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The Fibonacci Numbers and Its Amazing Applications

*Sudipta Sinha
Assistant Professor Department of Mathematics Burdwan Raj College,Burdwan,West Bengal,713104
Corresponding Author: Sudipta Sinha

Abstract: *Fibonacci sequence of numbers and the associated "Golden Ratio" are manifested in nature and in certain works of art. We observe that many of the natural things follow the Fibonacci sequence. It appears in biological settings such as branching in trees, phyllotaxis (the arrangement of leaves on a stem), the fruit sprouts of a pineapple, the flowering of an artichoke, an uncurling fern and the arrangement of a pine cone's bracts etc. At present Fibonacci numbers plays very important role in coding theory. Fibonacci numbers in different forms are widely applied in constructing security coding.*

Keywords: *Fibonacci Numbers, Golden ratio, Coding, Encryption, Decryption*

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I. Introduction

The Fibonacci numbers were first discovered by a man named Leonardo Pisano. He was known by his nickname, Fibonacci. The Fibonacci sequence is a sequence in which each term is the sum of the 2 numbers preceding it. The Fibonacci Numbers are defined by the recursive relation defined by the equations $F_n = F_{n-1} + F_{n-2}$ for all $n \geq 3$ where $F_1 = 1$; $F_2 = 1$ where Fn represents the nth Fibonacci number (n is called an index).
The Fibonacci sequence can elaborately written as {1,1,2,3,5,8,13,21,34,55,89,144,233…….}.
One of the most common experiments dealing with the Fibonacci sequence is his experiment with rabbits. Fibonacci put one male and one female rabbit in a field. Fibonacci supposed that the rabbits lived infinitely and every month a new pair of one male and one female was produced. Fibonacci asked how many would be formed in a year. Following the Fibonacci sequence perfectly the rabbits reproduction was determined...144 rabbits. Though unrealistic, the rabbit sequence allows people to attach a highly evolved series of complex numbers to an everyday, logical, comprehendible thought.Bortner and Peterson (2016) elaborately described the history and application of Fibonacci numbers.

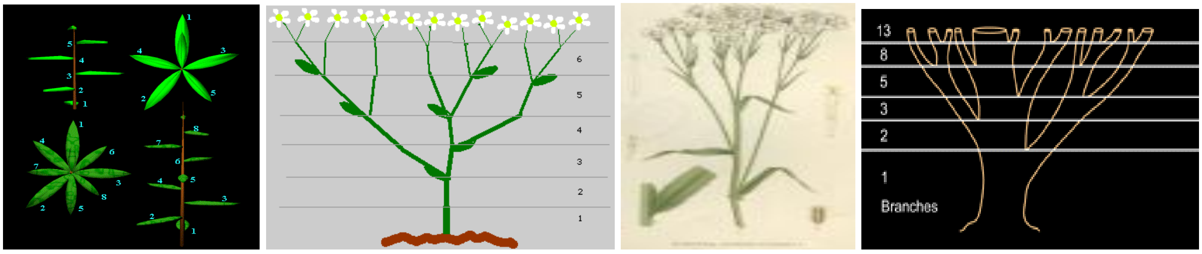
II. Fibonacci Sequence In Nature

Fibonacci can be found in nature not only in the famous rabbit experiment, but also in beautiful flowers (Internet access, 12). On the head of a sunflower and the seeds are packed in a certain way so that they follow the pattern of the Fibonacci sequence. This spiral prevents the seed of the sunflower from crowding themselves out, thus helping them with survival. The petals of flowers and other plants may also be related to the Fibonacci sequence in the way that they create new petals (Internet access, 10).

