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Efficient Algorithm for Constructing KU-algebras from Block Codes

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ABSTRACT. *In this paper, we will provide an algorithm which allows us to find a KU-algebra starting from a given binary block code.*

KEYWORDS: *KU-algebras, Block codes, partially ordered set.*

I. INTRODUCTION

The notion of BCK and BCI-algebras are first introduced by Imai and Iséki [6]. Later on, in 1984, Komori [8] introduced a notion of BCC-algebras, and Dudek ([3], [13]) redefined the notion of BCC-algebras by using a dual form of the ordinary definition in the sense of Komori. Accordingly, Dudek and Zhang [9] introduced a new notion of ideals in BCC-algebras and described connections between such ideals and congruences. Prabpayak and Leerawat [14] introduced a new algebraic structure which is called KU-algebra. They gave the concept of homomorphisms of KU-algebras and investigated some related properties. For more details, see ([13], [14]). Over the last 70 years, algebraic coding has become one of the most important and widely applied aspects of abstract algebra. Coding theory forms the basis of all modern communication systems, and is the key to another area of study, Information Theory, which lies in the intersection of probability and coding theory. Algebraic codes are now used in essentially all hardware-level implementations of smart and intelligent machines, such as scanners, optical devices, and telecom equipment. It is only with algebraic codes that we are able to communicate over long distances, or are able to achieve megabit bandwidth over a wireless channel. Coding theory is the study of methods for efficient and accurate transfer of information from one party to another. Various type of codes and their connections with other mathematical objects have been intensively studied. The idea of coding theory is to give a method of how to convert the information into bits, such that there are no mistakes in the received information, or such that at least some of them are corrected. On this account, encoding and decoding algorithms are used to convert and reconvert these bits properly. In Coding Theory, a block code is an error-correcting code which encodes data in blocks. In the paper [7], the authors introduced the notion of BCK-valued functions and investigate several properties. Moreover, they established block-codes by using the notion of BCK-valued functions. They show that every finite BCK-algebra determines a block-code constructed a finite binary block-codes associated to a finite BCK-algebra. In [5] provided an algorithm which allows to find a BCK-algebra starting from a given binary block code. In [16] the authors presented some new connections between BCK- algebras and binary block codes. Mostafa et al in [10] applied the code theory to KU- algebras and obtained some interesting results. In this paper, we provided an algorithm which allows to find a KU-algebra starting from a given binary block code.

II. Preliminaries

Now, we recall some known concepts related to KU-algebra from the literature, which are helpful in further study of this article.

Definition 2.1([13], [14]). (KU-algebra) Let X be a nonempty set with a binary operation $*$ and a constant 0. The triple $(X,*,0)$ is called a KU-algebra, if for all $x, y, z \in X$ the following axioms are satisfied.

(ku_1) $(x*y)*[(y*z))*(x*z)]=0$,

(ku_2) $x*0=0$,

(ku_3) $0*x=x$,

(ku_4) $x*y=0$ and $y*x=0$ implies $x=y$,

(ku_5) $x*x=0$.

On a KU-algebra X we can define a binary relation $\leq$ on X by putting $x \leq y \Leftrightarrow y * x = 0$. Then $(X, \leq)$ is a partially ordered set and 0 is its smallest element. Thus $(X, *, 0)$ satisfies the following conditions. For all $x, y, z \in X$

(ku_1) $(y * z) * (x * z) \leq (x * y)$,

(ku_2) $0 \leq x$,

(ku_3) $x \leq y, y \leq x$ implies $x = y$,

(ku_4) $y * x \leq x$.

Theorem2.2 ([13], [14]). In a KU-algebra X. The following axioms are satisfied. For all $x, y, z \in X$,

(1) $x \leq y$ imply $y * z \leq x * z$,

(2) $x * (y * z) = y * (x * z)$, for all $x, y, z \in X$,

(3) $((y * x) * x) \leq y$.

