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A Mathematical Model of the Marshall Olkin Exponential Weibull Distribution with the Sympathetic Activity and HPA Axis Activity in Post Traumatic Stress Disorder

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ABSTRACT: *The theoretical study for the effect of HPA-axis activity during sleep in post traumatic stress disorder (PTSD) with ACTH and Cortisol levels is investigated. A new four parameter model called the Marshall–Olkin exponential-weibull probability distributions is used to find the levels of ACTH and Cortisol with the comparisons of PTSD, Trauma controls (TC) and Healthy controls (HC).*

Keywords: *ACTH, Cortisol, PTSD, TC, HC, Marshall-Olkin Exponential-Weibull distribution.*

I. INTRODUCTION:

Patients with a post-traumatic stress disorder frequently suffer from sleep disturbances such as nightmares frequent awakening and sleep initiation and sleep maintenance insomnia. Sleep disturbances are of clinical relevance in PTSD and they are often therapy resistant and related to suicide risk. A relationship between nightmares and delayed sleep has been suggested with the increasing and decreasing levels of ACTH and Cortisol. For this application the hazard rate function is utilized to model and to measured the levels of Cortisol and ACTH in the Graphical representation of the MOEW Distribution.

APPLICATION:

Insomnia and nightmares may also exert a negative effect on daytime symptoms. Sleep fragmentation is related to blunted growth hormone secretion in PTSD which may be an underlying mechanism for compromised neuroplasticity and hippocampal functioning.

A major difficulty in research into PTSD – related sleep disturbances is that despite very frequent complaints of insomnia and night mares, objective sleep is only mildly disturbed according to polysomnography (PSG) which is the golden standard in sleep research ***[1].*** *The HPA-axis and the sympathetic nervous system(SNS) are well characterized in PTSD and have been associated with sleep regulation* ***[6].***

Heart rate is significantly higher throughout the night in patients with PTSD. They also exhibited more awakenings during sleep according to PSG than the two control groups ACTH was positively related to the number of awakenings ***[5].*** *ACTH secretion and Cortisol were inversely related to the amount of slow wave sleep. The ratio Cortisol : ACTH was decreased upon awakenings in PTSD compared with TCS. In TCS the natural Cortisol : ACTH ratio was increased compared with both PTSD and TCS. The relationship between ACTH secretion and awakenings in PTSD is a novel finding* ***[8].***

Graphical representation of the level of Cortisol and ACTH, PolysomnoGraphic (PSG) recordings and subjective sleep parameters simultaneously in veterans with PTSD, Trauma controls and Healthy Controls.