2.1 Petals on flowers

Probably most of us have never taken the time to examine very carefully the number or arrangement of petals on a flower. If we were to do so, we would find that the number of petals on a flower that still has all of its petals intact and has not lost any, for many flowers is a Fibonacci number (Internet access,8).
- 1 petal: white cally lily
- 3 petals: lily, iris
- 5 petals: buttercup, wild rose, larkspur, columbine (aquilegia)
- 8 petals: delphiniums
- 13 petals: ragwort, corn marigold, cineraria,
- 21 petals: aster, black-eyed susan, chicory
- 34 petals: plantain, pyrethrum
- 55, 89 petals: michaelmas daisies, the asteraceae family (Internet access,19)

Plants show the Fibonacci numbers in the arrangements of their leaves (Internet access,15). Three clockwise rotations, passing five leaves two counter-clockwise rotations. Sneezewort (Achillea ptarmica) also follows the Fibonacci numbers.



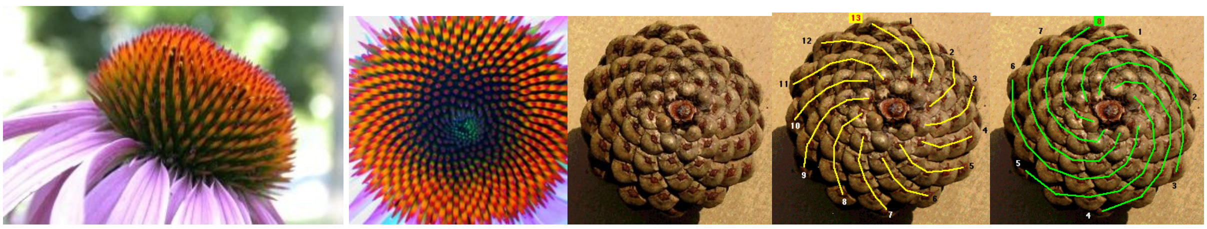
Schematic diagram (Sneezewort)

Why do these arrangements occur? In the case of leaf arrangement, or phyllotaxis, some of the cases may be related to maximizing the space for each leaf, or the average amount of light falling on each one.

These pictures are very common to us. We can see the flowers and the patterns of leaves just out of single step of our house. All of these follow the Fibonacci numbers.

