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Quantitative Digit in a Decoupled Positional System: A New Method of Understanding and Teaching Numbers

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Abstract: *The aim of this paper is to propose a new method of understanding and teaching numbers as spatial-temporal experience based on rational elements and components. The method contributes to the solution of the problem that with traditional methods of number representation much of their "inner life" remains hidden, inaccessible to learners. This requires an innovative approach in rethinking on the very basic level of the number system: numerical digits and their organization. The two main ideas of our discovery are: 1) quantitative representation of digits, and 2) decoupling the adjacent positions in a positional numeral system by means of rational phases.*
Keywords: *mathematical semiotics, quantitative digit, rational numeral*

I. Introduction

Numbers have a rich inner life, which is largely hidden in the traditional representation. The hidden properties relate to the exponents of the adjacent unique powers of two, i. e. components that every natural number is composed of, specifically 1) the position parity of the components, indicating if the exponent is even or odd, 2) the delta parity of the components, indicating whether the difference between the adjacent exponents is even or odd, 3) the logic of the component forms, indicating whether a component in relation to the form of the subsequent component is a logical affirmation or negation, and 4) the logic of component strands, indicating whether a component in relation to the alignment of the subsequent component is a logical affirmation or negation.

To make the intrinsic structure of the numbers more transparent, a new method of representation is required, which would make the individual elements and components of numbers accessible. We consider the main reason why the traditional number systems are insufficiently granting this kind of insight is attributable to the fact that digits used therewith differ by quality and not by quantity: 0, 1, 2, 3, etc. – all have the same weighting. To the same effect numeral symbols in this qualitative notation could be substituted by colors, e. g. in a base-2 notation white for 0 and black for 1. We propose an alternative approach, in which a unique geometrical magnitude (distance) is assigned to each digit, so that such distances can be expressed as ratios of the whole numbers, rendering the digits quantitative and rational[1].

II. Method

We found our method on the base-2 positional system, to which we apply the following modification: we assign magnitude 2 and "color quantity" 1 to the digit 0 (Fig. 1, left side), and magnitude 3 and "color quantity" 2 to the digit 1 (ibid. right side). The resulting weighted digits we call rational syllables[2], the short (magnitude 2) and the long (magnitude 3) syllable respectively.

Thus the first step of our method is to combine the quantitative and qualitative representation of the digits, which we call weighting. The second step is to "decouple" the adjacent positions of the positional number system by introducing the phase rules, as explained step by step below.

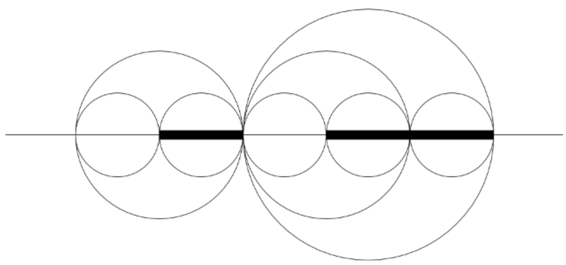

Figure 1: Base-2 quantitative digits

[1] In the sense of Euclid, 1980, p. 213, Book X, Definition 3.
[2] Using the terminology of Chandaḥśāstra by Pingala (Weber, 1863, p. 3).

III. Numbers In Spatial Mode

3.1 Bandwidth 1

The order of digits at the first position of the sequence of natural numbers[3] $a_n = a_{n-1} + 1$ in base-2 representation is 0, 1, 0, 1, 0, 1, etc. We retain this principle, but replace the qualitative digits 0 and 1 with our quantitative digits, respectively 2 and 3, see Fig. 2top row.

Figure 2: Two positions of quantitative digits

At the second position the order of digits is 0, 0, 1, 1, 0, 0, 1, 1, etc., and in qualitative representation 2, 2, 3, 3, 2, 2, 3, 3, etc., see ibid. bottom row. Two positions in base-2 system can represent a maximum of four unique numbers: 00 = 0, 01 = 1, 10 = 2, and 11 = 3.

Since in our quantitative system different "digits" can occupy different amount of space, we observe a mutual horizontal shift of positions representing number 2, see Fig. 2 the third line from the left on both rows: the long syllable of the bottom row starts one unit[4]"earlier" than the short syllable of the upper row. Syllables at all other positions start, as usual, synchronously.

The mutual horizontal shift of the individual positions of a rational numeral, caused by the weighted quantitative digits, we visualize by means of vertical and diagonal strokes, see Fig. 3.

Figure 3: Visualization of the decoupled positions

Since three of the four strokes have here identical geometrical shape (I-shape), they cannot be individually used for the representation of different numerical values, but rather, if we look at them in pairs: the combined form of the two I-strokes (Fig. 3, left side) differs sufficiently from the combined form of the diagonal Z-stroke and the I-stroke (ibid. right side).

