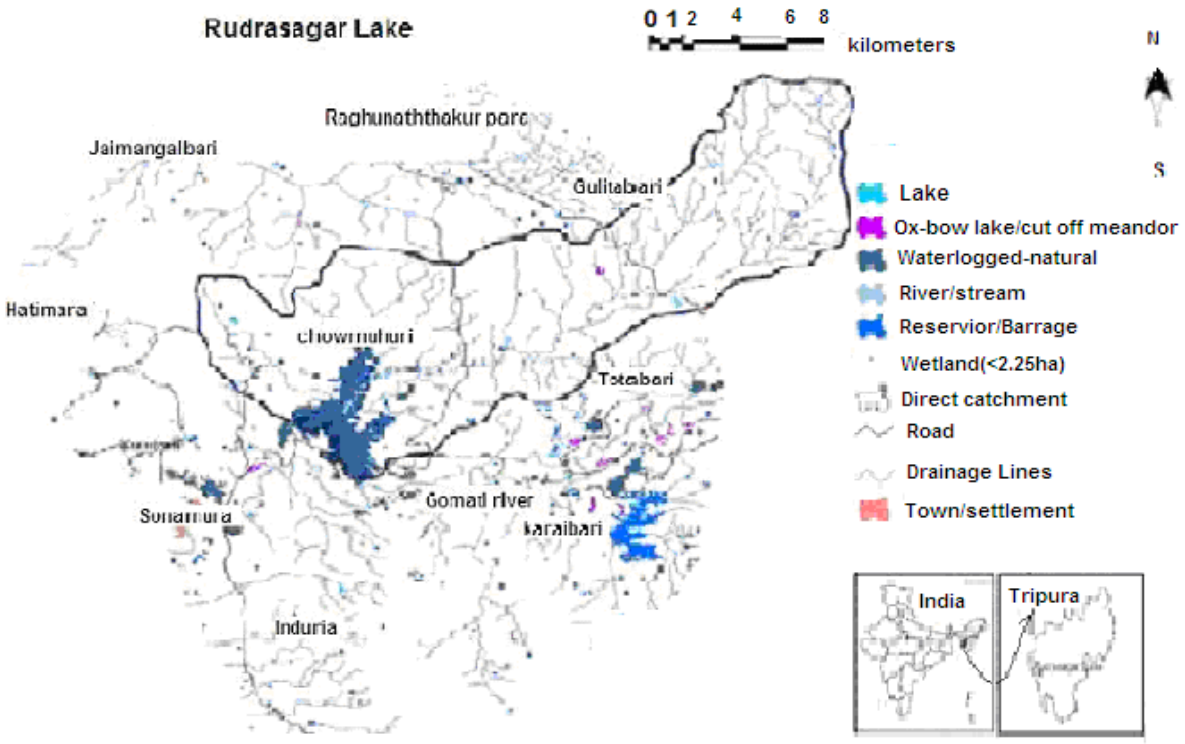

Figure 1.Map for Study Site of Rudrasagar lake based on Resourcesat-1 LISS-III(courtesy ISRO)

Sampling details : The observation station is selected considering the maximum water depth along the mid reach and considering the less human intervention of water of the lake. The air temperature over the lake surface and dissolved oxygen (DO) in the lake surface have, however, been measured using multipurpose water quality analyzer devices. Data are stored in memory cells of the devices which are then transferred to the database of the computer. The water quality analyzer devices are usually calibrated before the field work in every sampling day following the laboratory methods (APHA, 1989). The resolutions of air temperature over lake surface and dissolved oxygen measurement are 0.01^0c and 0.01mg/L respectively.

IV. THEORETICAL CONSIDERATIONS

In order to investigate the diurnal variation of air temperature over the lake Rudrasagar and dissolved oxygen concentration in its surface Fourier analysis procedure were adopted. The most salient features of Fourier analysis are as follows:

If a function f(t) is defined in the interval (0,T) and determined outside of this interval by f(t+T)=f(t),i.e if f(t) has a periodicity T , th [illegible]efined to be

$$f(t)=(A_0/2)+\sum_{n=1}^{\infty}[A_n Cos(2\pi nt/T)+B_n Sin(2\pi nt/T)]$$

where A_n and B_n are called Fourier co-efficient. The 1st term in the right hand side of the series (1) is simply the arithmetic average of f(t).