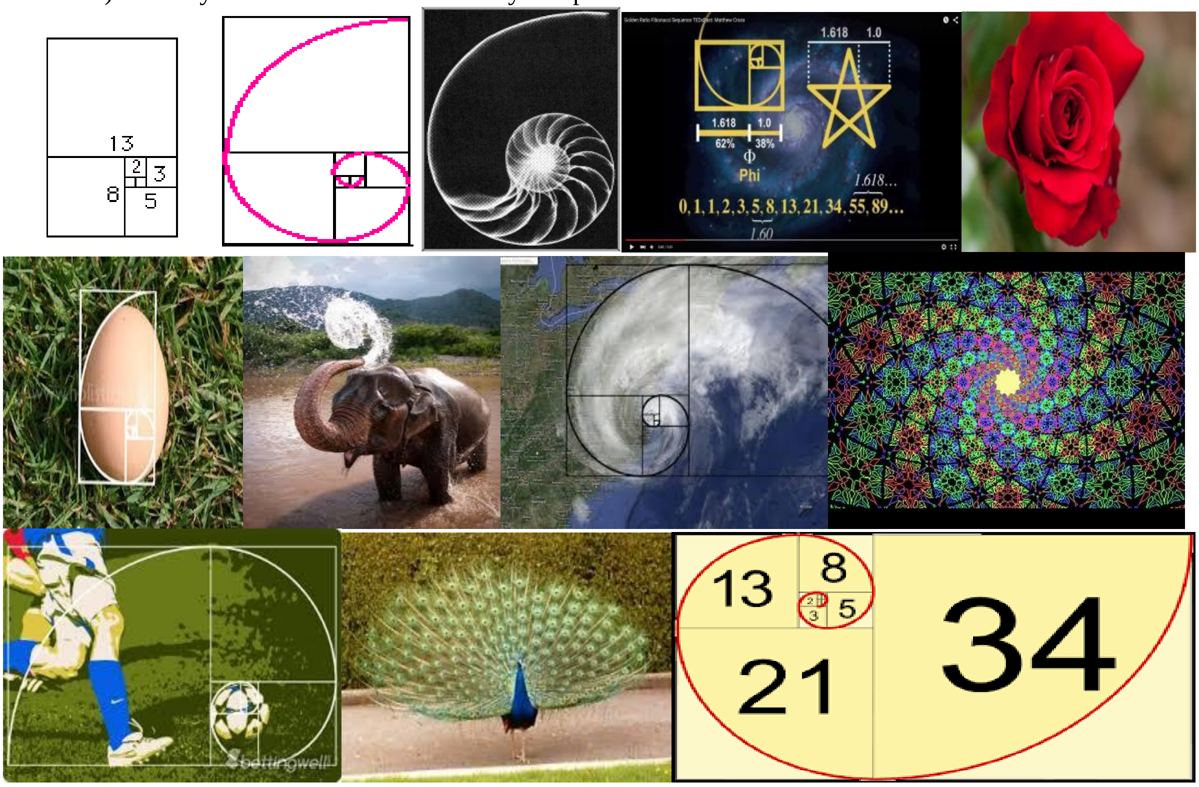
2.2 Fibonacci spiral

The Fibonacci numbers are found in the arrangement of seeds on flower heads (Internet access, 13). There are 55 spirals spiraling outwards and 34 spirals spiraling inwards in most daisy or sunflower blossoms (Internet access,14). Pinecones clearly show the Fibonacci spirals (Howard, 2004)



Fibonacci spiral can be found in cauliflower. The Fibonacci numbers can also be found in Pineapples and Bananas (Lin and Peng). Bananas have 3 or 5 flat sides and Pineapple scales have Fibonacci spirals in sets of 8, 13, and 21. Inside the fruit of many plants we can observe the presence of Fibonacci order.

Fibonacci spiral (Internet access, 9), (Internet access, 11) are also found in various fields associated in nature. It is seen snail, sea shells, waves, combination of colours; roses etc in so many things created in nature (Internet access 12).But very few of us have time to study this phenomenon.

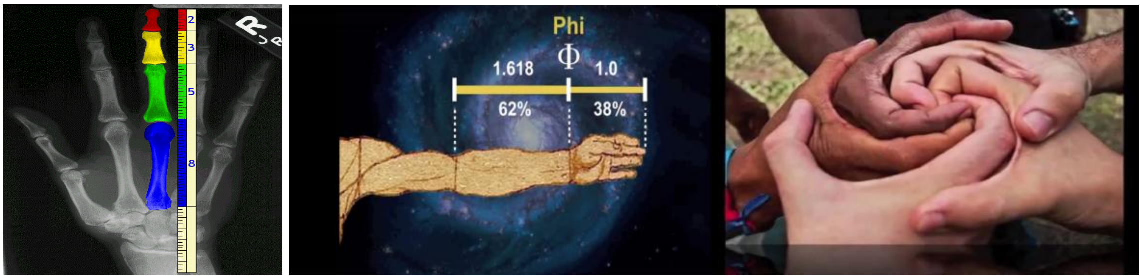


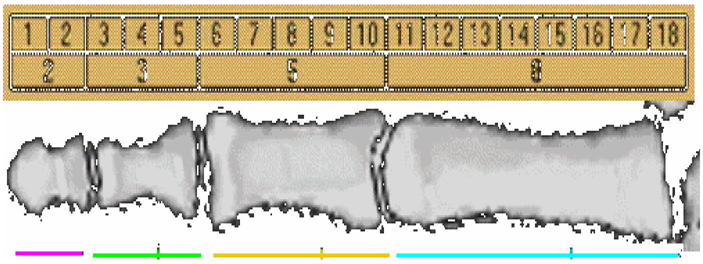
Nature isn't trying to use the Fibonacci numbers: they are appearing as a by-product of a deeper physical process. That is why the spirals are imperfect. The plant is responding to physical constraints, not to a mathematical rule.