Thus, we note that the bandwidth 2 (i. e. 2 positions) of the traditional qualitative system generates the bandwidth 1 of the new quantitative system. The left character, combining the two I-strokes (we call it the H-element, see 3.3.1), represents the rational numeral 0 (Fig. 4), and the right character, combining a Z-stroke and an I-stroke (we call it the J-element, see 3.3.1), stands for the rational numeral 1 (Fig. 4).

Figure 4: Rational numerals 0 and 1

3.2 Bandwidth 2

When we now increase the qualitative bandwidth to 3, we obtain at the first position (Fig. 5, top row) the sequence 0, 1, 0, 1, 0, 1, 0, 1 (or quantitatively 2, 3, 2, 3, 2, 3, 2, 3), at the second position (ibid., middle row) 0, 0, 1, 1, 0, 0, 1, 1 (or 2 quantitatively, 2, 3, 3, 2, 2, 3, 3), and at the third position (ibid., bottom row) 0, 0, 0, 0, 1, 1, 1, 1 (or quantitatively 2, 2, 2, 2, 3, 3, 3, 3).

If we now examine the fifth line from the left of each position (Fig. 5), we discover that the syllable of the bottom position is long, and of the middle and top position short. Further we note that the two short syllables have the same horizontal position, i. e. start synchronously, but the long syllable starts two units "too early".

Figure 5: Bandwidth 3 of the quantitative digits

[3] See also Frege, 1961, p. 117 f.

[4] In the sense of Euclid, 1980, p. 141, Book VII, Definition 1; cf. Frege, Die Grundlagen der Arithmetik, 1961, p. 39-44; see also Plato on original duality, the principle of dissimilarity, and the Limitless in Aristotle, Metaphysics, 2015, p. 26 (987b) and p. 332 (1081a); cf. also Heraclitus, Fragment 50, Diels, 1957, p. 26.

Should we use the vertical and diagonal strokes to visualize the mutual shift of the positions, we had to introduce a new type of stroke now: a diagonally stretched stroke. With the increasing of stretching distances in higher bandwidth this would lead to too many variations of the diagonal stroke, so to prevent that we need to define a special rule instead:

The length of a stroke must be less than 2 units[5].
The effect of this rule is that individual positions of the positional number system are decoupled of one another to form phases, each consisting of a specific type of rational elements (see 3.3.1).
If we now visualize the phases of the adjacent positions of the qualitative bandwidth 3 according to our phase rule, we note that the short syllables of the traditionalnumber 4 now belong to the rational numeral 2, and the long syllable to the rational numeral 1, see Fig. 6. Thus, the traditional positions of numbers are decoupled from each other, and connected in a new way to form rational phases.

Figure 6: Rational phases of the bandwidth 2

After visualization we obtain, as expected, a sequence of four distinct characters of quantitative bandwidth 2, the rational numerals 0, 1, 2, and 3 (see Fig. 7).

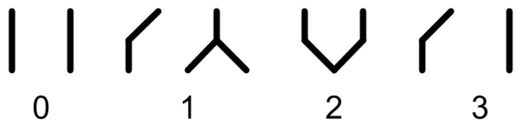

Figure 7: Rational numerals of the bandwidth 2

3.3 Higher Bandwidth
Following our method we can, by increasing the qualitative bandwidth, produce rational numerical sequences of arbitrary length.

Rational numerals 0 to 7:

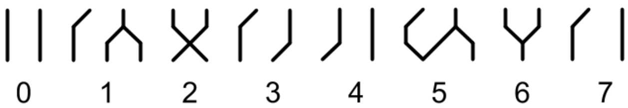

Figure 8: Rational numerals of the bandwidth 3

Rational numerals 0 to 15:

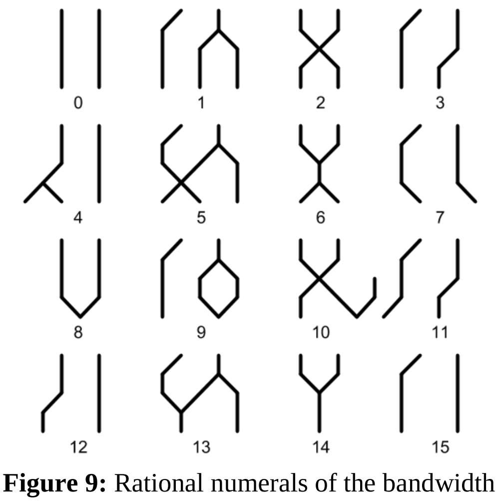

Figure 9: Rational numerals of the bandwidth 4

[5]In case if the horizontal (h) and vertical (v) units differ, the length of a stroke must be $\sqrt{h^2 + v^2}$ units or less.