If there are N equally spaced discrete observations exists in the given period T then the co-efficient are given by

$$A_n = (2/N)\sum_{i=1}^{N} f_i \cos(2n\pi i/T)$$

……..(i)

and

$$B_n = (2/N)\sum_{i=1}^{N} f_i \sin(2n\pi i/T)$$

……(ii)

The total variance σ^2,in the function f(t) is given by

$$\sigma^2 = (1/2)\sum_{n=1}^{\infty}(A^2_n + B^2_n)$$

…….(2)

If only 1st m harmonics are considered then the variance in the observed record will be given by

$$\sigma^2_m = (1/2)\sum_{n=1}^{m}(A^2_n + B^2_n)$$

…….(3)

V. RESULT AND DISCUSSION:

The observed diurnal variation of surface DO revels that there exists a peak value towards the end of day, as oxygen production rate by algae and aquatic plants due to photosynthetic process is much more than consumption rate. Mathematical models are attempted to generate for the study of diurnal variation of surface air temperature and DO in pre-monsoon period using Fourier analysis method.

Air temp diurnal model over the lake surface : A simple mathematical model is formulated permitting the observed diurnal variation of air temperature in the lake surface of Rudrasagar over the 24 hours window in a typical day of observation in pre-monsoon period. In this small window of observation time, the observed diurnal variation of air temperature in the lake surface along with its simulation is depicted in the fig 1.

The result of Fourier analysis applied to the recorded air temperature data over the lake surface and the variance for different harmonics is shown in table no.1 and table no.2 respectively. The predicted model of temperature as developed over 24 hours of observed time window can be represented by $\theta(t)= A_0+ A_i \text{ Cos } \{(2\pi it/T)-\varphi_i\}$, where i is the summation index according to Einstein 's summation convention and here it is taken from i=1 to 10 in the time domain of 24 hour. A_0 is the average value of hourly measurement. The values of A_i and φ_i have been displayed in table 1.

Table 1.Result of Fourier analysis for air temperature of the Rudrasagar lake

No. of Harmonic	**Amplitude(A_i)**	**Phase angle φ_i (in degree)**
1st	6.949969866	80.77471
2nd	0.811974308	49.42666
3rd	0.301456232	-73.6684
4th	0.180879239	-6.29009
5th	0.265663401	-17.4494
6th	0.10307358	58.94614
7th	0.186434761	78.61605
8th	0.164718298	-2.80805
9th	0.241191991	31.77036
10th	0.134900055	-34.5005

Table 2. Variance of air temperature of the Rudrasagar lake (in a typical pre-monsoon day)

No of harmonic	**variance**	**Percentage of total variance**
1st	24.15104057	97.96700197
2nd	0.329651138	1.33720672
3rd	0.04543793	0.184315776
4th	0.01635865	0.066357714
5th	0.035288521	0.143145411
6th	0.005312081	0.021548086
7th	0.01737896	0.070496532
8th	0.013566059	0.055029766
9th	0.029086788	0.117988516
10th	0.009099012	0.036909505
Total	**24.65221971**	

The above developed model is valid for that day of observation only. For the validity of the model at any pre-monsoon day hourly data has been collected through out the entire pre-monsoon period and average data sets has been prepared for the diurnal variation. Observed diurnal variation of air temperature based on prepared average data sets in pre-monsoon period along with the simulated graph is shown in fig 3.

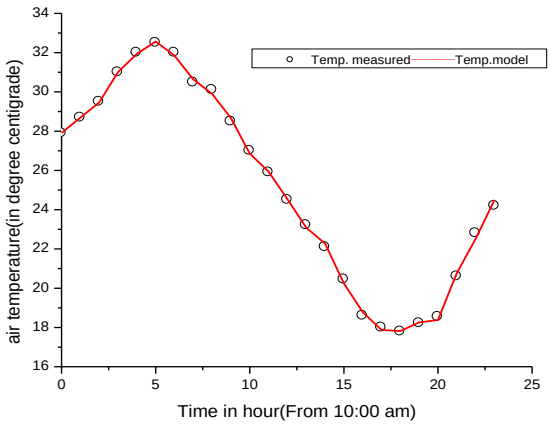

Figure 2. Observed diurnal variation of air temperature and model temperature in a typical pre-monsoon day(28.03.2013)

