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Analysis of Four-Bar Linkages Model using Regression

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ABSTRACT: ***FERDINAND FREUDENSTEIN*** *(1926-2006) is widely acknowledged to be the father of modern kinematics of mechanism and machines. The* **Freudenstein** *equation result through an analytical approach towards analysis and design of four link mechanisms which along which its variant's, are present in a large number of machines used in daily life. It has been developed method to enumerate and codify the solutions of type synthesis of linkage mechanisms with rotoidal and prismatic joints. The essence of mechanism synthesis is to find the mechanism for a given motion. Type Synthesis is the first stage of conceptual design of mechanisms, where the number, type and connectivity of links and joints are determined. It is followed by the Dimensional Synthesis stage, where the link lengths and pivot positions are computed to fulfill a given kinematic task. The latter and the subsequent stages of detailing design are very costly. Therefore, comparison of the result has been done by attaching a motor to the output shaft and without motor.*

Key Ward : *Link angle, Mechanism*

I. INTRODUCTION

Mechanisms and machines have been used since ancient times to reduce human effort, and, since the Industrial Revolution, they have entered and impacted almost all aspects of human society. In their most simplistic description, a mechanism is an assemblage of rigid links (or bars) connected by joints which allow relative motion between the connected links. One (input) link of the mechanism is actuated and another (output) link can be made to perform a desired, often intricate, motion. One of the first well-known examples of a mechanism is the Watt's straight-line linkage. This mechanism was designed by James Watt to pull and push the piston-rod in a double acting steam engine he had invented and which is credited to have started the Industrial Revolution. In modern times, mechanisms are present in a huge variety of gadgets, devices and systems – in bottle cork openers, in bicycles, in garage door opening system, in steering and braking system of a car, in construction equipment for moving dirt and other material, to move control surfaces of aircrafts, in spacecrafts to deploy solar panels and other appendages, in laparoscopic surgical tools, artificial prosthetic knees and other medical devices, to name a few. In the last 50 years mechanisms have been combined with advanced electronics, sensors, control systems and computing technologies, and this marriage has resulted in devices such as robots, micro-electro mechanical systems (MEMS) and other so-called intelligent products. In the nineteenth century and prior to 1950's, most mechanism analysis and design was done graphically. During the 1950's computers and algorithms for computing were being rapidly developed in USA and elsewhere. Freudenstein was amongst the first person to realize the potential of computers for analysis and design of mechanisms and machines and his analytical approach fitted perfectly well with the rapidly developing computing technologies. The primary mathematical tool for the analysis of a linkage is known as the kinematics equations of the system. This is a sequence of rigid body transformation along a serial chain within the linkage that locates a floating link relative to the ground frame. Each serial chain within the linkage that connects this floating link to ground provides a set of equations that must be satisfied by the configuration parameters of the system. The result is a set of non-linear equations that define the configuration parameters of the system for a set of values for the input parameters. Freudenstein introduced a method to use these equations for the design of a planar four-bar linkage to achieve a specified relation between the input parameters and the configuration of the linkage.

II. LITERATURE REVIEW

Literature review is an assignment of previous task done by some authors and collection of information or data from research papers published in journals to progress our task. It is a way through which we can find new ideas, concept. There are lot of literatures published before on the same task; only two papers are taken into consideration from which idea of the project is taken. **A.G. Erdman and S. Faik [1] (1999)** in this paper titled "A Generalized Performance Sensitivity Synthesis Methodology For Four Bar Mechanisms" published in a journal " Mechanism and Machine Theory" explained the term sensitivity coefficient The idea of sensitivity coefficient is defined and derived by Erdman and Faik. According to them, sensitivity coefficient is a mathematical expression which represents the change in output variable because of small change in the mechanism parameters like link length.