Example 2.3. Let $X = \{0, 1, 2, 3, 4\}$ be a set with a binary operation $*$ defined by the following table

*	0	1	2	3	4
0	0	1	2	3	4
1	0	0	0	3	0
2	0	1	0	3	0
3	0	0	0	0	0
4	0	1	0	3	0

Table (1)

Then $(X, *, 0)$ is a KU-algebra.

Definition 2.4[6]. (Sub-algebra) A non-empty subset S of a KU-algebra $(X, *, 0)$ is called KU-sub algebra of X if $x * y \in S$ whenever $x, y \in S$.

Definition 2.5[5]. A KU-algebra(X, ∗, 0) is said to be KU - commutative if it satisfies: ∀x, y∈ X, (y∗ x) ∗ x = (x∗ y) ∗ y.

Definition 2.6[11]. A KU-algebra(X, ∗, 0) is said to be KU -positive implicative, if it satisfies: (z∗ x) ∗ (z∗ y) = z∗ (x∗ y), for all x, y, z in X.

Definition 2.7[11]. A KU-algebra(X, ∗, 0) 0is called KU- implicative if x = (x∗ y) ∗ x, for all x, y in X.

Definition 2.8[14]. (Homomorphism) Let $(X, *, 0)$ and $(X', *', 0')$ be KU-algebras, a homomorphism is a map $f : X \to X'$ satisfying $f(x * y) = f(x) *' f(y)$ for all $x, y \in X$.

Theorem 2.9[14]. Let f be a homomorphism of KU-algebra X into KU-algebra X'. It follows that

(i) If 0 is the identity in X then $f(0)$ is the identity in X'.

(ii) If S is a KU-subalgebra of X then $f(S)$ is a KU-subalgebra of X'.

(iii) If S is a KU-subalgebra of X' then $f^{-1}(S)$ is a KU-subalgebra of X.

Definition 2.10. (Lexicographic) Let the two posets $(S_1, \leq_1)$ *and* $(S_2, \leq_2)$. The lexicographic order $\leq$ on the Cartesian product $S_1 \times S_2$ is defined by specifying that one pair is less than the other pair, i.e. $(x_1, x_2) \leq (y_1, y_2)$ iff $x_1 \leq_1 y_1$ *or* $x_1 = y_1$ *and* $x_2 \leq_2 y_2$

We obtain a partial ordering $\leq$ by adding equality to the ordering $\leq$ on $S_1 \times S_2$.

Example 2.11. (Lexicographic Order) Let $S_1 = \{a, b, c, ...\}$ and $\leq_1$ be the usual alphabetic order. Let $S_2 = \{0,1,2,3,....,10\}$ and $\leq_2$ be the usual alphabetic order be the usual partial order $\leq$, then $(a,8) \leq_1 (b,2)$, $\sin ce$ $a \leq_1 b$ *and* $(a,4) \leq_2 (a,10)$, $a = a$, $4 \leq_2 10$.

Now, we use some results literature from paper **[10]**.

In what follows let A and X denote a nonempty set and a KU-algebra respectively, unless otherwise specified.

Definition 2.12. A mapping $\tilde{A} : A \to X$ is called a KU-valued function (briefly, KU-function) on A.

Definition 2.13. A cut function of $\tilde{A}$, for $q \in X$ is defined to be a mapping $\tilde{A}_q : A \to \{0,1\}$ such that $(\forall x \in A)\tilde{A}_q(x) = 1 \Leftrightarrow \tilde{A}(x) * q = 0$.

Obviously, $\tilde{A}_q$ is the characteristic function of the following subset of A, called a cut subset or a q-cut of $\tilde{A}$.

Example 2.14. Let A ={x, y, z} and let X = {0, a, b, c, d} is a KU-algebra with the following Cayley table:

*	0	a	b	c	d
0	0	a	b	c	d
a	0	0	b	b	a
b	0	a	0	a	d
c	0	0	0	0	a
d	0	0	b	b	0

Table (2)

The function $\tilde{A} : A \to X$ given by $\tilde{A} = \begin{pmatrix} x & y & z \\ a & b & c \end{pmatrix}$ is a KU-function on A, and its cut subsets are

$A_0 = \Phi$, $A_a = \{x\}$, $A_b = \{y\}$, $A_c = A$, $A_d = \{x\}$

Let $x/\Theta = \{ y \in A ;\ x\Theta y \}$; for any $x \in A$, x/Θ is called equivalence class containing x.

Lemma 2.15. Let $\tilde{A} : A \to X$ be a KU- function on A. For every $x \in A$, we have $\tilde{A}(x) = \inf \{x/\Theta\}$, that is $\tilde{A}(x)$ the least element of the Θ to which it belongs.

Definition 2.16. Let $A = \{1,2,3,......, n\}$ and X be a finite KU-algebra. Then every KU-function $\tilde{A} : A \to X$ on A determines a binary block code V of length n in the following way: To every x/Θ, where $x \in A$, there corresponds a codeword $V_x = x_1 x_2 ..... x_n$ Such that

$x_i = x_j \Leftrightarrow \tilde{A}_x(i) = j$ *for* $i \in A$ *and* $j \in \{0,1\}$.

Let $V_x = x_1 x_2 ..... x_n$, $V_y = y_1 y_2 ..... y_n$ be two code words belonging to a binary block-code V.

Define an order relation $\leq_c$ on the set of code words belonging to a binary block- code V as follows:

$V_x \leq_c V_y \Leftrightarrow x_i \leq y_i$ *for* $i = 1,2,....,n$ ……………………………………… (1)

III. Basic Results

Suppose that $(X, \leq)$ be a finite partial ordered set with the minimum element θ . We define a binary relation $*$ on X as follows:

$$\begin{cases} (1) & \theta * x = x, x * x = \theta, \ \forall x \in X, \\ (2) & x * y = \theta \quad if \quad y \leq x, \quad x, y \in X, \\ (3) & x * y = y, \quad otherwise \end{cases} \qquad \text{……………………} (2)$$

Proposition3.1. The algebra $(X,*,\theta)$ is a KU-algebra through the previous notations.

Proof: Conditions (ku_2), (ku_3), (ku_4) and (ku_5) are satisfied. Now, we prove condition (ku_1) that is $(x * y) * ((y * z) * (x * z)) = 0$, for all x, y, z $\in$ X.

Case (1): at least one element is θ.

(1) $x = \theta$; $(\theta * y) * ((y * z) * (\theta * z)) = y * (z * z) = y * \theta = \theta$,

(2) $y = \theta$; $(x * \theta) * ((\theta * z) * (x * z)) = \theta * (z * z) = \theta * \theta = \theta$,

(3) $z = \theta$; $(x * y) * ((y * \theta) * (x * \theta)) = (x * y) * (\theta * \theta) = (x * y) * \theta = \theta$.

Case (2): one element is comparable with another.

(1) $x \leq y$; $(x * y) * ((y * z) * (x * z)) = y * (z * z) = y * \theta = \theta$,

(2) $x \leq z$; $(x * y) * ((y * z) * (x * z)) = y * (z * z) = y * \theta = \theta$,

(3) $y \leq x$; $(x * y) * ((y * z) * (x * z)) = \theta * (z * z) = \theta * \theta = \theta$,

(4) $y \leq z$; $(x * y) * ((y * z) * (x * z)) = y * (z * z) = y * \theta = \theta$.

(5) $z \leq x$; $(x * y) * ((y * z) * (x * z)) = y * (z * \theta) = y * \theta = \theta$,

(6) $z \leq y$; $(x * y) * ((y * z) * (x * z)) = y * (\theta * z) = y * z = \theta$,

Case (3): two elements are comparable with the third.

$x \leq y$ and $z \leq y$; $(x * y) * ((y * z) * (x * z)) = y * (\theta * z) = y * z = \theta$, etc.

Proposition3.2. a KU-algebra (X,*, 0) through the previous notations, is a non-positive implicative algebra.
Proof: we must prove condition in the above definition (2.6), that is $(z * x) * (z * y) = z * (x * y)$, $for \ all \ x, y, z \in X$.

Case (1): at least one element is θ.

(1) $x = \theta$; $(z * \theta) * (z * y) = \theta * y = y$ and $z * (\theta * y) = z * y = y$,

(2) $y = \theta$; $(z * x) * (z * \theta) = x * \theta = \theta$ and $z * (x * \theta) = z * \theta = \theta$,

(3) $z = \theta$; $(z * \theta) * (z * y) = \theta * y = y$ and $z * (\theta * y) = z * y = y$,

Case (2): one element is comparable with another.

(1) $x \leq y$; $(x * y) = y$ and $z * y = y$,

(2) $x \leq z$; $(\theta * y) = y$ and $z * y = y$,

(3) $y \leq x$; $(x * y) = y$ and $z * \theta = \theta$.

We have in Case 2 L.H.S $\neq$ R.H.S ,then a KU-algebra (X,*, 0) is a non-positive implicative algebra.

Proposition3.3. a KU-algebra (X,*, 0) through the previous notations, is an implicative and non-commutative.

We denote a KU-algebra with C_n , if it has n elements. Suppose that V is a binary block code with n codewords of length n, then we have M_v (the related matrix of the code V), where $M_v = (m_{i,j})_{i,j \in \{1,2,....,n\}} \in M_n(\{0,1\})$ with rows containing the codewords of V.

Theorem3.4. Let a matrix M_V be a lower triangular with $m_{ii} = 1, \forall i \in \{1,2,....,n\}$, and $1 x_{ik} ...... x_{in}$; $x_{ik} ...... x_{in} \in \{0,1\}$ in V. through the previous notations, we have a set A with n elements, a KU-algebra X, and a KU-function $f : A \rightarrow X$ such that f determines V.

Proof. We define on V the lexicographic order that denoted by $\leq_{lex}$,so we have (V,$\leq_{lex}$) is a totally ordered set. Suppose that $V = \{w_1, w_2, \ldots, w_m\}$, with $w_1 \leq_{lex} w_2 \leq_{lex} \ldots \leq_{lex} w_n$, then $w_1 = 10 \ldots 00$ (number of zeros are (n-1) times) and $w_n = 1x_{ik} \ldots x_{im}1$; $x_{ik} \ldots x_{im} \in \{0,1\}$. We define also a partial order $\leq$ on V, then $(V, \leq)$ is a partial ordered set with $w_1 \leq w_i$, i∈{1,2,……,n}, then $w_1 = 0$ and w_n is the maximal element in(V). If we define a binary relation $[2]$ on (V, $\leq$) as in Proposition 3.3. We have $X = (V, *, w_1)$ as a KU-algebra and V is isomorphic to C_n as KU-algebras. Then we consider $A = V$ and f: A→V , f (w) = w be the identity map as a KU-function, then f provides a family of binary block code

$V_{c_n} = \{f_r ; A \to \{0,1\};\ f_r(x) = 1$, if and only if $f(x) * r = 0, \forall x \in A, r \in X$.

Suppose that $w_k \in V, 1 \prec k \prec n$, $w_k = 1x_{ik} \ldots x_{iL}0000$, *where* $x_{ik} \ldots x_{iL} \in \{0,1\}$, and number of zeroes are (k-2).

$$\text{If } x_{ij} = \begin{cases} 1 & w_{ij} \leq w_k \to w_k * w_{ij} = 0 \\ 0 & w_k \leq w_{ij} \to w_k * w_{ij} = w_k,\ w_{ij}\ can't\ be\ compared \end{cases}$$

A binary block code as in the previous Theorem can be generated by two or more algebras (see examples 3.5, 3.6, and 3.7). But a KU-algebra generates a unique binary block code using the algorithm in [**10**].

Example 3.5. Let V = {1001, 1100, 1110, 1000} be a binary block code, using the lexicographic order, the code V can be written V = {1000, 1100, 1110, 1001} =$\{w_1, w_2, w_3, w_4\}$. With the following graph

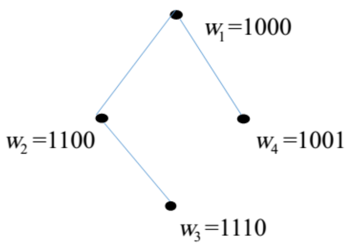

Fig. (1): A graph of code with 5 vertices and 7 edges.

In figure (1): $\{w_1, w_2, w_3, w_4\}$ are the set of vertices and $\{\{w_1, w_2\}, \{w_2, w_3\}, \{w_1, w_4\}\}$ are the set of edges.

By using the previous theorem we define the partial order ≤ on V, then we get $w_1 \leq w_i$, i∈{2,3,4} , $w_2 \leq w_3$, w_2 can't be compared with w_4 , and w_3 can't be compared with w_4 .then the operation [*] on V is defined by the following table:

*	w_1	w_2	w_3	w_4
w_1	w_1	w_2	w_3	w_4
w_2	w_1	w_1	w_3	w_4
w_3	w_1	w_1	w_1	w_4
w_4	w_1	w_2	w_3	w_1

Table (3)

Then, V with the operation [*] is a KU-algebra. The same binary block code V can be obtained from a KU-algebra $(A, \circ, \theta)$

∘	θ	a	b	c
Θ	θ	a	b	c
a	θ	θ	b	c
b	θ	θ	θ	c
c	θ	a	b	θ

Table (4)

With KU-function, f: V → V, f(x) = x.

Example 3.6. Let V = {101000, 110000,101100, 111111, 100000, 111010} be a binary block code. Using the lexicographic order, the code V can be written V = {100000, 110000, 101000, 101100,111010, 111111} $=\{w_1, w_2, w_3, w_4, w_5, w_6\}$ .With the following graph:

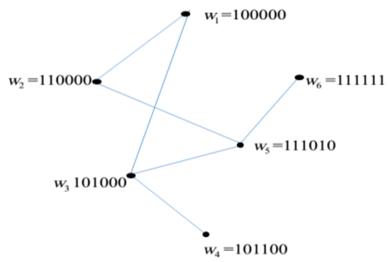

Fig. (2): A graph of code with 6 vertices and 6 edges

In figure (2): $\{w_1, w_2, w_3, w_4, w_5, w_6\}$ are the set of vertices and $\{\{w_1, w_2\}, \{w_2, w_5\}, \{w_1, w_3\}, \{w_5, w_6\}, \{w_3, w_4\}\}$ are the set of edges.

By using the previous theorem we define the partial order $\leq$ on V then we get $W_1 \leq W_i$, i∈ {2, 3, 4, 5, 6}, W_2 Can't be compared with W_3, W_2 can't be compared with W_4, $W_2 \leq W_5$, $W_2 \leq W_6$, $W_3 \leq W_4$, $W_3 \leq W_5$, $W_3 \leq W_6$, W_4 can't be compared with W_5, and $W_4 \leq W_6$ .The operation [*] on V is defined by the following table:

*	w_1	w_2	w_3	w_4	w_5	w_6
w_1	w_1	w_2	w_3	w_4	w_5	w_6
w_2	w_1	w_1	w_3	w_4	w_5	w_6
w_3	w_1	w_2	w_1	w_4	w_5	w_6
w_4	w_1	w_2	w_1	w_1	w_5	w_6
w_5	w_1	w_1	w_1	w_1	w_1	w_6
w_6	w_1	w_1	w_1	w_1	w_1	w_1

Table (5)

Then, V with the operation [*] is a KU-algebra. The same binary block code V can be obtained from a KU-algebra $(B, \circ, \theta)$

∘	θ	a	b	c	d	e
θ	θ	a	b	c	d	e
a	θ	θ	b	c	d	e
b	θ	a	θ	c	d	e
c	θ	a	θ	θ	d	e
d	θ	θ	θ	θ	θ	e
e	θ	θ	θ	θ	θ	θ

Table (6)

With KU-function, f: V → V, f(x) = x.

Example 3.7. Let V = {1010, 1100, 1011, 1000} be a binary block code, using the lexicographic order, the code V can be written V = {1000, 1100, 1010, 1011} $=\{w_1, w_2, w_3, w_4\}$. With the following graph

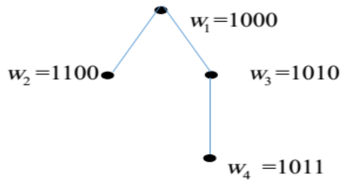

Fig. (3): A graph with 4 vertices and 3 edges.

In figure (3): $\{w_1, w_2, w_3, w_4\}$ are the set of vertices and $\{\{w_1, w_2\}, \{w_1, w_3\}, \{w_3, w_4\}\}$ are the set of edges.

By using the previous theorem we define the partial order ≤ on V, then we get $W_1 \le W_i$,i∈{2,3,4}, W_2 can't be compared with W_3 , W_2 can't be compared with W_4 ,and $W_3 \le W_4$ .The operation [*] on V is defined by the following table:

*	w_1	w_2	w_3	w_4
w_1	w_1	w_2	w_3	w_4
w_2	w_1	w_1	w_3	w_4
w_3	w_1	w_2	w_1	w_4
w_4	w_1	w_2	w_1	w_1

Table (7)

Then, V with the operation [*] is a KU-algebra.
The same binary block code V can be obtained from a KU-algebra $(C, \circ, \theta)$

∘	θ	a	B	c
θ	θ	a	B	C
a	θ	θ	B	c
b	θ	a	Θ	c
c	θ	a	Θ	θ

Table (8)

With KU-function, f: V → V, f(x) = x.

Proposition 3.8. Suppose that $C = (c_{i,j})_{\substack{i\in\{1,2,\ldots,n\} \\ j\in\{1,2,\ldots,m\}}} \in M_{n,m}(\{0,1\})$ is a matrix with rows lexicographic ordered in the ascending sense, so there is a matrix $S = (s_{i,j})_{i,j\in\{1,2,\ldots,q\}} \in M_q(\{0,1\})$, $q = n + m$, such that S is a lower triangular matrix, with $S_{ii} = 1, \forall i \in \{1,2,\ldots, q\}$ and C becomes a sub matrix of the matrix S.

Proof. Suppose that we add in the right side of the matrix C (from the left to the right) the new rows of the form $\underbrace{10\ldots00}_{n}$, $\underbrace{01\ldots00}_{n}$,........, $\underbrace{00\ldots01}_{n}$, so we have a new matrix S with n + m columns and n rows. Suppose that we add in the top of the matrix S the following n rows: $\underbrace{00\ldots0}_{n}\underbrace{10\ldots00}_{m}$, $\underbrace{00\ldots0}_{n+1}\underbrace{01\ldots00}_{m-1}$,......... , $\underbrace{000\ldots\ldots}_{n+m-1}1$ .We get the required matrix C.

Theorem 3.9. Let V be a binary block code with n codewords of length m , n≠m, or a block-code with n codewords of length n such that $1x_{ik} \ldots\ldots x_{in}$; $x_{ik} \ldots\ldots x_{in} \in \{0,1\}$ is not in V, or a block-code with n codewords of length n such that the matrix M_V is not lower triangular. Then Through the previous notations, we have a natural number q ≥ max {m, n}, a set A with m elements and a KU function f: A →Cq such that the obtained block code V_{Cn} contains the block code V with 1s as a first digit in its codewords .

Proof. Suppose that V=$\{w_1, w_2, w_3, w_4\}$, be a binary block code, with codewords of length m. We consider the codewords $w_1, w_2, \ldots\ldots, w_n$ lexicographic ordered $w_1 \leq_{lex} w_2 \leq_{lex} \ldots\ldots \leq_{lex} w_n$. Suppose that $M \in M_{n,m}(\{0,1\})$ be the associated matrix of V with the rows $w_1, w_2, \ldots\ldots, w_n$ in this order. By using Proposition 3.8, we lengthen the matrix M to a square matrix $M' \in M_q(\{0,1\})$, $q = n + m$, such that $M' = (m'_{i,j})_{i,j \in \{1,2,\ldots,q\}}$ is a lower triangular matrix with $m_{ii} = 1$, , for all i∈{1,2,...,q}.If the first column of the matrix M' is not 11….1(q-times), then we insert the column 11…..1(q+1 times) as a first column and the row 10….0 (number of zeroes =q-times) as a first row. Applying Theorem 3.2 for the matrix M', we obtain a KU-algebra $C_q = \{x_1, x_2, \ldots\ldots, x_q\}$,with $x_1 = \theta$ the zero of the algebra C_q and a binary block code V_{cq} . Supposing that the columns of the matrix M have in the new matrix M' with 1s as a first digit, so $A = \{x_{1j}, x_{2j}, \ldots\ldots, x_{mj}\} \subseteq C_q$. The KU-function $f : A \to C_q$, $f(x_{ij}) = x_{ij}$, $i \in \{1,2,\ldots\ldots, m\}$, determines the binary block-code V_{cq} such that the code V_{cq} contains the block code V with 1s as a first digit in its codewords.

Example 3.10. Let V = {01101, 00001, 00101, 01111} be a binary block code. By using the lexicographic order, the code V can be written V = {00001, 00101, 01101, 01111} =$\{w_1, w_2, w_3, w_4\}$. We organize the codewords in the associated matrix M, such that $M_V \in M_{4,5}(\{0,1\})$. we get,

$$M_V = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

By using proposition 3.8, we create a lower triangular matrix.

$$F = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \end{pmatrix} \text{ and}$$

$$S = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

The rows of the matrix S doesn't begin with 1.by using proposition 3.8, we add 11…..1(10-times) as a first column and the row 10….0 (number of zeroes is 9-times) as it don't exist in the first row of the matrix S. so we get the following:

$$S' = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

The binary block code $W = \{w_1, w_2, ..., w_{10}\}$, whose codewords are the rows of the matrix S' , determines a KU-algebra (X,*, w_1).

Suppose that $A = \{w_1, w_2, w_3, w_4, w_5, w_6\}$ and $f : A \to X$, $f(w_i) = w_i$,i∈{1,2,3,4,5,6} be a KU – function which determines the binary block code U= {100001, 110000, 101000, 100100, 100010, 100001, 100001,100101,101101,101111}.We have the code V included in the code U but it contains 1s as a first digit .

IV. Relationship between the ordered relation on KU-algebra and partial ordered set

Definition 4.1. Suppose that $(P, \leq)$ is a partially ordered set. For $q \in P$, we define a mapping $P_q : P \to \{0,1\}$ such that for each $b \in P$, we have $P_q(b) = 1$ if and only if $q \geq b$, Using this map, a codeword $v_x = x_1 x_2 .... x_n$ of a binary block-code V can be determined as follow: $x_i = J$ if and only if $P_x(i) = j$, *for* $i \in S$ *and* $j \in \{0,1\}$.

From a given partially ordered set we catch binary block codes as showing in the following examples:

Example 4.2. Suppose that P = {0, 1, 2, 3} is a set with a partial order over P as presented in the following figure (4)

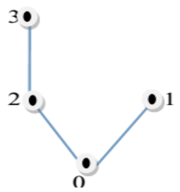

Fig. (4): partial order ($P, \leq$)

We catch the following table by using definition 4.1:

P_p	0	1	2	3
0	1	0	0	0
1	1	1	0	0
2	1	0	1	0
3	1	0	1	1

Table (9)

From table (9) we get the following code $V_1 = \{1000 \quad ,1100 \quad ,1010 \quad ,1011 \quad \}$, and we get following figure (5) from code V_1.

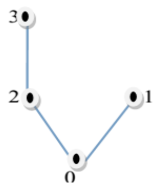

Fig. (5): order relation $(V_1, \leq_C)$

Example 4.3. Suppose that P= {0, 1, 2, 3, 4, 5} is a set with a partial order over P as presented in the following figure (6)

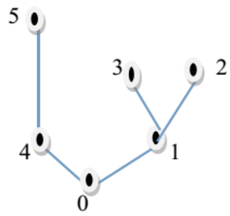

Fig. (6): partial order ($P, \leq$ **)**

We catch the following table by using definition4.1:

P_p	0	1	2	3	4	5
0	1	0	0	0	0	0
1	1	1	0	0	0	0
2	1	1	1	0	0	0
3	1	1	0	1	0	0
4	1	0	0	0	1	0
5	1	0	0	0	1	1

Table (10)

From table (10) we get the following code $V_2 = \{100000 \quad ,110000 \quad ,111000 \quad ,110100 \quad ,100010 \quad ,100011 \quad \}$, and we get following figure (7) from code V_2.

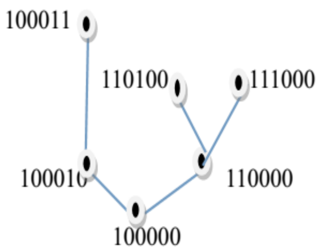

Fig. (7): order relation ($(V_2, \leq_C)$ **)**

Now, we generate binary block codes from KU-algebras by using definition 2.16 through the following examples.

Example 4.4. Suppose that X = {0, 1, 2, 3} is a KU-algebra and we represent the order on X as shown in figure (5).

*	0	1	2	3
0	0	1	2	3
1	0	0	2	3
2	0	1	0	3
3	0	1	0	0

Table (11)

Suppose that $\tilde{A} : X \to X$ is a KU-function on X given by $\tilde{A} = \begin{pmatrix} 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 3 \end{pmatrix}$. So, we have the following table.

$\tilde{A}_x$	0	1	2	3
$\tilde{A}_0$	1	0	0	0
$\tilde{A}_1$	1	1	0	0
$\tilde{A}_2$	1	0	1	0
$\tilde{A}_3$	1	0	1	1

Table (12)

We observe in table (12) the binary block code $V_3 = \{1000, 1100, 1010, 1011\}$ that equal the code V_1 in example 4.2.

Example 4.5. Suppose that X = {0, 1, 2, 3, 4, 5} is a KU-algebra and we represent the order on X as shown in figure (7).

*	0	1	2	3	4	5
0	0	1	2	3	4	5
1	0	0	2	3	4	5
2	0	0	0	3	4	5
3	0	0	2	0	4	5
4	0	1	2	3	0	5
5	0	1	2	3	0	0

Table (13)

Suppose that $\tilde{A} : X \to X$ is a KU-function on X given by $\tilde{A} = \begin{pmatrix} 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 3 \end{pmatrix}$. So, we have the following table.

$\tilde{A}_x$	0	1	2	3	4	5
$\tilde{A}_0$	1	0	0	0	0	0
$\tilde{A}_1$	1	1	0	0	0	0
$\tilde{A}_2$	1	1	1	0	0	0
$\tilde{A}_3$	1	1	0	1	0	0
$\tilde{A}_3$	1	0	0	0	1	0
$\tilde{A}_3$	1	0	0	0	1	1

Table (14)

We observe in table (12) the binary block code $V_3 = \{1000, 1100, 1010, 1011\}$ that equal the code V_1 in example 4.3.

We define a KU-algebra structure on a poset with 0 element and we have a code in Example 4.2 similar to the code in Example 4.4, also we have a code in Example 4.3 similar to the code in Example 4.5. The clear is that we use the order of KU-algebra only, not its properties. From the previous examples, we deduce that there is a one to one correspondence between the ordering relation $\leq$ and order relation $\leq_c$.

Proposition 4.6. There is a one to one correspondence between the ordered relation on KU-algebra and partial ordered set.

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