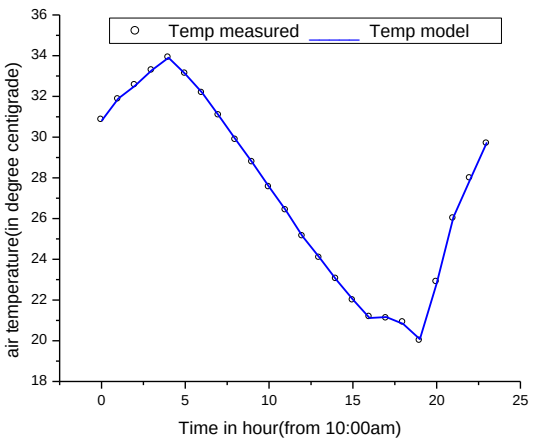

Figure 3. Observed diurnal variation of air temperature and model temperature in pre-monsoon period for the year 2013

The result of Fourier analysis applied to the prepared air temperature data sets over the lake surface and the variance for different harmonics are shown in table no.3 and table no.4 respectively. The predicted model for following the diurnal variation of surface temperature in any pre-monsoon day can also be represented by $\theta(t)= A_0+ A_i \text{ Cos } \{(2\pi it/T)-\varphi_i \}$, where i is the summation index according to Einstein 's summation convention and we have taken it from i=1 to 10 in the time domain of 24 hour. A_0 is the average value of corresponding hourly average measurements for all the pre-monsoon days data. The values of A_i and φ_i have been displayed in table 3.

Table 3.Result of Fourier analysis for air temperature over the Rudrasagar lake

No. of Harmonic	**Amplitude(A_i)**	**Phase angle φ_i (in degree)**
1st	6.221230755	62.59964
2nd	0.949851445	20.6676
3rd	0.468771936	-75.6642
4th	0.431833369	73.12665
5th	0.191085794	28.14315
6th	0.189022256	-85.623
7th	0.234557014	30.35356
8th	0.137387928	-45.8986
9th	0.139742231	53.6729
10th	0.124054462	38.02027

Table 4.Variance in air temperature above the Rudrasagar lake surface considering all the pre-monsoon days

No of harmonic	**variance**	**Percentage of total variance**
1st	19.35185605	96.29415521
2nd	0.451108884	2.244701943
3rd	0.109873564	0.546726991
4th	0.093240029	0.463959106
5th	0.01825689	0.090845644
6th	0.017864707	0.088894152
7th	0.027508496	0.13688131
8th	0.009437721	0.04696177
9th	0.009763946	0.04858505
10th	0.007694755	0.038288829
Total	**20.09660505**	

Model for diurnal variation of Surface DO of the lake : Mathematical modeling approach is also done for surface DO of lake water. The predicted model of DO as developed over 24 hours of observed time window can also be represented by $DO(t)= A_0+ A_i \text{ Cos } \{(2\pi it/T)-\varphi_i \}$, where i is the summation index according to Einstein 's summation convention and here it is taken from i=1 to 10 in the time domain of 24 hour. A_0 is the average value of hourly measurement. The result of Fourier analysis applied to the dissolved oxygen concentration data in the surface of lake and the variance for different harmonics are shown in table no.5 and table no.6 respectively.

Table 5.Result of Fourier analysis for Dissolved Oxygen in the Rudrasagar lake surface on atypical day of pre-monsoon season (28.03.2013)

No. of Harmonic	**Amplitude(A_i)**	**Phase angle φ_i (in degree)**
1st	2.7066	81.10473026
2nd	0.209134	0.069951252
3rd	0.270515	5.360215396
4th	0.139978	-41.27970379
5th	0.093739	-1.184651657
6th	0.07429	-6.458434929
7th	0.046673	-75.50865613
8th	0.041651	50.00215943
9th	0.029079	51.52010996
10th	0.010199	-6.784865815

Table 6.Variance of Dissolved Oxygen of the Rudrasagar lake

No of harmonic	variance	Percentage of total variance
1st	3.66284254	97.9191224
2nd	0.021868578	0.584614798
3rd	0.036589142	0.97814107
4th	0.009796884	0.261901055
5th	0.00439351	0.117452131
6th	0.002759492	0.073769758
7th	0.001089188	0.029117355
8th	0.000867398	0.023188243
9th	0.000422798	0.011302707
10th	5.20135E-05	0.001390481
Total	**3.740681545**	