The relationship for sensitivity coefficient is a very useful tool for analysis of errors in the given four bar chain. The relationship between the sensitivity to sensitivity of link lengths and the location of the moving pivot of four bar link mechanisms is investigated for the particular objectives of three and four positions synthesis maps with iso-sensitivity curves plotted in design solution space allows the designer to synthesis a planer mechanisms with desired sensitivity values or to optimize sensitivity from a set of acceptable design solutions. As a matter of fact, the performance of the assembly is measured by accuracy of the assembly specification which is a function of associated part tolerances. If the assembly specification has limits of variation in two or more directions, the correlation between these variations also impresses the limit of variation. He used the direct linearization method (DLM) to determine the bivariate distribution of the assembly specification, in terms of part tolerances. The mechanism consider in his paper consists of flexible parts and is subjected to external loading. The extra variations of each part dimensions due to its flexibilities will impose new variations for the assembly specification. The influence of loading on variation of the assembly specification is modeled by the finite elements method (FEM) using CALFEM toolbox of MATLAB software. First, the valid domains of DLM are recognized by means of Monte Carlo simulation and then, the percent contribution of each manufacturing variable in assembly specification is determined by DLM method. The simulation results in his paper confirm that after loading the mechanism, mass production rejects are remarkably increased. The paper proposes that by decreasing the tolerance limits of those manufacturing variables that have the highest contribution in the assembly specification, the number of rejects could be decreased significantly. A recent article **by Deepak and Ananthasuresh[2]** Mechanisms and machines have been used since ancient times to reduce human effort, and, since the Industrial Revolution, they have entered and impacted almost all aspects of human society. In their most simplistic description, a mechanism is an assemblage of rigid links (or bars) connected by joints which allow relative motion between the connected links. One (input) link of the mechanism is actuated and another (output) link can be made to perform a desired, often intricate, motion. One of the first well-known examples of a mechanism is the Watt's straight-line linkage. In modern times, mechanisms are present in a huge variety of gadgets, devices and systems – in bottle cork openers, in bicycles, in garage door opening system, in steering and braking system of a car, in construction equipment for moving dirt and other material, to move control surfaces of aircrafts, in spacecrafts to deploy solar panels and other appendages, in laparoscopic surgical tools, artificial prosthetic knees and other medical devices, to name a few. In the last 50 years mechanisms have been combined with advanced electronics, sensors, control systems and computing technologies, and this marriage has resulted in devices such as robots, micro-electro mechanical systems (MEMS) and other so-called intelligent products**.**

III. THEORETICAL ANALYSIS

A four bar mechanism is shown in figure-1: It is in equilibrium condition.a,b,C and Φ represent the magnitude of the links AB,BC,CD and DA respectively. Θ,β and ~~m~~ are the angles of AB, BC and DC respectively with the 'x' axis (taken along AD). Ad is the fixed link. AB is taken as the input link where DC is the output link. As in any configuration of mechanism the figure must enclosed. The link of mechanism can be considered as vectors. Thus vector displacement relationship as follows.

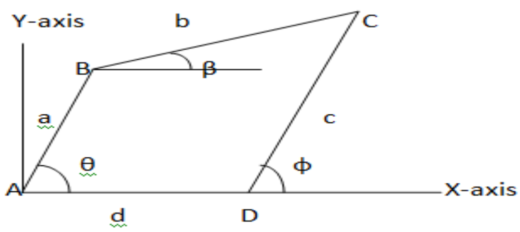


Fig-1 Theoretical four bar model

For equilibrium of mechanism, the sum of the components along 'x' axis and along 'y' axis must be equal to zero. First of all, taking the sum of the components along 'x' axis as shown in figure, we have

$a \cos\Theta + b \cos\beta - C \cos\Phi - d = 0$ …………………(1)

From this we can write,

$b \cos\beta = C \cos\Phi + d - a \cos\Theta$

squaring both sides,

$b^2 * \cos^2\beta = (C\cos\Phi + d - a\cos\Theta)^2$

$$=C^2 x\cos^2\Phi + d^2 + 2dCx\cos\Phi + a^2 x\cos^2\Theta$$
$$-2aCx\cos\Phi\, x\cos\Theta - 2xadx\cos\Theta \quad \ldots\ldots(2)$$