3.3.1 Rational Elements

Based on examination of the characters in FIG. 9, we can conclude that any rationalnumeral can be composed of not more than six distinct elements. These elements are:

1) H-element composed of two vertical strokes with the horizontal distance of 2 units; example see FIG. 9, number 2, top element,
2) A-element composed of two diagonal strokes that are connected at the top; example see ibid. number 6, bottom element,
3) Z- or S-element composed of two parallel diagonal strokes with the horizontal distance of 3 units; example see ibid. number 7, bottom element,
4) J-element composed of a diagonal stroke (Z-stroke) and a vertical stroke with the horizontal distance of 2 units at the top and 3 units at the bottom; example see ibid. number 1, top element,
5) V-element composed of two diagonal strokes that are connected at the bottom; example see ibid. number 8, bottom element, and
6) I-element composed of two vertical strokes with the horizontal distance of 3 units; example see ibid. number 12, bottom element.

The six rational elements combine to rational components. Since the detailed description of properties of the rational components is beyond the scope of this paper, we will limit it to remark that the vertical extent of the individual components of the rational numerals corresponds to the OEIS sequence A275536: "Differences of the exponents of the adjacent distinct powers of 2 in the binary representation of n (with -1 subtracted from the least exponent present) are concatenated as decimal digits in reverse order", see https://oeis.org/A275536.

IV. Numbers In Temporal Mode

Furthermore, rational numerals can be encoded and communicated (transmitted) in temporal mode. The example of numerals 0 to 3 (Fig. 10) of the quantitative bandwidth 3 are used below to demonstrate the encoding of spatial rational characters into temporal mode in two steps.

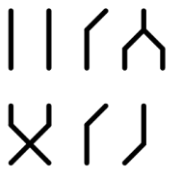

Figure 10: Rational numerals 0 to 3

In the first step we convert the rationalnumerals into the original "syllable" form. For this purpose we mark the individual positions by the corresponding quantitative digits (see Fig. 11, cf. Fig. 6):

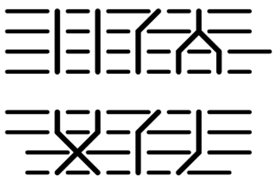

Figure 11: Rationalnumerals 0 to 3 with visualized quantitative digits

Next, we apply the rotation rule, which is:

Connect an available position (i. e. the horizontal gap between the syllables) to the closest available (i. e. beginning horizontally synchronously) position in downward direction,

If # 1 is not possible, connect to the furthest available position in upward direction,

If # 1 and # 2 are not possible, connect to the next position of the same level.

Fig. 12 shows the rational numerals 0 to 3 in temporal encoding:

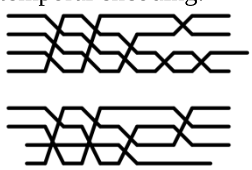

Figure 12: Rationalnumerals 0 to 3 in temporal encoding

Fig. 13 shows the rational numerals 0 to 15 in temporal encoding:

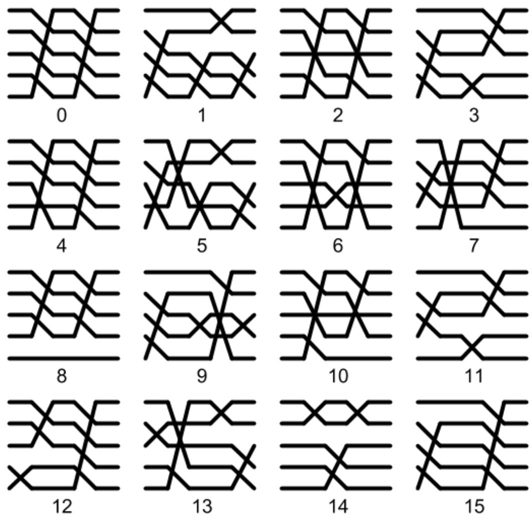

Figure 13: Rationalnumerals 0 to 15 in temporal encoding

V. Conclusion

The quantitative representation method described in this paper presents a new concept of numerals, i. e. as value with a sign, instead of sign with value. This concept opens up new areas of research not only in the education and practice of mathematics, but i. a. also in computer science, linguistics, musicology, art studies, and philosophy.

Knowledge of the elements of rational numerals can be extended in further training steps, which include: 1) learning how the elements combine to create components, 2) learning about the types of the components and the rules of logic to be considered in their application, and 3) exploring rational phases in terms of syntax and semantics.

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