Table 6 revels that the 1st harmonics accounts for 97.91 percent of total variance, the variance accounted for the higher harmonics are insignificant .Hence the diurnal variation of DO shows a cyclic variation and could be represented by 1st harmonic alone by a little cost in degree of validity.
The predicted model for following the diurnal variation of surface DO in any pre-monsoon day can also be represented by

$DO(t)= A_0+ A_i \text{ Cos } \{(2\pi it/T)-\varphi_i \}$, where i is the summation index according to Einstein 's summation convention and here it is taken from i=1 to 10 in the time domain of 24 hour. A_0 is the average value of the corresponding hourly average measurements for all the pre-monsoon days.

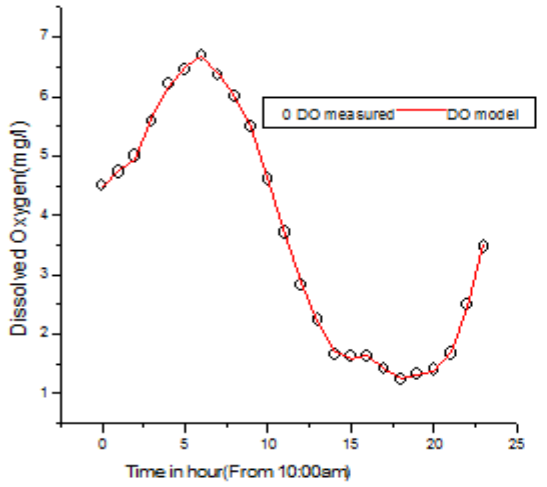

Figure 4.Observed Diurnal variation of measured DO and model DO in a typical fair pre-monsoon day (28.03.2013)

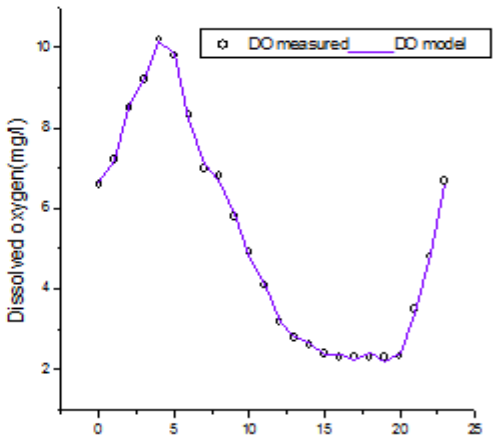

Figure 5. Observed diurnal variation of DO and model DO (considering entire pre-monsoon period for the year 2013)

The result of Fourier analysis applied to the dissolved oxygen concentration data in the surface of lake and the variance for different harmonics are shown in table no.7 and table no.8 respectively.

Table 7.Result of Fourier analysis of Dissolved Oxygen of Rudrasagar lake (in pre-monsoon season)

No. of Harmonic	**Amplitude(A_i)**	**Phase angle φ_i (in degree)**
1st	3.640069	61.57098663
2nd	0.637522	-77.08823115
3rd	0.076231	78.04649123
4th	0.404459	83.51452361
5th	0.298745	-65.39439039
6th	0.027618	-75.7340054
7th	0.09987	-7.075616916
8th	0.143968	56.9373872
9th	0.083681	-75.19162435
10th	0.092768	55.66154897

Table 8.Variance of Dissolved Oxygen of Rudrasagar lake

No of harmonic	variance	Percentage of total variance
1st	6.62505176	94.89943761
2nd	0.203217265	2.910951465
3rd	0.002905596	0.041620715
4th	0.081793715	1.171640292
5th	0.044624186	0.639211635
6th	0.000381385	0.005463078
7th	0.004986977	0.071435114
8th	0.010363448	0.148449465
9th	0.003501275	0.050153421
10th	0.004302972	0.061637201
Total	**6.981128578**	

Table 8 revels that the 1st harmonics accounts for 94.89 percent of total variance, the variance accounted for the higher harmonics are insignificant .Hence the diurnal variation of DO shows a cyclic variation and could be represented by 1st harmonic alone by a little cost in degree of validity.