Now taking the sum of the component along ‘y’ axis, we have

$$a\sin\Theta + b\sin\beta - C\sin\text{ᵯ} = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$
$$b\sin\beta = C\sin\text{ᵯ} - a\sin\Theta$$

squaring both sides,

$$b^2\sin^2\beta = (C\sin\text{ᵯ} - a\sin\Theta)^2$$
$$= C^2 x\sin^2\text{ᵯ} + a^2\sin^2\Theta - 2aCx\sin\text{ᵯ}\, x\sin\Theta \quad \ldots\ldots(4)$$

Adding equation,

$$b^2(\cos^2\beta + \sin^2\beta) = C^2(\cos^2\Phi + \sin^2\Phi) + d^2 + 2dC\cos\Phi$$
$$+a^2(\cos^2\Theta + \sin^2\Theta) - 2aC(\cos\Theta\cos\Phi + \sin\Theta\sin\Phi) - 2ad\cos\Theta$$

Or, $b^2 = C^2 + d^2 + a^2 + 2dC\cos\Phi - 2aC(\cos\Theta\cos\Phi + \sin\Theta\sin\Phi) - 2ad\cos\Theta$

Or, $2aC(\cos\Theta\cos\Phi + \sin\Theta\sin\Phi) = C^2 + d^2 + a^2 + 2dC\cos\Phi - b^2 - 2ad\cos\Theta$

Or, $\cos\Theta\cos\Phi + \sin\Theta\sin\Phi = (C^2 + d^2 + a^2 - b^2)/(2aC) + (d\cos\Phi)/a - (d\cos\Phi)/C$ …………(5)

Let, $b/a = \text{ȣ}$; $b/C = \text{Ђ}$; and $(C^2 + d^2 + a^2 - b^2)/(2aC) = \text{ʒ}$ …………(6)

Equation (5) may be written as

$$\cos\Theta\cos\Phi + \sin\Theta\sin\Phi = \text{ȣ}\cos\text{ᵯ} - \text{Ђ}\cos\Theta + \text{ʒ} \quad \ldots\ldots\ldots(7)$$

Or, $\cos(\Theta - \Phi) = \text{ȣ}\cos\Phi - \text{Ђ}\cos\Theta + \text{ʒ}$ …………(8)

The equations (7 & 8) are known as ***Freudenstein’s equation.***
With the related literature review and objective of this concern project, we will find that equation no (7) is the critical from of the ***Freudenstein’s equation***. Solving this equation, the value of ȣ, Ђ , ʒ and other constants are as follows

$\text{ȣ} = b/a$;

$\text{Ђ} = d/C$;

$\text{ʒ} = (C^2 + d^2 + a^2 - b^2)/(2aC)$;

$\text{₤} = (1 - \text{Ђ})\cos\Theta + \text{ʒ} - \text{ȣ}$
$\text{¥} = -2\sin\Theta$
$\text{€} = \text{ȣ} + \text{ʒ} - (1 + \text{Ђ})\cos\Theta$
$\tan(\text{ᵯ}/2) = \frac{-\text{¥} \pm \sqrt{\text{¥}^2 - 4\text{₤€}}}{2\text{₤}}$
$\text{ᵯ} = 2\tan^{-1}\left(\frac{-\text{¥} \pm \sqrt{\text{¥}^2 - 4\text{₤€}}}{2\text{₤}}\right)$

<u>For experimental analysis</u>

For experimental analysis of Four Bar Mechanism, a wooden model has been developed (Figure:2) and the length of the links have been taken as shortest link as 12 mm ,longest link as 30 mm , output link as 20 mm and fixed link as 24 mm respectively as per Grashop ‘s Crition and analysis has been performed manual and using synchronous motor of specification with 33 rpm .