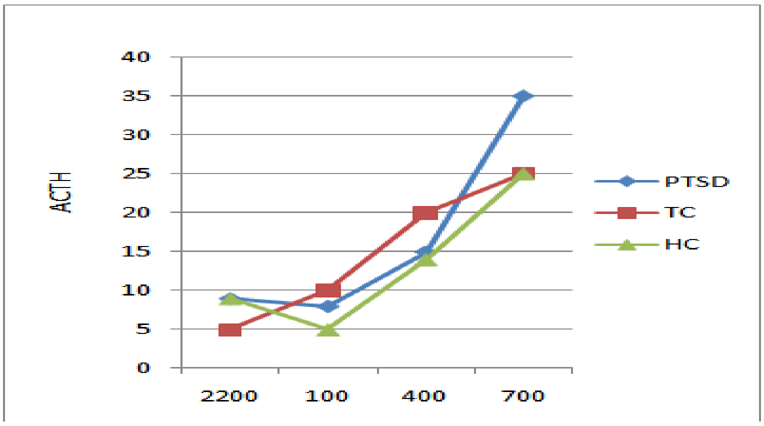

Fig.1

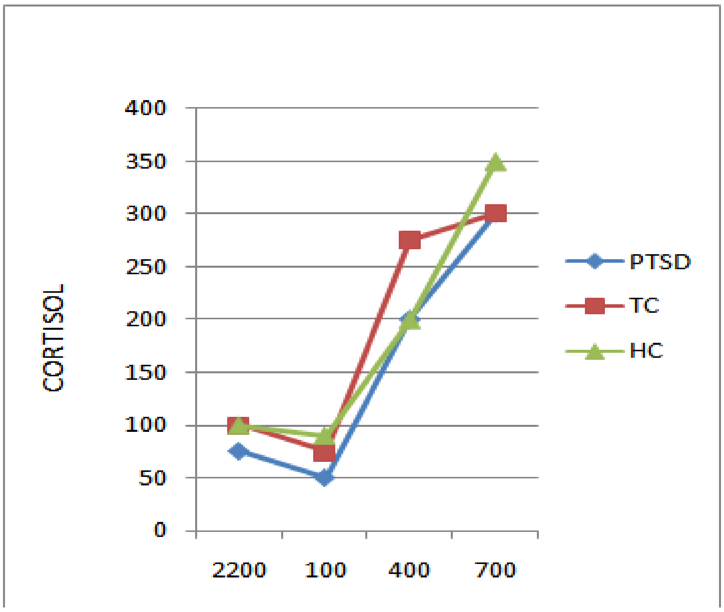

Fig.2

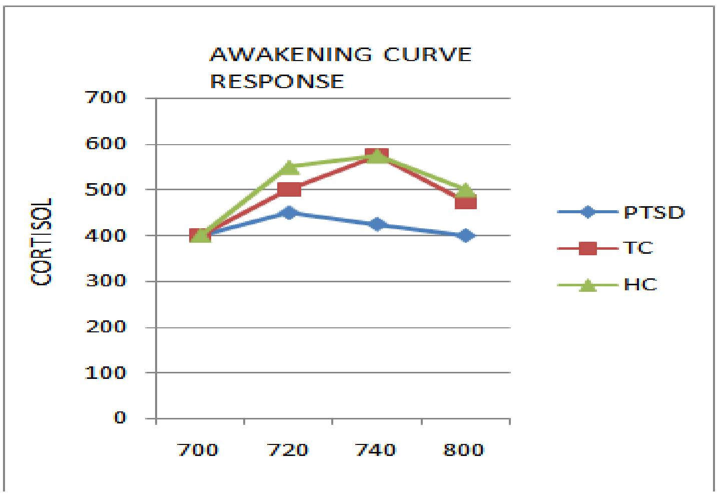

Fig.3

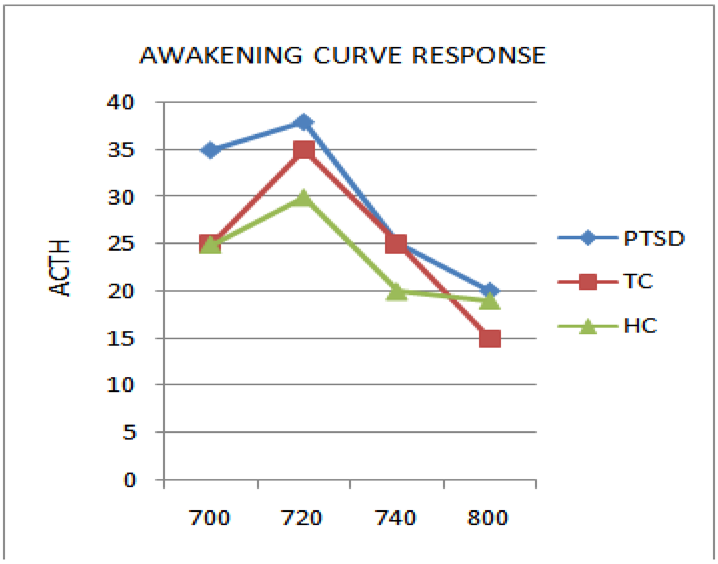

Fig.4

II. MATHEMATICAL MODEL:

The Weibull distribution is a popular life time distribution model in reliability engineering Marshall and Olkin introduced a method for adding a new parameter to an existing distribution which results in improved flexibility to model different types of data it is consider as baseline distribution having cumulative distribution function (CDF) G_6 ***[7],*** *with the related probability density function PDF g_b (x), being the Random – Nikodym derivative of the CDF G_b with respect to the ordinary Lebesgue measure****[2].*** *Then the associated Marshall – Olkin extended distribution CDF 'F' is given by*

$$F(x) = \frac{G_b(x)}{G_b(x) + \alpha \overline{G_b}(x)}$$

Where $\overline{G_b} = 1 - G_b$ *stands for the survival function of the baseline CDF G_b.*