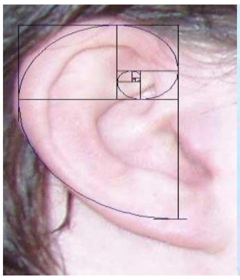
The basic idea is that the position of each new growth is about 222.5 degrees away from the previous one, because it provides, on average, the maximum space for all the shoots. This angle is called the **golden angle**, and it divides the complete 360 degree circle in the golden section, 0.618033989 . . . . which is described below.

2.3 Organs of human body

Humans exhibit Fibonacci characteristics. Every human has two hands, each one of these has five fingers and each finger has three parts which are separated by two knuckles (Internet access, 7). All of these numbers fit into the sequence. Moreover the lengths of bones in a hand are in Fibonacci numbers.







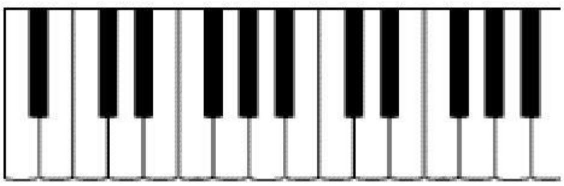
The cochlea of the inner ear forms a Golden Spiral

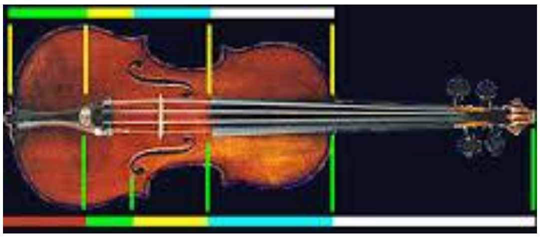
2.4 Fibonacci in Music

The Fibonacci sequence of numbers and the golden ratio are manifested in music widely.
The numbers are present in the octave, the foundational unit of melody and harmony.
Stradivarius used the golden ratio to make the greatest string instruments ever created.
Howat's(1983) research on Debussy's works shows that the composer used the golden ratio and Fibonacci numbers to structure his music. The *Fibonacci Composition* reveals the inherent aesthetic appeal of this mathematical phenomenon. Fibonacci numbers harmonize naturally and the exponential growth which the Fibonacci sequence typically defines in nature is made present in music by using Fibonacci notes. The intervals between keys on a piano of the same scales are Fibonacci numbers (Gend, 2014).

5 Black

3 B 2B





8 W & 5 B, 13 B&W

2.5 Fibonacci numbers in Pascal's Triangle

The Fibonacci Numbers are also applied in Pascal's Triangle. Entry is sum of the two numbers either side of it, but in the row above. Diagonal sums in Pascal's Triangle are the Fibonacci numbers. Fibonacci numbers can also be found using a formula

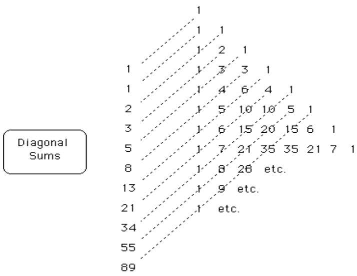
1

1 1

1 2 1

1 3 3 1

1 4 6 4 1



2.6 The Golden Section

Represented by the Greek letter Phi (ϕ) =1.6180339887.

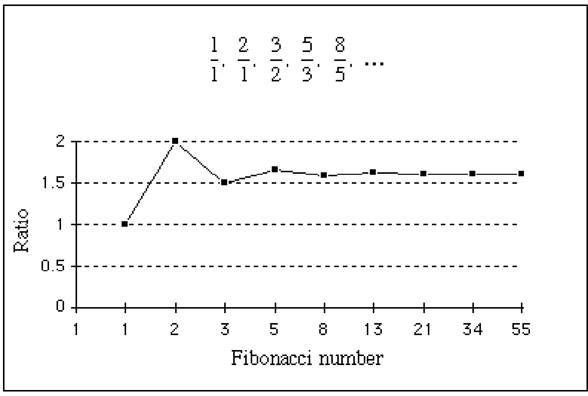
How did *1.6180339887*……. come from?

Let's look at the ratio of each number in The Fibonacci sequence to the one before it:

1/1 = 1	13/8 = 1.625	144/89 = 1.61798
2/1 = 2	21/13 = 1.61538	233/144 = 1.61806
3/2 = 1.5	34/21 = 1.61905	
5/3 = 1.666	55/34 = 1.61764	
8/5 = 1.6	89/55 = 1.61861	

If we keep going, we get an interesting number which mathematicians call "phi" (Golden Ratio or Gollden Ratio). It is denoted by ϕ and the value of ϕ *=1.6180339887*

$$\lim_{n\to\infty} \frac{F_{n+1}}{F_n} = 1.618$$



If you take two successive terms of the series, a, b, and $a+b$ then

$$\frac{b}{a} \cong \frac{a+b}{b}$$

$$\cong \frac{a}{b} + 1$$

We define the golden section, ϕ (*phi*), to be the limit of $\frac{b}{a}$, so:

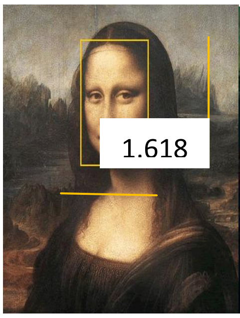
$$\phi = \frac{1}{\phi} + 1$$

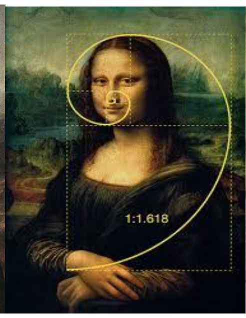
$$\phi^2 - \phi - 1 = 0$$

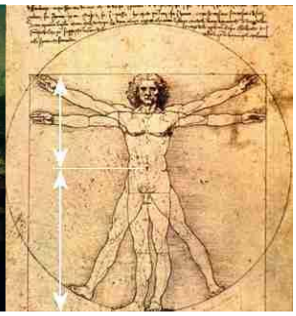
$$\phi = \frac{1+\sqrt{5}}{2} \approx 1.618$$

2.7 Some applications of Golden ratio

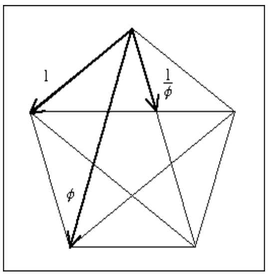
Leonardo da Vinci showed that in a 'perfect man' there were lots of measurements that followed the Golden Ratio. The Golden (Divine) Ratio has been talked about for thousands of years.

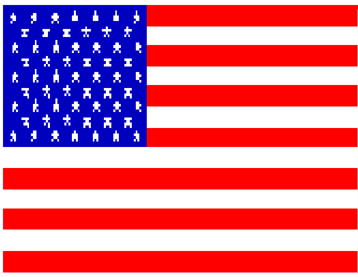







The Golden ratio is widely used in Geometry (Garg et al, 2014). It is the ratio of the side of a regular pentagon to its diagonal. The diagonals cut each other with the golden ratio (Stakov1989). Pentagram describes a star which forms parts of many flags. This five-point symmetry with Golden proportions is found in starfish which has five arms.

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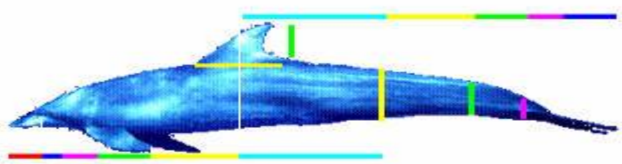
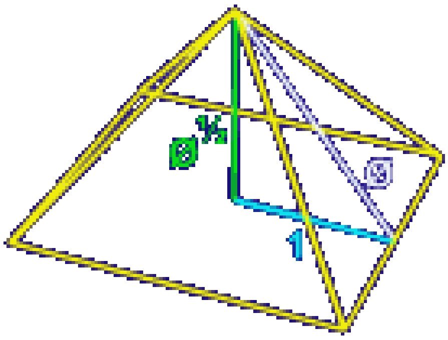
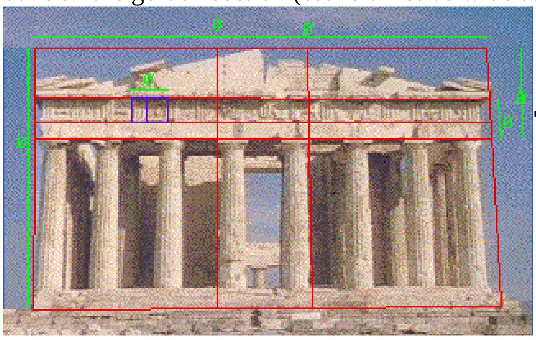
The eyes, fins and tail of the dolphin fall at Golden sections along the body.

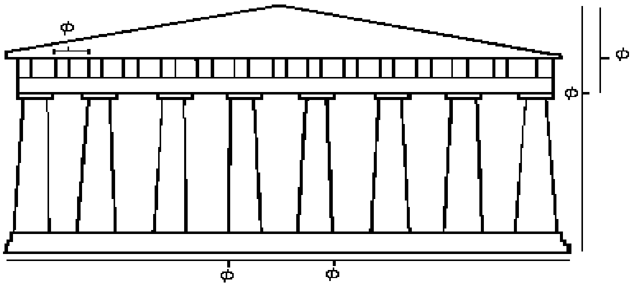


The Golden Ratio is also frequently seen in natural architecture also (Internet access, 18). It can be found in the great pyramid in Egypt. Perimeter of the pyramid, divided by twice its vertical height is the value of ϕ .



Golden section (Gend, 2014) appears in many of the proportions of the Parthenon in Greece. Front elevation is built on the golden section (0.618 times as wide as it is tall).





III. A beautiful example

Take any two consecutive numbers from this series as example 13 and 21 or 34 and 55.
Now smaller number is in miles = the other one in Kilometer or bigger number is in Kilometers = the smaller one in Miles (The other way around).
34 Miles = round (54.72) Kilometers = 55 Kilometers
21 Kilometers = round (13.05) Miles = 13 Miles
For distances which are not exact Fibonacci values you can always proceed by splitting the distance into two or more Fibonacci values.
As example, for converting 15 km into miles we can proceed as following:
15 km = 13 km + 2 km
13 km -> 8 mile
2 km -> 1 mile
15 km -> 8+1 = 9 mile
Another example, for converting 170km into miles we can proceed as:
170 km = 10*17 km
17 km = 13 km + 2 km + 2 km = 8 + 1 + 1 miles = 10 miles (approximately)
Now, 170 km = 10*10 miles = 100 miles (approximately)
So, either way we can proceed. For bigger numbers we can proceed as above.
(Ref: Sudip Maji, B.C.Roy Engineering College)

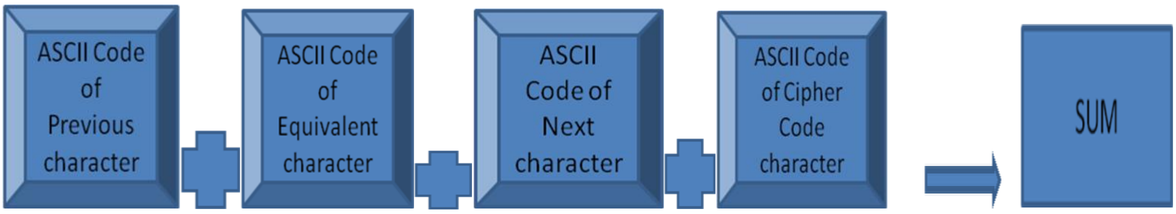
IV. Fibonacci in Coding

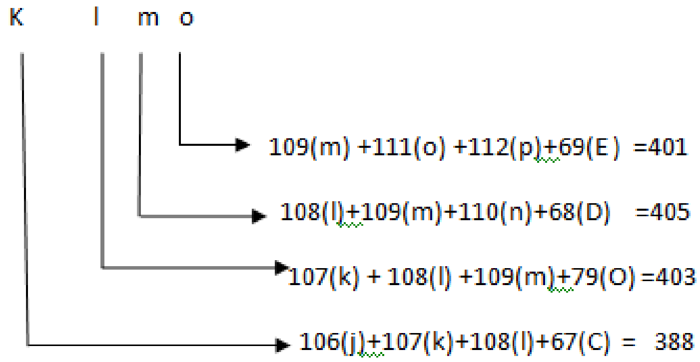
Recently Fibonacci sequence and golden ratio are of great interest to the researchers in many fields of science including high energy physics, quantum mechanics, Cryptography and Coding. Raghu and Ravishankar(2015) developed a paper of application classical encryption techniques for securing data.(Raphael and Sundaram,2012) showed that communication may be secured by the use of Fibonacci numbers. Similar application of Fibonacci in Cryptography is described here by a Simple Illustration.
Suppose that Original Message"CODE" to be Encrypted. It is sent through an unsecured channel. Security key is chosen based on the Fibonacci number. Any one character may be chosen as a first security key to generate cipher text and then Fibonacci sequence can be used. Agarwal et al (2015) used Fibonacci sequence for encryption data.

4.1 Method of Encryption

(a) For instance, let the first security key chosen be ‘k’.
Plain Text: **C O D E**
Characters: **k l m o** p q r s t u v w x y z a b c d e f g h I j k l ….
Fibonacci : **1 2 3 5** ……..
Cipher Text: **k l m o**
Cipher Text is converted into Unicode symbols and saved in a text file. The text file is transmitted over the transmission medium. It is the first level of security.
(b) Cipher text to Unicode
In the second level of security, the ASCII code of each character obtained from the cipher text plus the ASCII code of its previous character, and next character is added to the ASCII code of the equivalent character in the original message. Here, ASCII codes of four characters are used as a security key to further encode the characters available in the cipher text to Unicode symbols.

For instance,





By looking at the symbols in a text file no unknown persons can identify what it is and the message cannot be retrieved unless the re-trivial procedure is known.Mukherjee and Samanta(2014) developed a paper where they used Fibonacci numbers in hiding image cryptography.

4.2 Decryption method

The Decryption process follows a reverse process of Encryption. Recipient extracted each symbol from the received text file and mapped to find its hexadecimal value .Obtained value is converted into a decimal value to find out the plain text using the key. Without knowledge of the key an unknown person cannot understand the existence of any secret message.

V. Conclusion

The Fibonacci numbers are Nature’s numbering system. They appear everywhere in Nature, from the leaf arrangement in plants, to the pattern of the florets of a flower, the bracts of a pinecone, or the scales of a pineapple. The Fibonacci numbers are therefore applicable to the growth of every living thing, including a single cell, a grain of wheat, a hive of bees, and even all of mankind. Nature follows the Fibonacci numbers astonishingly. But very little we observe the beauty of nature. The Great poet Rabindranath Tagore also noted this. If we study the pattern of various natural things minutely we observe that many of the natural things around us follow the Fibonacci numbers in real life which creates strange among us. The study of nature is very important for the learners. It increases the inquisitiveness among the learners. The topic is chosen so that learners could be interested towards the study of nature around them. .Security in communication system is an interesting topic at present as India is going towards digitalization. A little bit of concept for securing data is also provided in this model. Let us finish by the words of Leonardo da Vinci "Learn how to see, Realize that everything connects to everything else".

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