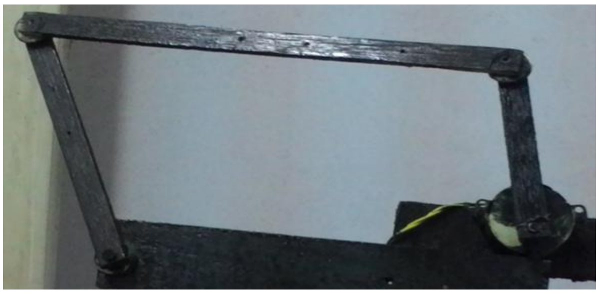


Fig 2: Wooden model of four bar mechanism

IV. RESULT AND DISCUSSION

The variation of input angle subtend by input link or crank with fixed link has been taken 0 to 180^0 with step of 15^0 and magnitude of output angle subtend by output link has been derived by manual and automatic(motor) details are given in Table -1 along with theoretical value of output angle using Fredustien Equation and also percentage of deviation shown in this table. It has observed that maximum deviation is more than 12 % in three case and rest are less than 5%.

Table -1 Theoretical value of output angle and percentage of deviation

Input Angle	Output Angle Theoretical	Output Angle Experimental	Output Angle Exp. with Motor	% deviation without motor	% deviation with motor
0	42	44	46	4.761904762	9.5238095
15	34	35	30	2.941176471	-11.764706
30	38	36	32	-5.263157895	-15.789474
45	44.9	45	39	0.222717149	-13.140312
60	55.3	55	56	-0.542495479	1.2658228
75	65.24	66	67	1.164929491	2.6977315
90	70.27	78	72	11.00042692	2.4619325
105	85.09	87	83	2.244682101	-2.4562228
120	94.43	95	93	0.60362173	-1.5143493
135	103	102	99	-0.970873786	-3.8834951
150	111.1	108	108	-2.790279028	-2.790279
165	117.9	116	115	-1.611535199	-2.4597116
180	123.59	120	119	-2.904765758	-3.7138927

Where Θ is input angle in degree subtends by input link (Shortest link or crank) to the fixed link, θ_{prog} and θ_{Exp} are the output angle in degree subtends by output link to fixed link as theoretical and experimental. The Fig-(3) shows the variation of output angle to the input angle where * stands for experimental value of output angle without motor and + stands for theoretical value by using Freudenstein's ***equation.***

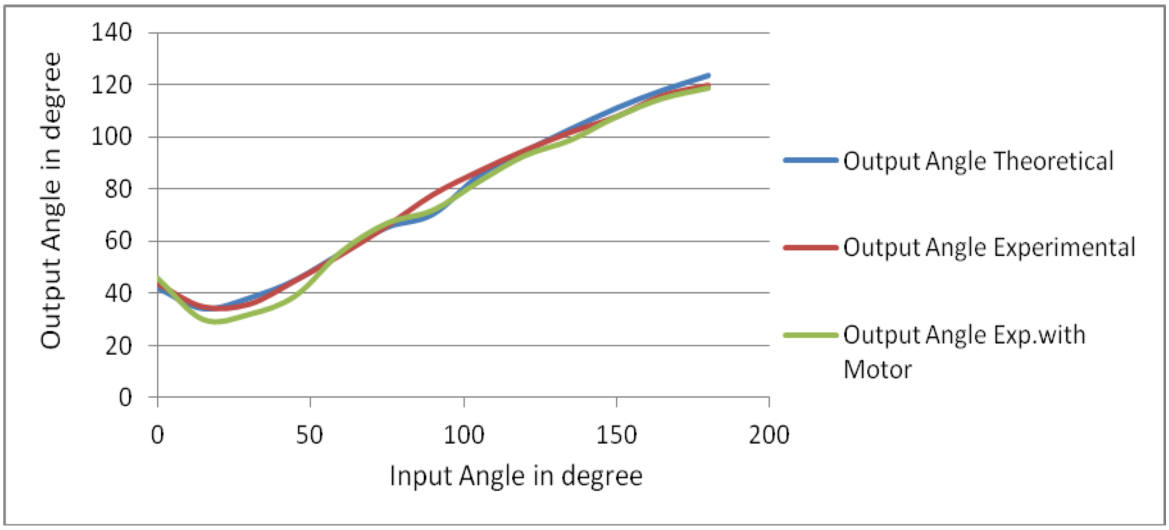


Fig –(3) showing the output angle of theoretical and experimental value.