Accordingly , the baseline PDF g_b (related to G_b) the Marshal – Olkin PDF becomes

$$f(x) = \frac{\alpha g_b(x)}{\left[G_b(x) + \alpha \overline{G_b}(x)\right]^2}$$

Cordeiro introduced a type of exponential weibull distribution by considering the baseline CDF

$$G_b(x) = \left[1 - e^{-\lambda x - \beta x^k}\right] I_{(0,\infty)}(x), \quad \lambda > 0,\ \beta > 0,\ k > 0$$

With the related PDF

$$g_b(x) = \left(\lambda + \beta k x^{k-1}\right) e^{-\lambda x - \beta x^k} . I_{(0,\infty)}(x)$$

Let $\theta = (\lambda, \beta, k, \alpha)$ *a vector parameter having positive coordinates. The random variable* ξ *defined on an fixed probability space* Ω, ξ, p *posses the Marshall –Olkin exponential-Weibull distribution****[3,4]*** *when its CDF and PDF are respectively given by*

$$F(x) = \frac{1 - e^{-(\lambda x + \beta x^k)}}{1 - (1 - \alpha) e^{-(\lambda x + \beta x^k)}} . I_{(0,\infty)}(x)$$

$$f(x) = \frac{\alpha \left(\lambda + \beta k x^{k-1}\right) e^{-(\lambda x + \beta x^k)}}{1 - (1 - \alpha) e^{-(\lambda x + \beta x^k)}} . I_{(0,\infty)}(x) \quad \lambda, \beta, k, \alpha > 0$$

$\xi \approx MOEW \ (\theta)$ with $\theta = (\lambda, \beta, k, \alpha)$ *to indicate that the random variable* ξ

This distribution using MOEW distribution is that, if $X_1, X_2, \ldots\ldots$ *be an i.i.d random variable from G(x) and N be a random variable with probability mass function* $\theta = (1-\theta^{n-1})$ *for n=1,2,3…… and define* $m_N = \min\{X_1, X_2 \ldots X_N\}$ *then*

$$P[m_N \le x] = 1 - \sum_{n=1}^{\infty} P[M_N \ge x/N = n]$$

$$P[N = n] = \frac{G(x)}{G(x) + \theta \overline{G(x)}}$$

Graphical representation of effect of introducing the parameter α *and the hazard rate function*

$$h(x) = \frac{\lambda + \beta k x^{k-1}}{1-(1-\alpha)e^{-(\lambda x + \beta x^k)}} . I_{(0,\infty)}(x)$$

III. RESULT:

By using hazard rate function

$$h(x) = \frac{\lambda + \beta k x^{k-1}}{1-(1-\alpha)e^{-(\lambda x + \beta x^k)}} . I_{(0,\infty)}(x)$$

$(\lambda, \beta, \mathrm{k}, \alpha)$ *are the parameters. The secretion of nocturnal and awakening curve response of ACTH and Cortisol levels of PTSD, TC, HC due to human stress correlated with h(x) and the levels shows the significance of the control groups compared with PTSD.*

