International Journal of Engineering Science Invention
ISSN (Online): 2319 – 6734, ISSN (Print): 2319 – 6726
www.ijesi.org ||Volume 5 Issue 5|| May 2016 || PP.77-83

Radial Heat Transport in Packed Beds-III: Correlations of Effective Transport Parameters at High Pressure

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ABSTRACT: *The reliability and accuracy of experimental with predictions data of two models ("MC model" Marshall and Coberly model, [1] and modified model by Ibrahim et al. [2] are investigated for the effective radial thermal conductivity (K_{er}), and the wall heat transfer coefficient (h_w) in packed beds in the absence of chemical reactions. The results were evaluated by the modified mathematical model as to the boundary bed inlet temperature; (T_o) number of terms of the solution series and number of experimental points used in the estimate. Very satisfactory was attained between the predicted and measured temperature profiles for a range of experiments. These cover a range of tube to (equivalent) particle diameter ratios from d_t/d_p = 4 to 10; Reynolds numbers ranged between 3.8-218 for particle, and elevated pressure from 11 to 20 bar for particle catalyst pellets. In all cases the fluid flowing throughout the bed has been air. The results indicate to the choice of the inlet boundary condition can have a large impact on the values of obtained parameters. And model parameters have been shown to be dependent on the pressure inside the reactor.*
The following correlations for both (h_w) and (K_{er}) respectively under a given conditions obtained by using multiple regressions of our results that based on the modified mathematical model:
$Nu_w = 67.9Re^{0.883}(d_t/d_p)^{-0.635}(P/P_o)^{-1.354}$
$K_{er} = 0.2396 + 0.0041Re$
The results accuracy of these correlations obtained from the modified mathematical model are more than the results accuracy of correlations obtained from MC model with respect to experimental data; these accuracy of both correlations reach up to 91% and 65% for (h_w) and (K_{er}) respectively; which these results indicate to the reliability of the modified mathematical model to obtained the parameters of heat transport in packed bed reactors.
Keywords: *Heat, Packed Beds, Pseudo-Homogeneous Model, Parameters Correlations, Pressure Effect*

I. Introduction

Diabatic packed bed tubes with fluid flow are frequently used in industries as adsorbers or chemical reactors. A major problem with such equipments is runway to very high or low temperatures. It is important to prevent these and order to develop such equipment reliable prediction of heat transfer in the randomly arranged packed bed is necessary to study the behavior of such equipments by computer simulation. For this purpose a good mathematical model of packed beds is needed, capable of predicting the temperature and concentration profiles in the packed bed specially the reactors, given a certain reactor design and operating conditions. Here the description of heat transport in wall-heating packed beds at low tube-to-particle diameter ratio, using the pseudo-homogeneous two dimensional continuum models is discussed.

II. Models Equations

The system studied is gas flow (air), heating down in a wall-heating packed tube packing with a pellet catalyst [3]. To model the heat transport in the experimental equipment as used, we made the following assumptions:

1. The system is in steady state and no chemical reaction carried out.
2. The system is considered to be pseudo-homogeneous.
3. There is no axial dispersion of heat.
4. There is no free convection or radiation of heat.
5. There is no variation of superficial gas velocity.
6. The pressure is constant throughout the packed bed.
7. The physical properties of the gas and solid are independent of temperature.

A heat balance an infinitesimally small ring yields the following pseudo-homogeneous two dimensional models:

$$GCp \frac{\partial T}{\partial z} = K_{er} \left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \left(\frac{\partial T}{\partial r} \right) \right) \qquad \text{........ (1)}$$

Several studies have been presented on gas flow discussing experimental, statistical and theoretical aspects depending on equation (1) and the results are quite conclusive [3-9]. These studies usually contain effective heat transfer coefficients (K_{er} and h_w), which are preferably obtained from experimental at atmospheric pressure.