The Fig –(4) showing the magnitude of output angle of theoretical (+) and experimental value of the output angle without motor (Blue). and experimental value of the output angle with motor (Red) It is emphasized that the deviations of these value are very close to each other except in one – two cases . The maximum deviation is about 16% in one case to 13% to 11% in one-two cases and in other case it is below 5%.

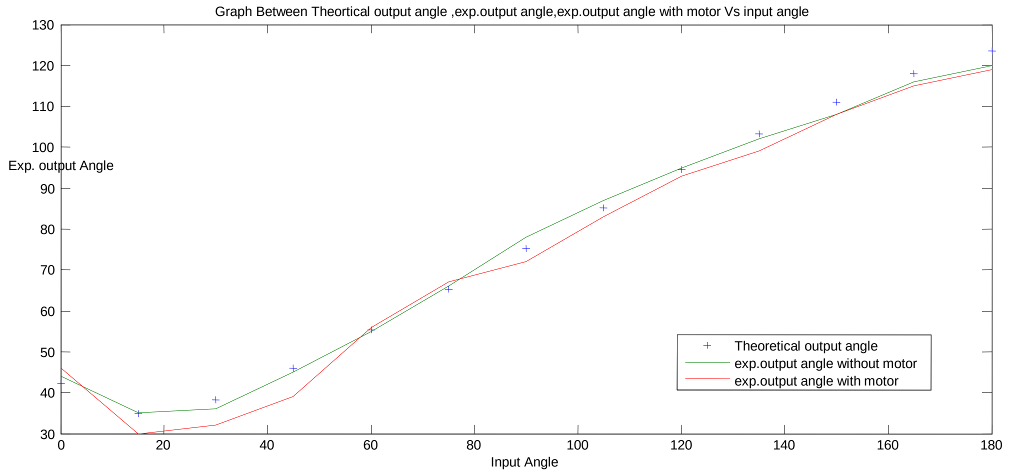


Fig –(4) showing the output angle of theoretical and experimental value with or without motor

V. MATHEMATICAL REGRESSION:

In statistics, regression analysis is a statically progress for estimating the relationships among variables. It includes many techniques for modeling and analyzing several variables, when the focus is on the relationship between a dependent variable and independent variable. More specifically, regression analysis helps one understand how to typical value of the dependent variable changes when any one of the independent variables is varied, while the other independent variables are held fixed. Most commonly, regression analysis estimates the conditional expectation of the dependent variable given the independent variable-that is ,the average value of the dependent variable when the independent variable are fixed. In all cases, the estimation target is a function of the independent variables called the regression function. In regression analysis it is also of interest to the dependent variables around the regression function which can be described by a probability distribution. In this regression analysis it is chosen as output angle(Φ) is function of input angle (Θ) ,and mathematical regression is set as n order polynomial as $\Phi = f(\Theta^n)$ where n is index of coefficient of the polynomial .For linear regression n = 1 and it is defined as : $\Phi = (a\Theta+b)$, where a & b are constant and Θ is chosen extreme lower and upper value of input angle as 0 to 180^0 , so therefore the value of a and b is obtained as 0.452 and 42.1478 respectively. In this model the percentage of deviation is is found in two cases more than 40% out of thirteen data. For reducing the deviation ,the degree of polynomial is increased n = 2,3 ,and so on, It is found that for n=2, : $\Phi = (a\Theta^2+b\Theta+c)$, when Θ varies 0,90,&$180^0$, the value of a, b & c are respectively $9.38\text{x}10^{-4}$, 0.283 & 42.1478.The deviation decrease and reduce to less than 40% .But the degree of polynomial increase n= 3 , $\Phi= (a\Theta^3+b\Theta^2+c\Theta+d\ )$, when Θ varies 0,60,120,180^0 , the value of a, b & c is respectively$2.84\text{x}10^{-5}$, 0.0088 & 0.21132. and deviation becomes reduced to 20% in one case and other are less than 10%. Further the increasing the degree of polynomial, it is found that deviation almost same as n=3 .So this analysis emphasizes that the output angle (Φ =~~m~~) is related to input angle (Θ) as three degree of polynomial .Details are given in Table -2 .

