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On $(LCS)_n$-manifold admitting M-projective curvature tensor

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Abstract: *The objective of this paper is to study M-projective flat, M-projective semisymmetric, M-projective Ricci pseudosymmetric $(LCS)_n$-manifold. Further we study $(LCS)_n$-manifold satisfying $W^* \cdot R = 0$ and $W^* \cdot S = 0$.*
***Key Words**: $(LCS)_n$-manifold, M-projective curvature tensor, M-projective flat, M-projective semisymmetric, M-projective Ricci pseudosymmetric.*
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I. Introduction

In 2003, Shaikh [15] introduced and studied Lorentzian concircular structure manifolds (briefly $(LCS)_n$-manifolds) with an example, which generalizes the notion of LP-Sasakian manifolds introduced by Matsumoto [8]. Also Shaikh et al. ([16, 17, 18, 19]), Prakasha [13], Yadav [26] studied various types of $(LCS)_n$-manifolds by imposing curvature restrictions.

In 1926, the concept of local symmetry of a Riemannian manifold was started by Cartan [1]. This notion has been used in several directions by many authors such as recurrent manifolds by Walker [25], semi-symmetric manifold by Szabo [21], pseudosymmetric manifold by Chaki [2], pseudosymmetric spaces by Deszcz [6], weakly symmetric manifold by Tamassy and Binh [23], weakly symmetric Riemannian spaces by Selberg [14]. Despite, the notions of pseudosymmetric and weak symmetry respectively by Chaki and Deszcz and Selberge and Tamassy and Binh are different. As a mild version of local symmetry, Takahashi [22] introduced the notion of ϕ-symmetry on a Sasakian manifold. In 2003, De et al. [5] introduced the concept of ϕ-recurrent Sasakian manifold, which generalizes the notion of ϕ-symmetry.

In 1971, Pokhariyal and Mishra [12] defined a tensor field W^* on a Riemannian manifold given by

(1.1) $$W^*(X,Y)Z = R(X,Y)Z - \frac{1}{2(n-1)}[S(Y,Z)X - S(X,Z)Y + g(Y,Z)QX - g(X,Z)QY]$$

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Such a tensor field W^* is known as M-projective curvature tensor. Ojha [10, 11] studied M-projective curvature tensor on Sasakian and Kaehler manifold. The properties of M-projective curvature tensor were also studied on different manifolds by Chaubey [3, 4], Venkatesha [24] and others.
Motivated by the above studies, we made an attempt to study M-projective curvature tensor on $(LCS)_n$-manifold.

The present paper is organized as follows: Section 2 is equipped with some preliminaries of $(LCS)_n$-manifold. In section 3, we have proved that if an n-dimensional $(LCS)_n$-manifold $M^n (n > 1)$ is M-projective flat if and only if the manifold is of constant scalar curvature $(\alpha^2 - \rho) \neq 0$. Section 4 deals with the study of $(LCS)_n$-manifold satisfying $W^* \cdot R = 0$. We study M-projective semisymmetric $(LCS)_n$-manifold in section 5. Section 6 is devoted to the study of M-projective Ricci-pseudosymmetric $(LCS)_n$-manifold. In the last section, we study $(LCS)_n$-manifold satisfying $W^* \cdot S = 0$.

II. Preliminaries

An n-dimensional Lorentzian manifold M^n is a smooth connected para-compact Hausdorff manifold with a Lorentzian metric g of type $(0,2)$ such that for each point $p \in M$, the tensor $g_p : T_p(M^n) \times T_p(M^n) \longrightarrow R$ is a non-degenerate inner product of signature $(-,+,+,\dots,+)$, where $T_p(M^n)$ denotes the tangent space of M^n at p and R is the real number space [15, 9]. In a Lorentzian manifold (M^n, g), a vector field P defined by

$$g(X,P) = A(X),$$

for any vector field $X \in \chi(M^n), (\chi(M^n),$ being the Lie algebra of vector fields on M^n) is said to be a concircular vector field [20] if

$$(\nabla_X A)(Y) = \alpha[g(X,Y) + \omega(X)A(Y)],$$

where α is a non-zero scalar function, A is a 1-form and ω is a closed 1-form.
Let M^n be a Lorentzian manifold admitting a unit time like concircular vector field ξ, called the characteristic vector field of the manifold. Then we have

(2.1) $$g(\xi,\xi) = -1.$$

Since ξ is a unit concircular vector field, there exists a non-zero 1-form η such that for