In the available literature the known methods for simultaneous estimation of both values considered. The theoretical basis of all those methods was formed on the integration of Eqn. (1) for the following boundary conditions:

$$z = 0, \quad \text{all } r, \; T = T_o \qquad \text{........ (2)}$$

$$r = 0, \quad \text{all } z, \; \frac{\partial T}{\partial r} = 0 \qquad \text{........ (3)}$$

$$r = R_t, \quad \text{all } z, \; -K_{er} \frac{\partial T}{\partial r} = h_w (T_w - T) \qquad \text{........ (4)}$$

In dimensionless form; Eqn. (1) becomes:

$$\frac{\partial T'}{\partial \zeta} = \alpha' \frac{1}{r'} \frac{\partial}{\partial r'} \left(r' \frac{\partial T'}{\partial r'} \right) \qquad \text{........ (5)}$$

The boundary conditions in Eqns. (2), (3) and (4) respectively become:

$$\zeta = 0, \quad \text{all } r', \; T' = 1 \qquad \text{........ (6)}$$

$$r' = 0, \quad \text{all } \zeta, \; \frac{\partial T'}{\partial r'} = 0 \qquad \text{........ (7)}$$

$$r' = 1, \quad \text{all } \zeta, \; -\frac{\partial T'}{\partial r'} = \beta_i T' \qquad \text{........ (8)}$$

The integration of Eqn. (5) was performed on the assumption that the wall temperature (T_w) is uniform and also the inlet gas temperature (T_o) be uniform over the whole inlet cross section of the bed.
Marshall and *Coberly* [1] have first made use of the equation:

$$T'(r', \zeta) = 2 \sum_{n_j}^{\infty} \frac{J_o\left(r'\lambda_{n_j}\right) exp\left(-\alpha' r' \lambda_{n_j}^2\right)}{\lambda_{n_j} J_1\left(\lambda_{n_j}\right) \left[\left(\lambda_{n_j} / \beta_i\right)^2 + 1\right]} \qquad \text{........ (9)}$$

Eqn. (9) result of integration of Eqn. (5) for boundary conditions (6), (7) and (8) and on the assumption the bed depth is fairly high. J_o and J_1 are the Bessel functions of the first kind of order zero and one, respectively, and (λ_{n_j}) are the roots to:

$$\lambda_{n_j} J_1(\lambda_{n_j}) = \beta_i J_o(\lambda_{n_j}) \qquad \text{........ (10)}$$

The temperature (T') in these equations is made dimensionless according to:

$$T' = \frac{T_w - T_{r,z}}{T_w - T_o} \qquad \text{........ (11)}$$

In Eqn. (11) (T_o) is the inlet temperature of the gas at the bed entrance, taken constant over the radius. In the experimental set-up this temperature is taken as the average of three temperatures at the bed entrance, measured at three different radial locations of $r' \approx 0$, 0.39 and 0.79, respectively [3].

It is also possible to take the first measured radial temperature profile in the packed bed as the inlet temperature profile and use the location of this profile as the bed entrance point. If this radial temperature profile is described by a polynomial function [2, 10, 11] the following model equations are found:

$$\theta(r',\delta) = 2\sum_{n_j=1}^{\infty}\left[\frac{\left(A\lambda_{n_j}^4 - B\lambda_{n_j}^2 + C\,\frac{\lambda_{n_j}^4}{\beta_i} + (9d - b\lambda_{n_j}^2)\,\frac{0.91973}{J_1(\lambda_{n_j})}\right) J_o(\lambda_{n_j} r')}{\lambda_{n_j}^5 J_1(\lambda_{n_j})\left[(\lambda_{n_j}/\beta_i)^2 + 1\right]}\exp(-\alpha'\lambda_{n_j}^2\delta)\right] \quad \text{........ (12)}$$

where:
$A = (a+b+c+d)$
$B = (4c+9d)$
$C = (b+2c+3d)$
λ_{n_j} is found from Eqn. (10).

Eqn. (12) is solved with modified boundary conditions that may be writing as:

$$\theta(r') = a + b(r') + c(r')^2 + d(r')^3 \quad \text{at} \quad \delta = 0 \quad \text{for all } r' \text{ values.} \quad \text{........ (13)}$$

$$\frac{\partial\theta}{\partial r'} = 0 \quad \text{at} \quad r' = 0, \text{ for all } \delta \text{ values .} \quad \text{........ (14)}$$

$$\frac{\partial\theta}{\partial r'} = -\beta_i\,\theta \quad \text{at} \quad r' = 1, \text{ for all } \delta \text{ values.} \quad \text{........ (15)}$$

In these Equations θ and δ are given by:

$$\theta = \frac{T'}{T_o'} \quad \text{and} \quad \delta = \frac{\zeta - \zeta_o}{1-\zeta_o} \quad \text{........ (16)}$$

Eqn. (12) describes the axial and radial temperature profiles with two model parameters α' and β_i containing the effective radial heat conductivity K_{er} and the wall heat transfer coefficient h_w respectively. Values for K_{er}, h_w, a, b, c and d, for the two models are obtained by fitting Eqns. (9), (12) and (13) to experimentally determined temperature profiles [10].

III. Experimental Set-Up

The set-up to measure radial and axial temperature profiles contains the heat exchanger reactor tube constructed from stainless steel. The tube reactor are filled with a packing and heating from the wall with steam, flowing through a jacket, flows upwards through the tube reactor and is heating down at the wall. The lower section or calming section (50.88 *mm* inside diameter, and 360 *mm* long) was surrounded by a jacket through which a steam was passed through the jacket under pressure. Stainless steel rasching rings 5.56 *mm* diameters were packed in order to obtain uniform velocity and temperature profile to the set section. The two sections are separated from each other by a gasket of low thermal conductivity (insulating rubber gasket) in order to minimize any heat escape by conduction from test section to the calming section through the tube wall. The test section (50.88 *mm* inside diameter and 750 *mm* in length) contains a stainless steel jacket, in which the steam was passed through the jacket under pressure to serve heating the reactor tube. In the steady state, the radial temperature profile is measured in the packed bed by eight thermocouples (0.5 *mm* diameter) were placed above the packing at different distance from the center of the cross through the middle of hypodermic tube. A ninth thermocouple was attached to the stainless steel tube holder at the wall to record the inside wall temperature reactor. The temperature of the air at the inlet of the packed bed, the pressure before and after the bed and the air flow rate are also measured. During the measurements, the bed can easily to repacked as appropriate to take into account the angular temperature fluctuations. Combination of the averaged radial temperature profiles, as obtained for different axial positions at the same experimental conditions, yields temperature field for the whole tube. All experiments reported here have been performed with air at high pressure (11 and 20.7 *bar*). For more detailed description of the set-up and the measuring procedure [3].

IV. Choice Of The Inlet Boundary Condition

By using Eqns. (9) and (12) it is possible to calculate temperature profiles for wall-heating packed bed, with flat or polynomial shaped inlet temperature profile, respectively. In Figures (1) examples of such profiles are given for arbitrary chosen values of the model parameters. In these figures the difference in temperature profiles can be seen. Also it can be seen that the temperature at the wall of the tube approximately identical with experimental readings in case a polynomial inlet temperature profile used, whereas it does not do so for the model with a flat inlet temperature profile (*MC* model), for the same conditions.

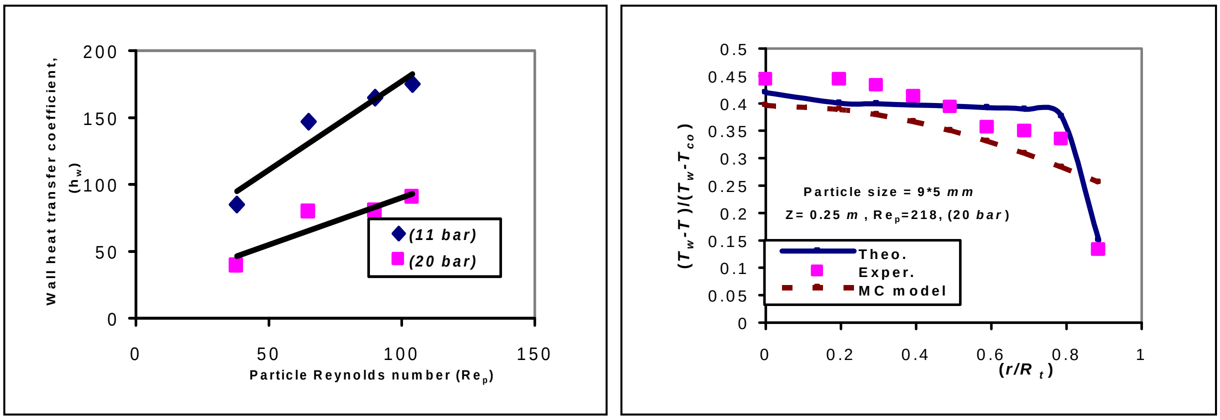


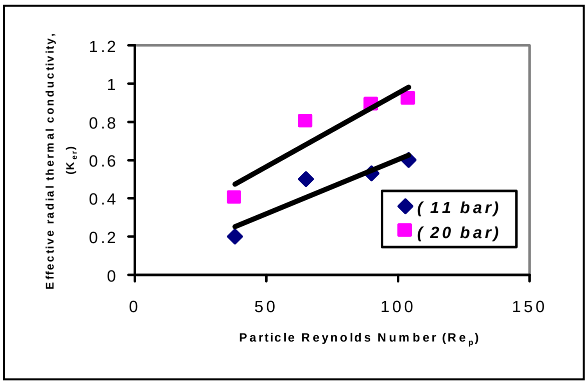
Figure (1): Radial temperature distribution for a flat and polynomial shaped inlet temperature profile and comparison them with experimental data for arbitrary conditions

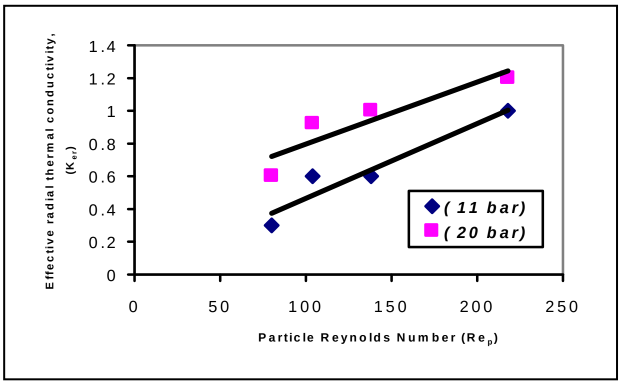
These deviations of temperatures profiles between two models with respect to experimental data reflecting on the variation of the parameters values extracted from these models when fitting with experimentally determined temperature profiles. This deviation resulting from the change of the inlet boundary condition. For more discussion of the influence of this choice of the inlet boundary condition [2].

V. Results And Discussion

5.1 Correlations for the Effective Transport Coefficients

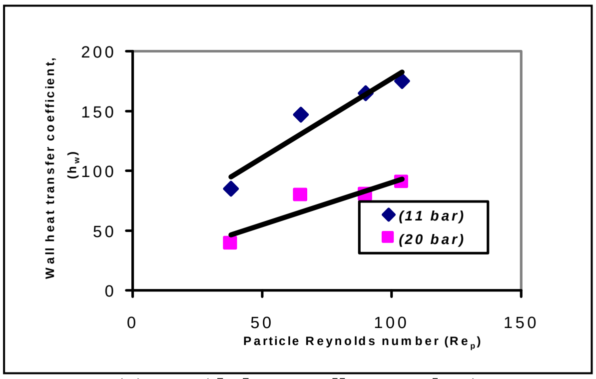
Values for the K_{er} and h_w, for two models, were obtained by fitting Eqns. (9) and (12) to experimentally determine temperature profiles. In Figures (2) and (3) values K_{er} and h_w are obtained for different gas flow rates, pressure and packing diameter evaluated by using Eqn. (12) (modified mathematical model).

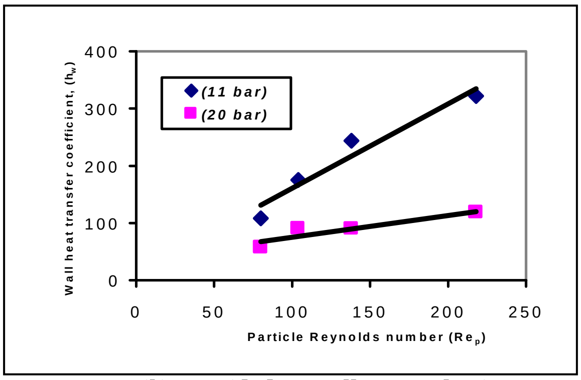




(*a*) For (d_t/d_p=10 pellets catalyst) **(*b*) For (d_t/d_p=4 pellets catalyst)**

Figure (2): Variation of the Effective Radial Thermal Conductivity with Particle Reynolds number and elevated of pressure.





(*a*) For (d_t/d_p=10 pellets catalyst) **(*b*) For (d_t/d_p=4 pellets catalyst)**

Figure (3): Variation of the Wall Heat transfer coefficient with Particle Reynolds number and elevated of pressure.

Other pellet packing sizes investigated, from these figures it can be seen that the pressure effect largely the values of K_{er} and h_w also the particles diameter have affect on these values. So; by taking into our accounts these conditions and their effects on these parameters to obtain general correlations of them. The correlations obtaining by using the multiple regressions of the values of K_{er} and h_w that extracted from Eqn. (12) (modified mathematical model) illustrate as Eqns. (17) and (18). Whereas the confidence intervals are 91% and 65% for both equations respectively:

$$Nu_w = 67.91\, Re_p^{0.883} \left(\frac{d_t}{d_p}\right)^{-0.635} \left(\frac{P}{P_o}\right)^{-1.354} \quad \text{……..} \quad (17)$$

$$K_{er} = 0.2393 + 0.0041\, Re \quad \text{……..} \quad (18)$$

Also, by using the same procedure of regression, correlate the values of the K_{er} and h_w extracted from Eqn. (9), (*MC* model) illustrate at Eqns. (19) and (20); with confidence intervals reach up to 74.2% and 15% for both equations respectively:

$$Nu_w = 6.41\, Re^{1.699} \left(\frac{d_t}{d_p}\right)^{-0.197} \left(\frac{P}{P_o}\right)^{-2.4854} \quad \text{……..} \quad (19)$$

$$K_{er} = 0.4947 + 0.0018\, Re \quad \text{……..} \quad (20)$$

All these correlations obtained for air flow through the packed bed and at the same criterion conditions:
($38 < Re < 218$), ($10 < P < 20$ bar) and ($4 < (d_t/d_p) < 10$).

Together with the correlations for K_{er} and h_w as given above, Eqns. (17) and (18) together and Eqns. (19) and (20) together also, obtained from modified mathematical model and *MC* model respectively for all the available data. For more discussions of these correlations and other results [11].

5.2 Effect of the Inlet Boundary Condition

In the previous section we assumed the radial gas inlet temperature to be ploynomially shaped, (Eqn. (13). However, often the inelt temperature condition used is uniform in the initial cross section of the bed (Eqn.(6)) is generally not valid; for there always exists a tempearture gradient along the radius, which especially large near the wall [2, 12, 13]. In order to fufill this condition as best as possible it is necessary to consider the initial profile as the one existing just at the entrance to the bed; (for a more general discussion of this condition presented by *Ibrahim et al.* [2].

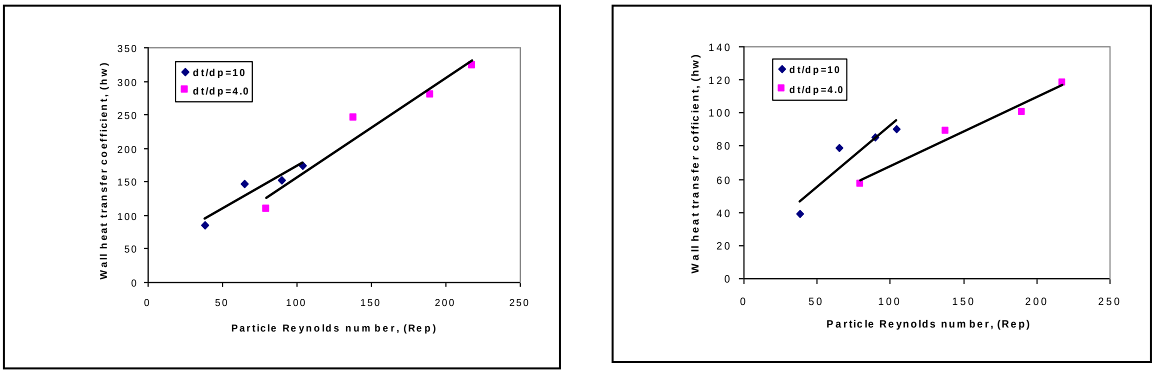
The expermintal measurments of the radial temperature profiles above the bed for its different and the data of the radial and axial temperature profiles resulting of *MC* model, are the reason of further uncertinly in estimation of K_{er} and h_w values and make the model rather time-consuming [11, 13]. At the same time the results indictes to the good temperature profiles obatined from tme modified mathematical model with respect to expermintal data.

5.3 Effect Of High Pressure

From Figure (3), wenote that; the effect of pressure greater than atmospheric on the readial themal conducitivty (K_{er}) at different particle Reynolds number, the plots suggest that the radial thermal conductiovity is a strong effects increased with increasing pressure. Also from Figure (2), the wall heat transfer coefficients (h_w) increased by the increase of particle Rynolds number at the same pressure. Also, at the same Ryenolds number the wall heat transfer coefficient decreased by the increase of pressure. This can be explained due to the variation of gas properties with respect to elevated pressure, (More discussion of the effect of elevated pressures on the parameters of heat transfer in packed beds [3].

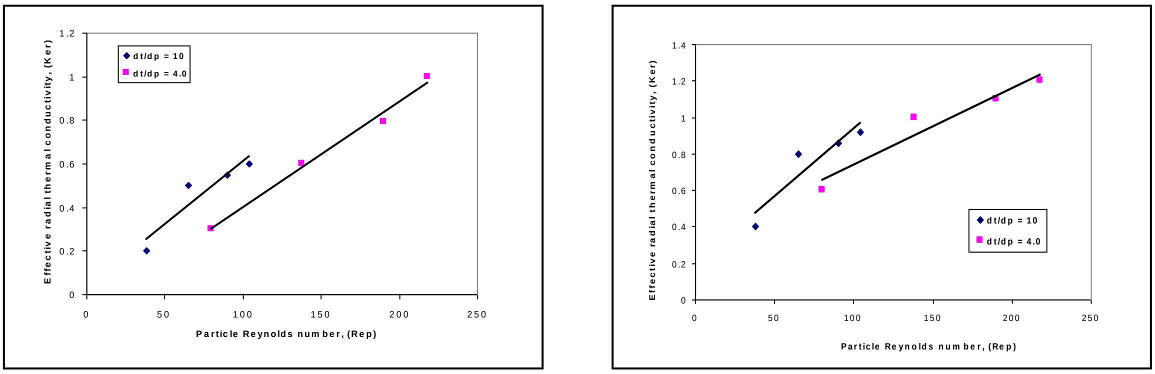
5.4 Effect of Radial Velocity Profile

The results are plotted in Figure (4), it can be noted that the influence of tube-particle diameter ratio (d_t/d_p) upon the the wall heat transfer coefficient (h_w), whereas, an increase in the ratio of d_t/d_p leads to decrease particle Reynolds number followed by decreasing in the wall heat transfer coefficient. The reason for this influence is the increase in contact area between the pellets and the tube wall. An increase in the ratios of d_t/d_p means that more pellets are adjacent to the tube wall, and thus givinig a greater area for heat transfer by pellet conduction and a smaller area for heat transfer by gas convection. So, the effective radial thermal conductivity (K_{er}), will be increaseing with increase of d_t/d_p, generally this effects may be noted in Figure (5).



(*a*) For (*P*= 11 *bar*) **(*b*) For (*P*=20 *bar*)**

Figure (4): Effect of tube-particle diameter ratios upon the wall heat transfer coefficient of catalyst pellets at elevated pressure



(*a*) For (*P*= 11 *bar*) **(*b*) For (*P*=20 *bar*)**

Figure (5): Effect of tube-particle diameter ratios upon the effective radial thermal conductivity of catalyst pellets at elevated pressure

VI. Conclusions

One should be careful in chossing the inlet boundary condition, when modelling the heat transport in packed beds. If the inelt temperature profile is assumed to be radially flat, whereas the actual profile is curved, an apparent dependence of the heat transports coefficents on the inlet boundary condition of the temperature. The effective radial heat conductivity and the wall heat transfer coefficient are found to be strongly affected by the pressures throughout the bed. The results accuracy of the correlations obtained from the modified mathematical model are more than the results accuracy of correlations obtained from *MC* model with respect to experimental data; which these results indicate to the reliability of the modified mathematical model to obtained the parameters of heat transport in packed bed reactors.

Nomenclature

A_p	External surface area of a pellet.	(m^2)
a,b,c and d	Constants of Equation (13) determined from the first experimentally temperature profiles.	*(---)*
Cp	Specific heat at constant pressure of the gas.	$(J\ kg^{-1}\ K^{-1})$
d_t	Tube diameter.	*(m)*
d_p	Equivalent diameter of particle,($6V_p / A_p$).	*(m)*
G	Fluid mass velocity per unit area.	$(kg\ m^{-2}\ sec^{-1})$
h_w	Wall heat transfer coefficient.	$(W\ m^{-2}\ K^{-1})$
J_o ,J_1	Zeroth-order and first order Bessel functions of the first kind.	*(--)*
K_{er}	Effective radial thermal conductivity.	$(W\ m^{-1}\ K^{-1})$
L	Length of the bed.	*(m)*
P	Pressure through packed bed.	*(bar)*
P_o	Atmospheric pressure.	*(bar)*
R_t	Tube radius.	*(m)*
R	Radial coordinates.	*(m)*