Table -2 : Mathematical Regression Table

Θ	~~m~~	~~m~~$_{exp}$	~~m~~$_{expmotor}$	Linear	Quadratic	Cubic	% of deviation linear (n=1)	% of deviation quadratic (n=2)	% of deviation cubic (n=3)
0	42	44	46	42.1478	42.1478	42.1478	0.3519048	0.3519048	0.351905
15	34	35	30	49	46.6	40.86	44.117647	37.058824	20.17647
30	38	36	32	55.7	51.48	41	46.578947	35.473684	7.894737
45	44.9	45	39	62.48	56.78	48.225	39.153675	26.458797	7.405345
60	55.3	55	56	69.26	62.5	55.01	25.244123	13.019892	-0.52441
75	65.24	66	67	76.04	68.68	65.46	16.554261	5.2728387	0.337216
90	70.27	78	72	82.88	75.21	76.54	17.945069	7.030027	8.922727
105	85.09	87	83	89.6	82.2	84.102	5.3002703	-3.396404	-1.16112
120	94.43	95	93	96.38	89.61	94.43	2.0650217	-5.10431	0
135	103	102	99	103.17	97.44	104	0.1650485	-5.398058	0.970874
150	111.1	108	108	109.94	105.7	112	-1.044104	-4.860486	0.810081
165	117.9	116	115	116.72	114.37	119	-1.000848	-2.994063	0.932994
180	123.59	120	119	123.5	123.47	123.6	-0.072821	-0.097095	0.008091

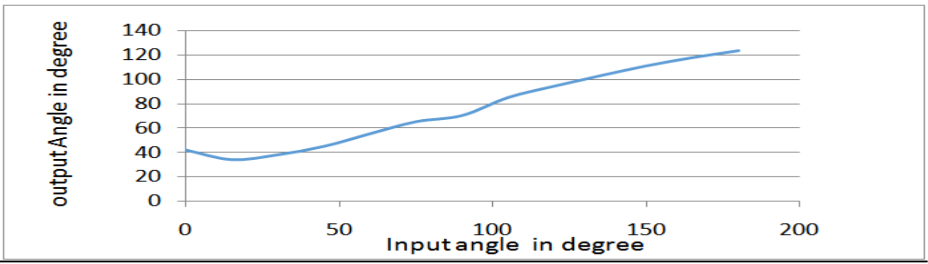
The variation of the degree of polynomial vs input angle are given in Fig-5 to Fig- 8 describe the First figure theoretical value of output angle (ᵯ) vs input angle of crank (Θ) ,subsequent Fig (5) to Fig (7) showing the variation of the output angle with increasing the order of degree of polynomial (ᵯ = Θ^N) as linear quadratic and cubic and found that cubic degree of polynomial is very close to the theoretical solution and also observed that error in one case 20% and 7% in two cases and rest are less than 1 % .It is therefore emphasized that cubic order of the polynomial gives satisfactory result which is given as

$$ᵯ = ᵯ_s + a_n \Delta\Theta^n + a_{n-1} \Delta\Theta^{n-1} + + a_{n-2} \Delta\Theta^{n-2} + \ldots\ldots\ldots\ldots + a_0 \text{---------------------} \quad (i)$$

Where, $\Delta\Theta^n = (\Theta^n - \Theta_s^n)$ & $\Delta\Theta^2 = (\Theta^2 - \Theta_s^2)$ $\Delta\Theta^3 = (\Theta^3 - \Theta_s^3)$ and an,an-1 are constant, the value of n (Degree of polynomial is odd number except one and it varies 3≤n≤ any odd number e,g 3, 5,7,9 etc .The deviation is gradually reduces by using odd degree of polynomial of input angle

The deviation of output angle between theoretical and given by above equation is 1-2% in all cases. So by increasing the odd degree of polynomial more than three, the better result can be obtained with less error. Depending upon the range of input angle, the degree of polynomial can be selected or chosen and the constant of polynomial can be found out by equi-spacing of angle intervals as discussed above, the result is very accurate and feasible. The variation of the output angle given by mathematical regression and theoretical value of its has been plotted vs. input angle emphasizes that the linear , quadratic and cubic (n=3) regression are showing in Fig- 5 to Fig-7 .

The Fig-5 showing the theoretical value of output angle (derived from Fredustien "s Equation) vs input angle in degree .



Fig(5) – Variation of theoretical output angle(ᵯ) with input angle (Θ)

The variation of the polynomial for linear (n=1), quadratic (n=2) and cubic (n=3) are given in Fig-6 to Fig-8.

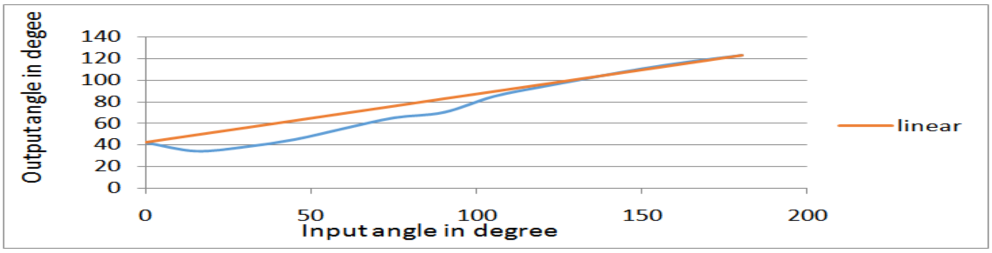


Fig (6) - GRAPH OF AND ᵯ WITH A LINEAR regression

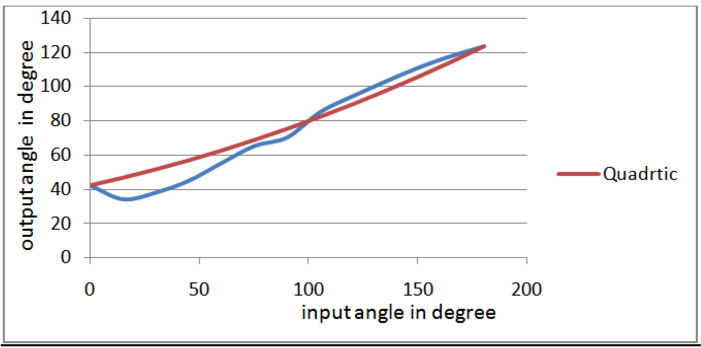


Fig (7) - GRAPH OF Θ AND ᵯ WITH A QUADRTIC regression

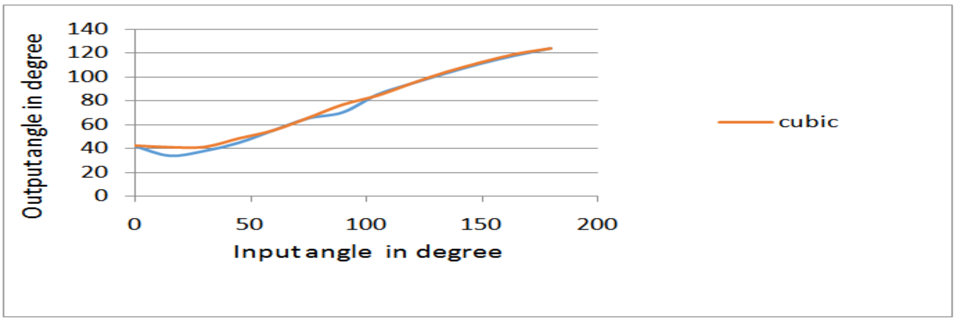


Fig (8) - GRAPH OF Θ AND m WITH CUBIC regression

If the order of odd degree of polynomial is increased as five, seven etc, the error become very less or negligible and the results becomes more accurate and feasible. The Table-3 shows the magnitude of the output angle for the odd degree of polynomial i,e n=3,5,7 and so on. It is observed that error are very less at n=7 and further if degree of polynomial increase, error becomes zero.

Table -3 : Mathematical Regression odd-degree of polynomial Table

Input angle	Output angle(Theo.)	Cubic order	Fifth order	seventh order
0	42	42.1478	42.1478	42.1478
15	34	40.86	36.08	34.12
30	38	41	39.12	38.106
45	44.9	48.225	45.88	45
60	55.3	55.01	55.01	55
75	65.24	65.46	65.46	65.46
90	70.27	76.54	72.86	70.65
105	85.09	84.102	84.82	84.89
120	94.43	94.43	94.43	94.43
135	103	104	104	103.67
150	111.1	112	111.7	111
165	117.9	119	118.1	117.68
180	123.59	123.6	123.6	123.6

The Fig-8 shows the variation of regression is almost fitted with theoretical value of the output angle with input angle .So from the analysis it is concluded that the minimum order of degree of polynomial is three and depending upon the range of input angle, the degree of polynomial can be increased higher order of polynomial so that the deviation is very close to zero.

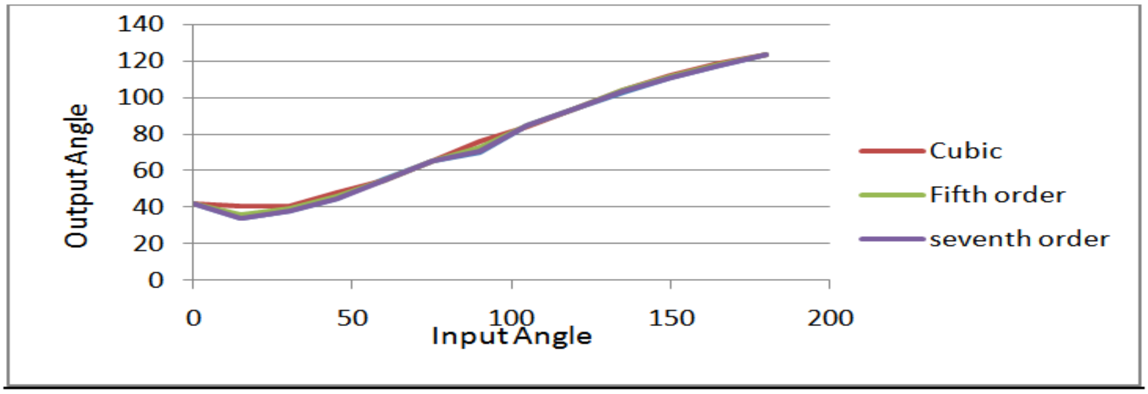


Fig (9) - GRAPH OF Θ AND m with cubic , fifth and seventh order regression

VI. Conclusion

It is observed that the variation between input and output angle have been observed and their result Without considering angular velocity of input link and found the maximum deviation was about 10% in one case and in other case it was below 1%. But the result With considering angular velocity of input link, the maximum deviation was about 13% in one case and in other case it was below 2%.The comparison of the result between theoretical value of the output angle with same given by mathematical regression is very close and deviation showing very less in above article and it is also seen that due to lack of suffocating device the error or deviation in some cases is very high. The wooden model having wooden link also causes of some percentage of deviations in the result due lack mobility at the joining point. For regression analysis the degree of odd polynomial (n=3) has been best fitted .Moreover depending the size of input value ,the degree of polynomial of polynomial may be increased higher order i,e n=5,7 for feasible result.

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