Figure 6.represents the diurnal variation of observed and predicted air temperature in the pre-monsoon period of the current year 2014, where as the observed and predicted diurnal variation of DO in lake water has been displayed in fig 7.Table. 9 and Table.10 display the statistical comparative study of the observed and modeled values of the two quantities satisfactorily.

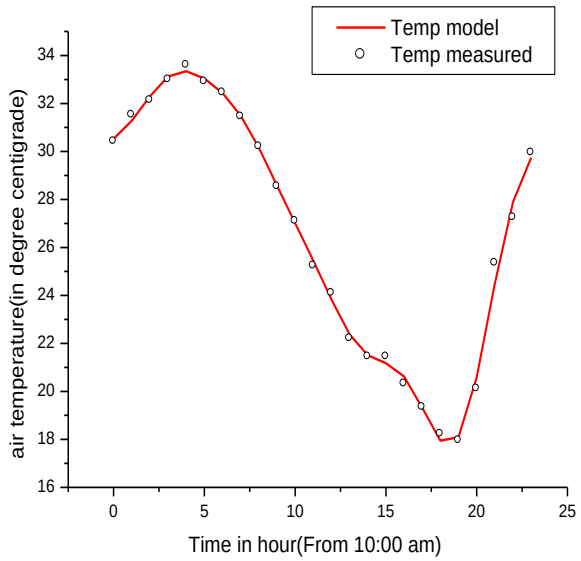

Figure 6.Observed and predicted diurnal variation of air temperature in pre-monsoon period in Rudrasagar lake, 2014

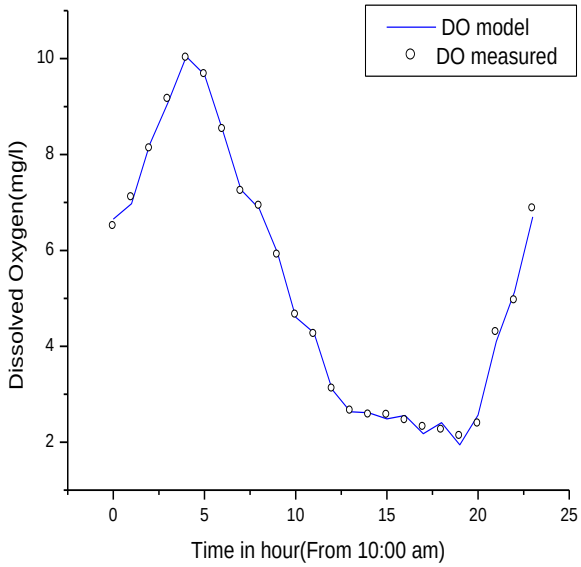

Figure 7.Predicted DO and observed surface DO in lake water in Pre-monsoon period, 2014

Table 9.Staistical comparative study for observed and modeled Temperature in pre-monsoon period

Premon-soon	Observed Diurnal variation (Temp. in Degree centigrade)			Modeled Diurnal variation (Temp.in Degree centigrade)		
	Mean	Variance	Standard Deviation	Mean	Variance	Standard Deviation
2013	**27.305**	**20.902**	**4.5719**	**27.306**	**20.89**	**4.5716**
2014	**26.51213**	**28.10735**	**5.3016**	**26.51217**	**28.0187**	**5.2932**

Table 10.Statistical comparative study for observed and modeled surface DO in pre-monsoon period

Premon-soon	Observed Diurnal variation (DO in mg/l)			Modeled Diurnal variation (DO in mg/l)		
	Mean	Variance	Standard Deviation	Mean	Variance	Standard Deviation
2013	5.245417	7.284165	2.698919	5.245458	7.278619	2.697892
2014	5.272083	7.074365	2.659768	5.272083	7.062391	2.657516

VI. CONCLUSION

Though considerable works have already been done on various Indian lakes, no such work on the lake Rudrasagar has been reported earlier. In the preliminary level, the model we have developed in this paper is valid only for pre-monsoon period of the lake Rudrasagar .However as this model has only regional interest ,a generalized model applicable for other seasons will be generated and the sinusoidal nature of the diurnal curve will be tested at different locations for other lakes in our next phase of investigation. It is worth mentioning that lack of high quality, complete, long term datasets for surface DO and surface air temperature, model development for long time validity has become a challenging task for this lake. More and more extensive research work is also required to validate such model.

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