(2.2) $$g(X,\xi) = \eta(X),$$

the equation of the following form holds

(2.3) $$(\nabla_X \eta)(Y) = \alpha[g(X,Y) + \eta(X)\eta(Y)], \quad (\alpha \neq 0)$$

for all vector fields X and Y. Here ∇ denotes the covariant differentiation with respect to the Lorentzian metric g and α is a non-zero scalar function satisfying

(2.4) $$(\nabla_X \alpha) = (X\alpha) = d\alpha(X) = \rho\eta(X),$$

ρ being a certain scalar function given by

$$\rho = -(\xi\alpha).$$

If we put

(2.5) $$\phi X = \frac{1}{\alpha}\nabla_X \xi$$

then from (2.3) and (2.5) we have

(2.6) $$\phi^2 X = X + \eta(X)\xi,$$

(2.7) $$\eta(\xi) = -1, \quad \phi\xi = 0, \quad \eta(\phi X) = 0, \quad g(\phi X, \phi Y) = g(X,Y) + \eta(X)\eta(Y),$$

from which it follows that ϕ is a symmetric $(1,1)$-tensor, called the structure tensor of the manifold. Thus the Lorentzian manifold M together with the unit timelike concircular vector field ξ, its associated 1-form η and $(1,1)$-tensor field ϕ is said to be a Lorentzian concircular structure manifold (briefly $(LCS)_n$-manifold) [15]. Especially, if we take $\alpha = 1$, then we obtain the LP-Sasakian structure of Matsumoto [8].
In a $(LCS)_n$-manifold, the following relations hold [15]:

(2.8) $$\eta(R(X,Y)Z) = (\alpha^2 - \rho)[g(Y,Z)\eta(X) - g(X,Z)\eta(Y)],$$

(2.9) $$R(X,Y)\xi = (\alpha^2 - \rho)[\eta(Y)X - \eta(X)Y],$$

(2.10) $$R(X,\xi)Z = (\alpha^2 - \rho)[\eta(Z)X - g(X,Z)\xi],$$

(2.11) $$R(\xi,X)Y = (\alpha^2 - \rho)[g(X,Y)\xi - \eta(Y)X],$$

(2.12) $$R(\xi,X)\xi = (\alpha^2 - \rho)[X + \eta(X)\xi],$$

(2.13) $$S(X,\xi) = (n-1)(\alpha^2 - \rho)\eta(X), Q\xi = (n-1)(\alpha^2 - \rho)\xi,$$

(2.14) $$(\nabla_X \phi)(Y) = \alpha[g(X,Y)\xi + 2\eta(X)\eta(Y)\xi + \eta(Y)X],$$

(2.15) $$S(\phi X, \phi Y) = S(X,Y) + (n-1)(\alpha^2 - \rho)\eta(X)\eta(Y),$$

for all vector fields X, Y, Z and R, S respectively denotes the curvature tensor and the Ricci tensor of the manifold.

A $(LCS)_n$ manifold M^n is said to be a η-Einstein manifold if the following condition

(2.16) $S(X,Y) = \alpha g(X,Y) + \beta\eta(X)\eta(Y),$

holds on M^n. Here α and β are smooth functions. If $\beta = 0$ then the manifold reduces to an Einstein manifold.
From (1.1), we have

(2.17) $$\eta(W^*(\xi,Y)Z) = \frac{1}{2(n-1)}S(Y,Z) - \frac{1}{2}g(Y,Z),$$

(2.18) $$\eta(W^*(X,\xi)Z) = -\frac{1}{2(n-1)}S(X,Z) - \frac{3}{2}(\alpha^2-\rho)g(X,Z),$$

(2.19) $$\eta(W^*(X,Y)\xi) = 0,$$

(2.20) $W^*(X,Y)\xi = 0,\ \ W^*(X,\xi)\xi = 0,\ \ W^*(\xi,\xi)Z = 0,$

(2.21) $$W^*(X,\xi,Z,T) = \frac{1}{2(n-1)}S(X,Z)\eta(T) - \frac{1}{2(n-1)}S(X,T)\eta(Z) + \frac{1}{2}(\alpha^2-\rho)g(X,T)\eta(Z) - \frac{1}{2}(\alpha^2-\rho)g(X,Z)\eta(T),$$

(2.22) $$W^*(X,\xi,Z,\xi) = -\frac{1}{2(n-1)}S(X,Z) - \frac{1}{2}(\alpha^2-\rho)g(X,Z),$$

(2.23) $$W^*(X,\xi)Z = \frac{1}{2(n-1)}S(X,Z)\xi - \frac{1}{2}(\alpha^2-\rho)g(X,Z)\xi,$$

(2.24) $$(\nabla_U S)(X,\xi) = (n-1)\alpha(\alpha^2-\rho)[g(U,X) + \eta(U)\eta(X)] - \alpha S(X,\phi U).$$

III. $(LCS)_n$-manifold satisfying $W^* = 0$

Definition 3.1: An n-dimensional, $(n > 3)$ $(LCS)_n$-manifold M^n is said to be M-projective flat if the M-projective curvature tensor $W^* = 0$.

Theorem 3.1: An n-dimensional $(LCS)_n$-manifold M^n is M-projective flat if and only if the manifold is of constant scalar curvature $(\alpha^2 - \rho) \neq 0$.
Proof: Suppose $(LCS)_n$-manifold is M-projective flat. Then from (1.1), we get

(3.1) $$R(X,Y)Z = \frac{1}{2(n-1)}[S(Y,Z)X - S(X,Z)Y + g(Y,Z)QX - g(X,Z)QY].$$

By replacing $Z = \xi$ in (3.1) and making use of (2.7), (2.9) and (2.13), we obtain

(3.2) $$\eta(Y)QX - \eta(X)QY = (n-1)(\alpha^2-\rho)[\eta(Y)X - \eta(X)Y].$$

Again treating $Y = \xi$ in (3.2) and using (2.7) and (2.13), we get

(3.3) $$QX = (n-1)(\alpha^2-\rho)X,$$

which implies

(3.4) $$S(X,Y) = (n-1)(\alpha^2-\rho)g(X,Y).$$

In view of (3.3) and (3.4), (3.1) gives

(3.5) $$R(X,Y)Z = (\alpha^2-\rho)[g(Y,Z)X - g(X,Z)Y].$$

Conversely,
From the above equations (3.3), (3.4) and (3.5), (1.1) gives

(3.6) $$W^*(X,Y)Z = 0,$$

provided $(\alpha^2 - \rho) \neq 0$.
This completes the proof of the theorem.

IV. $(LCS)_n$-manifold satisfying $W^* \cdot R = 0$

Let M^n be an n-dimensional $(n > 3)$ $(LCS)_n$-manifold satisfying $W^* \cdot R = 0$. Then we have

(4.1) $$(W^*(\xi,Y)\cdot R)(U,V)T = W^*(\xi,Y)R(U,V)T - R(W^*(\xi,Y)U,V)T$$
$$-R(U,W^*(\xi,Y)V)T - R(U,V)W^*(\xi,Y)T = 0.$$

Theorem 4.1: If n-dimensional $(LCS)_n$-manifold satisfies $(W^*(\xi,X)\cdot R) = 0$, then the manifold is Einstein manifold with scalar curvature $r = n(n-1)(\alpha^2 - \rho)$, provided
$(\alpha^2 - \rho) \neq 0$.
Proof: By taking $T = \xi$ and using (2.9), the expression (4.1) reduces to

(4.2) $$(\alpha^2 - \rho)[\eta(W^*(\xi,Y)U)V - \eta(W^*(\xi,Y)V)U] = 0.$$

As $(\alpha^2 - \rho) \neq 0$, taking inner product of (4.2) with ξ and then using (2.17), we obtain

(4.3) $$(\alpha^2 - \rho)[\frac{1}{2(n-1)}S(Y,U)\eta(V) - \frac{1}{2(n-1)}S(Y,V)\eta(U) + \frac{1}{2}(\alpha^2 - \rho)g(Y,V)\eta(U)$$
$$-\frac{1}{2}(\alpha^2 - \rho)g(Y,U)\eta(V)] = 0.$$

Replacing V by ξ in (4.3), we have

(4.4) $$S(Y,U) = (n-1)(\alpha^2 - \rho)g(Y,U).$$

Contracting (4.4), we get

(4.5) $$r = n(n-1)(\alpha^2 - \rho).$$

Hence, the theorem.

V. M-projective semisymmetric $(LCS)_n$-manifold

An $(LCS)_n$-manifold is said to be M-projective symmetric if $\nabla W^* = 0$, and it is called M-projective semisymmetric if

(5.1) $$(R(X,Y)\cdot W^*)(U,V)G = 0.$$

Theorem 5.1: If in an n-dimensional $(LCS)_n$-manifold, the relation $R \cdot W^* = 0$ holds with the condition $(\alpha^2 - \rho) \neq 0$, then the manifold is Einstein manifold and the scalar curvature r of such a manifold is given by $r = (\alpha^2 - \rho)\frac{n(n-1)(n+4)}{(n+1)}$, provided $(\alpha^2 - \rho) \neq 0$.
Proof: Let M^n be a M-projective semisymmetric. Then from (5.1), we have

(5.2) $$R(\xi,Y)W^*(U,V)G - W^*(R(\xi,Y)U,V)G - W^*(U,R(\xi,Y)V)G$$
$$-W^*(U,V)R(\xi,Y)G = 0.$$

In view of (2.11) the above expression reduces to,

(5.3) $$(\alpha^2 - \rho)[g(Y,W^*(U,V)G)\xi - \eta(W^*(U,V)G)Y - g(Y,U)W^*(\xi,V)G$$

$$+\eta(U)W^*(Y,V)G - g(Y,V)W^*(U,\xi)G + \eta(V)W^*(U,Y)G$$
$$-g(Y,G)W^*(U,V)\xi + \eta(G)W^*(U,V)Y] = 0.$$

Taking inner product of the above equation with ξ and then using (2.2) and (2.7), we get

(5.4) $$(\alpha^2-\rho)[-g(Y,W^*(U,V)G) - \eta(W^*(U,V)G)\eta(Y) - g(Y,U)\eta(W^*(\xi,V)G)$$
$$+\eta(U)\eta(W^*(Y,V)G) - g(Y,V)\eta(W^*(U,\xi)G) + \eta(V)\eta(W^*(U,Y)G)$$
$$-g(Y,G)\eta(W^*(U,V)\xi) + \eta(G)\eta(W^*(U,V)Y)] = 0,$$

which on using (2.8), (2.17) - (2.19), gives

(5.5) $$(\alpha^2-\rho)[-R(U,V,G,Y) + \tfrac{1}{2(n-1)}\{S(V,G)g(U,Y) - S(U,G)g(V,Y)$$
$$+g(V,G)S(U,Y) - g(U,G)S(Y,V) + S(V,G)\eta(Y)\eta(U) - S(U,G)\eta(Y)\eta(V)$$
$$-g(Y,U)S(V,G) + (n-1)(\alpha^2-\rho)g(V,G)\eta(Y)\eta(U) - (n-1)(\alpha^2-\rho)$$
$$g(U,G)\eta(V)\eta(Y) - S(V,G)\eta(U)\eta(V) + S(Y,G)\eta(U)\eta(V) - (n-1)(\alpha^2-\rho)$$
$$g(V,G)\eta(Y)\eta(U) + (n-1)(\alpha^2-\rho)g(Y,G)\eta(V)\eta(U) + S(U,G)g(Y,V)$$
$$-S(G,U)\eta(Y)\eta(V) + S(Y,U)\eta(G)\eta(V) - S(V,Y)\eta(U)\eta(G) - (n-1)(\alpha^2-\rho)$$
$$g(G,U)\eta(U)\eta(V) + (n-1)(\alpha^2-\rho)g(Y,U)\eta(G)\eta(V) - (n-1)(\alpha^2-\rho)$$
$$g(V,Y)\eta(U)\eta(G) + S(U,Y)\eta(G)\eta(V) + (n-1)(\alpha^2-\rho)g(U,Y)\eta(V)\eta(G)\}$$
$$+2(\alpha^2-\rho)g(U,G)\eta(Y)\eta(V) - 2(\alpha^2-\rho)g(U,Y)\eta(G)\eta(V) + \frac{1}{2}(\alpha^2-\rho)$$
$$g(Y,U)g(V,G) - (\alpha^2-\rho)g(Y,G)\eta(V)\eta(U) + \frac{3}{2}(\alpha^2-\rho)g(U,G)g(Y,V)$$
$$+(\alpha^2-\rho)g(V,Y)\eta(U)\eta(G)] = 0.$$

Contracting (5.5), we obtain

(5.6) $$S(U,Y) = (\alpha^2-\rho)\tfrac{(n-1)(n+4)}{(n+1)}g(U,Y).$$

Hence the theorem.

VI. M-projective Ricci pseudosymmetric $(LCS)_n$-manifold

Definition 6.1: An n-dimensional Riemannian manifold (M^n, g) is said to be Ricci pseudosymmetric [7] if the condition

(6.1) $$(R(U,V)\cdot S)(Z,T) = L_S[((U\wedge V)\cdot S)(Z,T)],$$

holds on $U_S = \{x \in M : S \neq \frac{r}{n}g \ \ at\ x\}$, where L_S is some function on U_S.

Definition 6.2: An n-dimensional $(LCS)_n$-manifold (M^n, g) is said to be M-projectively Ricci pseudosymmetric if M-projective curvature tensor W^* satisfies

$$(W^*(U,V)\cdot S)(Z,T) = L_S[((U\wedge V)\cdot S)(Z,T)],$$

Or

(6.2) $$S(W^*(U,V)Z,T) + S(Z,W^*(U,V)T) = L_S[S((U\wedge V)Z,T) + S(Z,(U\wedge V)T)].$$

Theorem 6.1: If an $(LCS)_n$-manifold M^n is M-projective Ricci pseudosymmetric with restrictions $V = T = \xi$, then either $L_S = \frac{(\alpha^2-\rho)}{2}$ or the manifold is Einstein manifold, provided $(\alpha^2-\rho) \neq 0$.

Proof: Restricting $V = T = \xi$ in (6.2) and making use of (1.1), (2.7), (2.10), (2.13), we obtain

$$[L_S - \frac{(\alpha^2-\rho)}{2}]\{-S(U,Z) + (n-1)(\alpha^2-\rho)g(U,Z)\} = 0, \tag{6.3}$$

which implies either $L_S = \frac{(\alpha^2-\rho)}{2}$ or

$$S(U,Z) = (n-1)(\alpha^2-\rho)g(U,Z). \tag{6.4}$$

Thus, we proved the theorem.

VII. $(LCS)_n$-manifold satisfying the condition $W^* \cdot S = 0$

Let us suppose that M^n satisfies $W^* \cdot S = 0$.
Then

$$S(W^*(X,Y)Z,U) + S(Z,W^*(X,Y)U) = 0. \tag{7.1}$$

Theorem 7.1: If $(LCS)_n$-manifold satisfies the condition $W^* \cdot S = 0$, then it is Einstein manifold, provided $(\alpha^2 - \rho) \neq 0$.

Proof: Putting $X = \xi$ and in view of (1.1), the equation (7.1) gives

$$(\alpha^2-\rho)\{-\tfrac{1}{2}S(Y,Z)\eta(U) - \tfrac{1}{2}S(Y,U)\eta(Z) + \tfrac{1}{2}(n-1)(\alpha^2-\rho)g(Y,Z)\eta(U) \tag{7.2}$$
$$+\tfrac{1}{2}(n-1)(\alpha^2-\rho)g(Y,U)\eta(Z)\} = 0.$$

As $(\alpha^2-\rho) \neq 0$, by treating $U = \xi$ in (7.2), we obtain

$$S(Y,Z) = (n-1)(\alpha^2-\rho)g(Y,Z). \tag{7.3}$$

Hence, the theorem.

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References

[1] E. Cartan, Sur une classes remarquable despaces de Riemann, Bull. Soc. Math. France, 54, 214-264,1926.
[2] M. C. Chaki, On weakly symmetric manifolds, Analel stiintificae Ale Univeritatii Alenandru Ioan CuzzaIasi Romania, 33, 53-58, 1987.
[3] Chaubey S. K. and Ojha R. H, On the M-projective curvature tensor of a Kenmotsu manifold, Differential Geomerty-Dynamical Systems, 12, 52-60, 2010.
[4] Chaubey S. K, Some properties of LP-Sasakian manifolds equipped with M-Projective curvature tensor, Bulletin of Mathematical Analysis and Applications 3(4), 50-58, 2011.
[5] U. C. De, A. A. Shaikh and S. Biswas, On ϕ-recurrent Sasakian manifolds, Novi Sad J. Math, 33, 13-48,2003.
[6] R. Deszcz, On pseudosymmetric spaces, Acta Math. Hungar., 53, 481- 482, 1992.
[7] R. Deszcz, On pseudosymmetric spaces, Bull. Soci. Math., Belg.Ser., A 44(1), 1-34, 1992.
[8] K. Matsumoto, On Lorentzian almost paracontact manifolds, Bull. Yamagata Univ. Nat. Sci., 12, 151-156, 1989.
[9] B. O. Neill, Semi-Riemannian geometry, Academic Press, New York, 1983.
[10] Ojha R. H, A note on the m-projective curvature tensor, Indian J. pure Applied Math. 8(12), 1531-1534,1975.
[11] Ojha R. H, On Sasakian manifold, