International Journal of Engineering Science Invention
ISSN (Online): 2319 – 6734, ISSN (Print): 2319 – 6726
www.ijesi.org ||Volume 6 Issue 8|| August 2017 || PP. 29-33

Chromatic Number of Some S-Valued Graphs

T.V.G. Shriprkash* and M. Chandramouleeswaran**
**Kurinji College Of Engineering And Technology MANAPPARAI – 621 307*
Tamil Nadu. India
*** Saiva Bhanu Kshatriya College Aruppukottai – 626 101*
Tamil Nadu, India.

ABSTRACT : *In [3], the authors introduced the notion of semiring valued graphs. In [1], theauthors introduced the notion of regularity on S- Valued graphs . In [4], we have introducedcolouring on S-valued graphs. In [5],we have introduced the notion of K-colouring on S-Valued graphs. In this paper, we study the upper bounds for chromatic number of some S-valued graphs.*

Keywords:*Semiring,S-valued graph, colouring, K-colouring,Chromatic-number,*
AMS *subject classification:05C25, 16Y60*

Date of Submission: 17-07-2017 Date of acceptance: 18-08-2017

I. Introduction

An assignment of colors to the vertices of a graph so that no two adjacent vertices get the same colour is called a colouring of the graph. For each color, the set of points which get the samecolour is independent and is called a colourclass. A colouring of a graph G usingatmost n colours is called a n-colouring. The chromatic number $\chi(G)$of a graph G is the minimum number of colours needed to colour G. A graph G is called, n-colourableif
$\chi(G)$≤n.

The problem of colouring in crisp graph is dealt in [2] by Jensen. In [3], the authors introduced the notion ofsemiring valued graphs. In [1], the authors introduced the notion of regularity on S-valued graphs. Motivated by this, in [4], we have introducedthe notion of coloring on S-valued graphs. In [5], we introduced the notion of K-colouring on S-valued graphs. In this paper, we study the chromatic number of some S-valued graphs.

II. Preliminaries

In this section, we recall some basic definitions that are required for our work in the sequel.

Definition 2.1

A semiring (S, +,·) is an algebraic system with a non-empty set S together with two binary operators + and · such that

(1) (S, +, 0) is a monoid.
(2) (S,·) is a semi group.
(3) For all a, b, c ∈ S, a · (b + c) = a · b + a · c and (a + b) · c = a · c + b · c.
(4) 0 · x = x · 0 = 0 for all x ∈ S.

Definition 2.2

Let (S, +,·) be a semiring. $\preceq$ is said to be a canonical preorder if for a, b ∈ S, a $\preceq$ b if and if there exists c ∈ S such that a + c = b.

Definition 2.3 [6]

Let $(S_1, +, .)$ and $(S_2, +, .)$ be given two semirings. A mapping $\beta : S_1 \rightarrow S_2$ is a semiring homomorphism if $\beta(0_{S_1}) = \beta(0_{S_2})$; β (a+b) = β (a) + β (b) ; β(ab) = β(a) β(b) ∉∀ a, b ∈ S_1.

Remark 2.4. If the semiring contains multiplicative identity then $\beta(1_{S_1}) = 1_{S_2}$ must be satisfied.

Definition 2.5 [6]

Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be given two graphs. A mapping $\alpha: V_1 \rightarrow V_2$ is said to be a graph Homomorphism if $(u, v) \in E_1 \Rightarrow (\alpha(u), \alpha(v)) \in E_2$.

Remark 2.6A graph homomorphism is an edge preserving map . It need not be 1-1, onto (or) both.

Definition 2.7 [2]

A k – vertex colouring of a graph G is an assignment of k – colours to the vertices of G such that no two adjacent vertices receive the same colour.

Definition 2.8 [2]

A graph G that required k – different colours for its colouring and not less number of colours is called a k – chromatic graph and the number k is called the chromatic number of G, denoted by (G). That is $\chi(G)$ = k.

Theorem 2.8 (a):

G is r-colorable iff there is a homomorphism from G to K_r where K_r is a complete graph. of r vertices. .

Definition 2.9 [3]

Let G = (V, E) be a given graph with V, E $\neq \phi$. For any semiring (S,+, ·) a Semiring - valued graph (or S-valued graph) G^S is defined to be the graph

$G^S = (V, E, \sigma, \psi)$ where $\sigma : V \to S$ and $\psi: E \to S$ is defined to be

$$\psi(x,y) = \begin{cases} \min\{\sigma(x),\sigma(y)\} & if\ \sigma(x) \preccurlyeq \sigma(y) or\ \sigma(y) \preccurlyeq \sigma(x) \\ 0 & otherwise \end{cases}$$

for every unordered pair (x, y) of $E \subseteq V \times V$. We call σ, a S-vertex set and ψ a S-edge set of S-valued graph G^S.

Definition 2.10[4]

Consider the S-valued graph G^S. A colouring f on G^S is said to be equi-weight (or vertex regular) proper colouringif for all v $\in$V, σ (v) have equal value in S and c(v) $\in$ C differ for adjacent vertices.

Definition 2.11[4]

A colouringf : $V \times V \to S \times C$ is said to be proper weight-unicolouring, if ∀ v $\in$V and c(v) $\in$ C is the same, but σ(v) $\in$ S differ for adjacent vertices.

Definition 2.12[4]

Consider the S-valued graph G^S. A colouring f on G^S is said to be total proper colouring if for all v $\in$V, σ (v) $\in$ S and c(v) $\in$ C differ for adjacent vertices.

Definition 2.13[4]

Let G^S be a S-valued graph. The vertex chromatic number of G^S, denoted by($\chi_S(G^S)$, is defined to be $\chi_S(G^S) = (\min_{v \in V} \sigma(v), \min |C|)$

Definition 2.14[4]

A S-valued graph G^S is said to be k-colourable, if it has a proper vertex regular or total proper colouring such that $|C|$ = k.

Definition 2.15. [6]

Let $G_1^{S_1} = (V_1, E_1, \sigma_1, \psi_1)$ and $G_2^{S_2} = (V_2, E_2, \sigma_2, \psi_2)$ be given two S-values graphs.

A mapping $\phi = (\alpha, \beta) : G_1^{S_1} \to G_2^{S_2}$ is a S – valued vertex homomorphism if

(i) $\alpha : V_1 \to V_2$ is a graph homomorphism.

(ii) $\beta : S_1 \to S_2$ is a semiring homomorphism with $\beta(\sigma_1(v)) = \sigma_2(\alpha(v))\ \forall\ v \in V_1$.

Definition 2.16. [6]

Let $G_1^{S_1} = (V_1, E_1, \sigma_1, \psi_1)$ and $G_2^{S_2} = (V_2, E_2, \sigma_2, \psi_2)$ be given two S-valued graphs.

A mapping $\phi = (\alpha, \beta) : G_1^{S_1} \to G_2^{S_2}$ is a S – valued edge homomorphism if

(i) $\alpha : V_1 \to V_2$ is a graph homomorphism.

(ii) $\beta : S_1 \to S_2$ is a semiring homomorphism

with $\beta(\psi_1(v_i, v_j)) = \psi_2(\alpha(v_i), \alpha(v_j))\ \forall\ (v_i, v_j) \in E_1$.

3.CHROMATIC NUMBER OF SOME S-VALUED GRAPHS.

In this section, we are going to find the upper bounds of chromatic number of some S-valued graphs.

Theorem:3.1

Let $T^S = \{ V, E, \sigma, \psi\}$ be a s – valued tree with $|V| = n \geq 3$. Then T^S is 2 – chromatic. Thus,

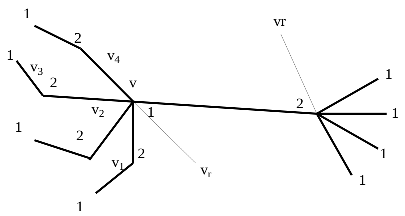
$$\chi_S(T^S) = (\min_{v \in V} \sigma(v), 2)$$

Proof:

Let T^S = { V, E,σ,ψ} be a S – valued tree with n≥2 vertices. Assume that $T^S$is rooted at vertex v. Assign color 1 to v. Then assign color 2 to all vertices which are adjacent to v .Let v_1,v_2,….v_r be the vertices which have been assigned color 2. Now assign color 1 to all the vertices which are adjacent to v_1,v_2,….v_r. Continue this process till every vertex in T^S has been assigned the color. We observe that in T^S all vertices at odd distance from v have color 2 and vertices at even distance from v have color 1. Therefore along any path in T^S, the vertices are of alternating colors. Since there is one and only one path between any two vertices in a tree, no two adjacent vertices have the same color. Thus T^S is colored with two colors. Hence T^S is 2- chromatic. Since T^S is S-valued tree, by definition

$\chi_S(T^S)$= $(\min_{v\in V}\sigma(v)$, min|C|)

= $(\min_{v\in V}\sigma(v)$, 2)



Theorem: 3.2

For any S – valued vertex regular wheel graph W^S with S – vertex set {a} ,a ∈S. $\chi_S(W^S) = \begin{cases}(a,3)\ if\ |V|\ is\ odd\\(a,4)\ if\ |V|\ is\ even\end{cases}$

Proof:

Case: (i)

$W^S_{2m+1}$is the join of even cycle C^S_{2n} and complete graph K_1^S. The crisp graph C_{2m} can be coloured withone colour, the cycle can be coloured with 2 colours. Therefore the join, the crisp graph W^S_{2m+1} is coloured by three colors. Therefore $\chi_S(W^S_{2m+1})$= $(\min_{v\in V}\sigma(v)$, 3) = (a,3).

Case: (ii)

Let W^S_{2m}be vertex regular graph with S – vertex set {a} , a∈S. Therefore, σ(v) = a ∀ $v \in V$. Now$W^S_{2m}$is the join of odd cycleC^S_{2m+1} and the complete graph K_1^S, the crisp graph C_{2m+1} is colored by 3 colors and the centre K_1^S is coloured by one color. This implies even order wheel W^S_{2m} is colored by 4 colors.

∴$\chi_S(W^S_{2m})$= $(\min_{v\in V}\sigma(v)$, 4) = (a,4).

Theorem:3.3

For a S- valued graph $\chi_S(C^S_n)$ with n ≥ 3,

$$\chi_S(C^S_n) = \begin{cases}\{(\min_{v\in V}\sigma(v), 3)\}\ if\ n\ is\ odd\\\{(\min_{v\in V}\sigma(v),\ 2)\}\ if\ n\ is\ even\end{cases}$$

Proof:

Let C_n^S be a S – cycle of length n. let v_1,v_2, ...,v_n be the vertices in C_n^S with σ - values. σ(v_i) ∈S (1 ≤ i ≤ n)

Assume that n ≥ 3 ,

For vertices $v \in V$ with odd indices assign color c_1, for vertices with even indices assign c_2. If n is an even, no adjacent vertex get the same color.$\chi_S(C^S_n)$) = $(\min_{v\in V}\sigma(v)$,2)

If n is odd the vertices v_1 and v_n are adjacent and have the same color c_1. Also v_{n-1} will have color c_2. Hence we need to assign a third color c_3 to v_n.

∴$\chi_S(C^S_n)$) = $(\min_{1\le i\le n}\sigma(v_i)$, 3).

Thus for the cycle C_n^s with vertices v_1, v_2, … v_n we have.

$$\chi_S(C^S_n) = \begin{cases}\{(\min_{1\le i\le n}\sigma(v_i), 3)\}\ if\ n\ is\ odd\\\{(\min_{1\le i\le n}\sigma(v_i),\ 2)\}\ if\ n\ is\ even\end{cases}.$$

Hence the proof.

Theorem: 3.4

A S-valued graph G^S is *l*-colorable if and only if there is a S- valued vertex homomorphism from $G^S$to K_l^S .

Proof:

Let G^S=(V,E,σ,χ)be *l*-colorable and let it be colored by 1,2,3,...,*l* colors.Let v_i∈V be the vertex colored by i in G^S. Since G^S is *l*-colorable, its underlying graph G must be *l*-colorable. By theorem 2.8(a), we see that there is a homomorphism α : V(G) →V(K_l) defined by α(v_i)=k_i,

$1 \le i \le l$,where $k_i$is the vertex colored by i in K_l^S .Let $v \in V$ be arbitrary. Therefore $v = v_i$ for some i proving that $\alpha(v_i) \in V(K_l)$.

That is, $\alpha(v) = \alpha(v_i) \in V(K_l)$ for all $v \in V$. Since α is graph homomorphism from $V(G) \rightarrow V(K_l)$, it preserves edges.

That is, for any $(v_i, v_j) \in E(G)$, $(\alpha(v_i), \alpha(v_j)) \in E(K_l)$.

Now, define a semiring homomorphism β: $S \rightarrow S$ by $\beta(\sigma(v_i)) = \sigma(\alpha(v_i))$. Then $\beta(\sigma(v_i)) = \sigma(\alpha(v_i)) = \sigma(k_i)$ for all i.Thusβ preserves S-values.

If suppose some vertices of K_l are not in image set, let their weights be 'a', for some $a \in S$.Then K_l is a complete graph with S-vertex set $\{(k_i),a\}$, $1 \le i \le l$.

Thus we have K_l^S as a S-valued graph colored by$l$ colors with α and β as a crisp graph and semiringhomorphisms respectively.Therefore by definition of S- valued vertex homomorphism, $\varphi = (\alpha, \beta)$: $G^S \rightarrow K_l^S$ form a S- valued vertex homomorphism.

Conversely,

Let $\varphi = (\alpha, \beta)$:$G^S \rightarrow K_l^S$ be a S-valued vertex homomorphism. Then $\alpha : G \rightarrow K_l$ is graph homomorphism. Therefore by theorem 2.8.a, G is$l$-colorable.Since β preserves S-values, G^S is l-colorable.

Cor.1:

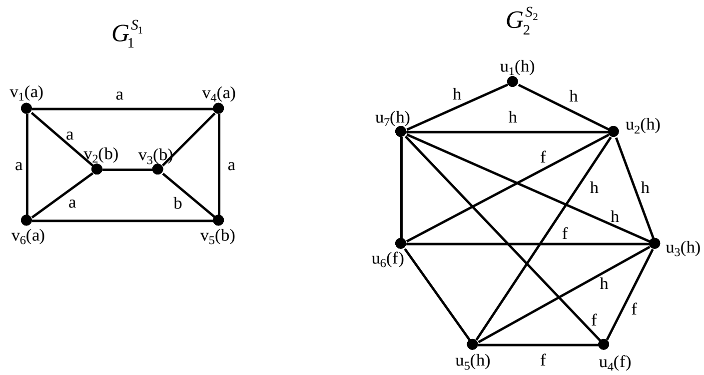
If G^S is l-colorable then there is a S-valued edge homomorphism from $G^S$to K_l^S .

Proof:

Since every S-valued vertex homomorphism is a S-valued edge homomorphism, by the above result, it follows.

Remark:3.11

Converse of the above corollary is not true because every S-valued edge homomorphism is nota S-valued vertex homomorphism, in general. For example, consider a semiringhomomorphism $\beta : S_1 \rightarrow S_2$ in example 3.2. Let $G_1^{S_1}$ and $G_2^{S_2}$ be as follows:



Define $\alpha : V_1 \mapsto V_2$ by $v_1 \mapsto u_3$; $v_2 \mapsto u_5$;$v_3 \mapsto u_2$;$v_4 \mapsto u_6$; $v_5 \mapsto u_7$; $v_6 \mapsto u_4$.

Clearly, $(v_1, v_4) \mapsto (u_3,u_6)$; $(v_1, v_2) \mapsto (u_3,u_5)$; $(v_1, v_6) \mapsto (u_3,u_4)$; $(v_2, v_3) \mapsto (u_5,u_2)$; $(v_2, v_6) \mapsto (u_5,u_4)$; $(v_3, v_4) \mapsto (u_2,u_6)$; $(v_3, v_5) \mapsto (u_2,u_7)$; $(v_4, v_5) \mapsto (u_6,u_7)$; $(v_5, v_6) \mapsto (u_7,u_4)$.

$\Rightarrow (v_iv_j) \in E_1 \Rightarrow (\alpha(v_i), \alpha(v_j)) \in E_2 \forall (v_i, v_j) \in E_1$.

Therefore α is a graph homomorphism.

Now $\beta(\psi_1(v_1, v_4)) = \beta(a) = f = \psi_2(\alpha(v_1), \alpha(v_4)) = \psi_2(u_3, u_6)$

$\beta(\psi_1(v_1, v_2)) = \beta(a) = f = \psi_2(\alpha(v_1), \alpha(v_2)) = \psi_2(u_3, u_5)$

$\beta(\psi_1(v_1, v_6)) = \beta(a) = f = \psi_2(\alpha(v_1), \alpha(v_6)) = \psi_2(u_3, u_4)$

$\beta(\psi_1(v_2, v_3)) = \beta(a) = f = \psi_2(\alpha(v_2), \alpha(v_3)) = \psi_2(u_5, u_2)$

$\beta(\psi_1(v_2, v_6)) = \beta(a) = f = \psi_2(\alpha(v_2), \alpha(v_6)) = \psi_2(u_5, u_4)$

$\beta(\psi_1(v_3, v_4)) = \beta(a) = f = \psi_2(\alpha(v_3), \alpha(v_4)) = \psi_2(u_2, u_6)$

$\beta(\psi_1(v_3, v_5)) = \beta(b) = h = \psi_2(\alpha(v_3), \alpha(v_5)) = \psi_2(u_2, u_7)$

$\beta(\psi_1(v_4, v_5)) = \beta(a) = f = \psi_2 (\alpha(v_4), \alpha(v_5)) = \psi_2 (u_6, u_7)$
$\beta(\psi_1(v_5, v_6)) = \beta(a) = f = \psi_2 (\alpha(v_5), \alpha(v_6)) = \psi_2 (u_7, u_4)$
$\Rightarrow\beta (\psi_1(v_i, v_j)) =\psi_2 (\alpha(v_i), \alpha(v_j)) \forall (v_i, v_j) \in E_1$ .... (*).
$\Rightarrow\beta$is a semiringhomomophism satisfying equation (*).
Therefore $\varphi = (\alpha, \beta) : G_1^{S_1} \to G_2^{S_2}$ is a S – valued edge homomorphism.
Now, Inparticular,$\sigma_1 (v_1) = a \Rightarrow \beta (\sigma_1(u_1)) = \beta (\sigma_1(u_1)) = \beta (a) = f$ and
$\sigma_2 (\alpha (v_1)) = \sigma_2 (u_3) = h$.
Therefore $\beta (\sigma_1(v_1)) \neq \sigma_2 (\alpha(V_1))$.
$\Rightarrow \phi = (\alpha, \beta)$ is not a S – valued vertex homomorphism.
It isa S – valued edge homomorphism but not a S – valued vertex homomorphism.

Cor.2:

A S- valued graph G^S is *l*-colorable iff there is a S-valued semi homomorphism from $G^S$to K_l^S .

Proof:

Since every S-valued vertex homomorphism is a S- valued semi homomorphism and every S-valued semi homomorphism is both S-valued vertex and edge homomorphisms, this corollary holds.

III. CONCLUSION

In this paper, we have discussedthe Chromatic number for some S-valued graphs.Further investigation will be done on bounds for chromatic numbers of S-valued graphs.

References

[1] Jayalakshmi. S, Rajkumar. M and Chandramouleeswaran.M : Regularity on S-valued Graphs,GJPAM, Vol. 11(4), (2015), pp-2971-2978.
[2] JensenT.R. Toft.: Graph coloring problems. John-Wiley & Sons, New York, 1995.
[3] Rajkumar, M, Jeyalakshmi, S. and Chandramouleeswaran. M.:Semiring Valued Graphs.IJMSEA Vol. 9(3), (2015), 141-152
[4] Shriprakash T.V.G and Chandramouleeswaran.M :Colouring on S-Valued graphs, International journal of pure and Applied Mathematics, 112(5) (2017), 123-129 doi:10.12732/ijpam.v112i5.14
[5] Shriprakash T.V.G and Chandramouleeswaran.M :k-colorableS-valued graphs.International J.ofmath.Sci.&Engg.Appls.(IJMSEA) ISSN 0973-9424, Vol.11 No. I (April, 2017)
[6] Rajkumar M and Chandramoule.M : S-valved semi homomorphism (Print).
[7] Jonathan Golan :Semirings and Their Applications, Kluwer Academic Publishers, London.
[8] Vandiver. H. S: Note on a simple type of algebra in which the cancellation law of addition does not hold, Bull. Amer. Math. Soc.,Vol 40, 1934, 916 -920.

International Journal of Engineering Science Invention (IJESI) is UGC approved Journal with Sl. No. 3822, Journal no. 43302.

T.V.G. Shriprkash. "Chromatic Number of Some S-Valued Graphs." International Journal of Engineering Science Invention (IJESI), vol. 6, no. 8, 2017, pp. 29–33.