$T_{r,z}$	Temperature at any position through the bed..	*(^oC)*
T_w	Wall temperature tube.	*(^oC)*
T_o	Inlet temperature to the reactor.	*(^oC)*
$T_{c.}$	Center temperature of the first experimentally determined radial temperature profile.	*(^oC)*
V_p	Volume of particle.	*(m^3)*
Z	Axial coordinate (bed depth).	*(m)*
z_o	The axial position of the first experimentally determined radial temperature profile.	*(m)*
μ_g	Dynamic viscosity of the gas.	*(Pa.sec)*

List Of Dimensionless Group Numbers:

$T' \quad = (T_w - T)/(T_w - T_o)$ (--)

β_i Biot number = ($(h_w * R_t)/K_{er}$). (--)

$r' \quad = r/R_t$. (--)

$\zeta \quad = z/L$. (--)

$$\alpha' = \frac{K_{er} * L}{G * Cp * R_t^2} .$$ (--)

$$\theta = \frac{(T_w - T)/(T_w - T_o)}{(T_w - T_C)/(T_w - T_o)} = \frac{T_w - T}{T_w - T_c} .$$ (--)

$$\delta = \frac{\frac{Z}{L} - \frac{Z_o}{L}}{1 - \frac{Z_o}{L}} = \frac{\zeta - \zeta_o}{1 - \zeta_o} .$$

(--)

R_{ep} Particle Reynolds number = ($\frac{Gd_p}{\mu_g}$). (--)

References

[1]. Coberly C.A., and Marshall W.R.; *Temperature gradient in gas stream flowing through fixed granular beds*; Chem. Engng. Prog.; 47; 141-150 (1951).

[2]. Ibrahim H.G.; Al-Meshragi M.A.; Alshuiref A.A.; Alagta A.R.; and Edali M.A.; *Radial heat transport in packed beds-II: Mathematical modeling of heat transfer through packed beds at elevated pressure*; Int. J. of Engng. and Applied Scis., 4(2); 24-36 (2013).

[3]. Al-Meshragi M.; Elakrami H. A.; and Ibrahim H.G. (2009); *Radial heat transport in packed beds-I: Experimental investigation of heat transfer coefficients of pellets and monolith catalysts at atmospheric and high pressure*; J. of Chemistry and Chemical Engng.; 3(6); 1-14.

[4]. Gunn D.G., and Khalid M.; *Thermal dispersion and wall heat transfer in packed beds*; Chem. Engng. Sci., 30; 261-267 (1975).

[5]. Wellauer T.; Cresswell D.L., and Newson E.J.; *Heat transfer in packed bed reactor tubes suitable for selective oxidation*; ACS Symp. Ser. No. 169, 527-542 (1982).

[6]. Tsotsas E., and Schlünder E.U.; *Heat transfer in packed beds with fluid flow: remarks on the meaning and the calculation of a heat transfer coefficient at the wall*; Chem. Engng. Sci.; 45; 819 (1990).

[7]. Freiwald M.G., and Paterson W.R.; *Accuracy of model predictions and reliability of experimental data for heat transfer in packed beds*; Chem. Engng. Sci.; 47; 1545-1560 (1992).

[8]. Jorge L.M.M., Jorge R.M.M., Fujii, F., and Giudici R.; *Evaluation of heat transfer in a catalytic fixed bed reactor at high temperature*; Braz. J. Chem. Engng.; 16; 1-19 (1999).

[9]. Thomeo J.C., and Freire J.T.; *Heat transfer in fixed bed: a model non-linearity approach*; Chem. Engng. Sci.; 55; 2329-2338 (2000).

[10]. Ibrahim H.G.; *Wall Heat Transfer Modeling in Fixed Bed Reactor at High Pressure*; LAP LAMBERT Academic Publishing GmbH & Co. KG, Germany (2010).

[11]. Ibrahim H.G.M; *Wall heat transfer modeling in fixed bed reactor at high pressure*; M.Sc. Thesis; Al-Mergheb University; Al-Khoms; Libya (2005).

[12]. Ziolkowski D.; *Effective thermal conductivity and wall heat transfer coefficient*; Acad. Polon. Sci.; 14; 221-226 (1970).

[13]. Thomeo J.C.m, and Grace J.R.; *Heat transfer in packed beds: experimental evaluation of one-phase water flow*; Braz. J. Chem. Engng.; 21; 1-15 (2004).