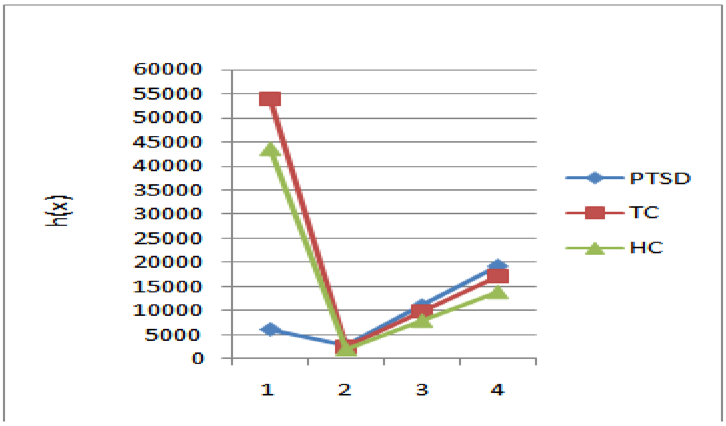

Fig.5

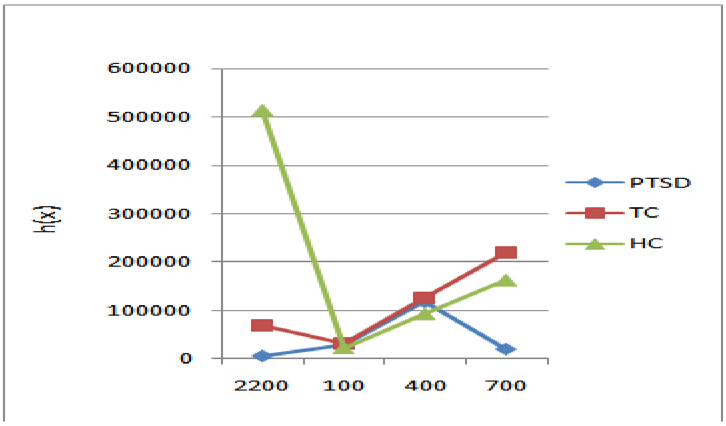

Fig.6

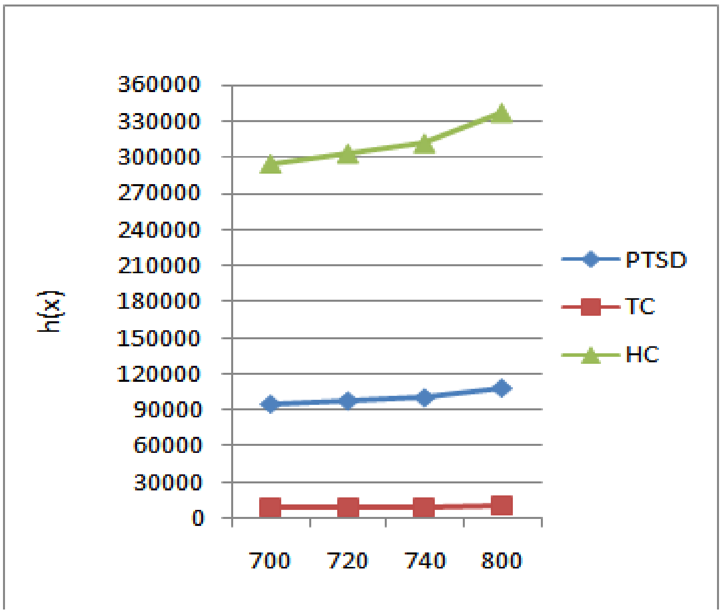

Fig.7

Fig.7

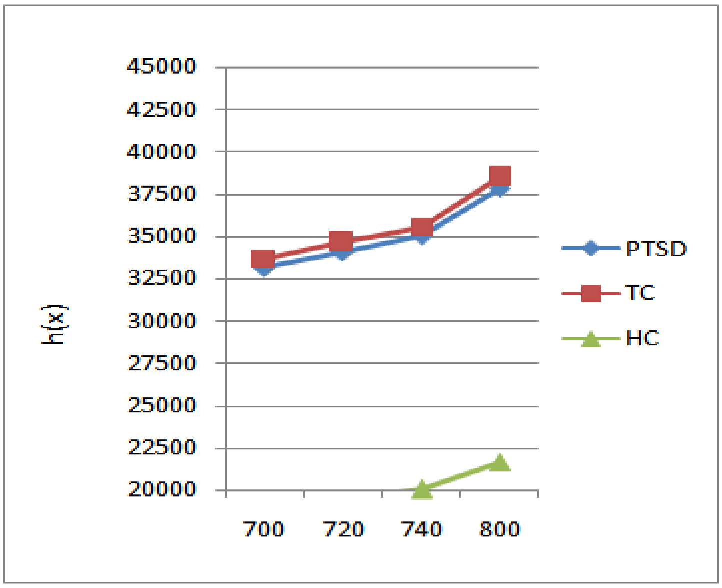

Fig.8

Fig.8

IV. CONCLUSION:

This study suggests that hypothalamic- pituitary –adrenal axis activity is related to sleep fragmentation in PTSD. Activity of the sympathetic nervous system (SNS) is increased during sleep in PTSD. The successive time intervals and sample values of ACTH and Cortisol are used to estimate the parameters support to find the hazard rate to obtain the levels of ACTH and Cortisol to explore the potential causal relationship between sleep problems and the activity of the HPA axis and SNS in PTSD.

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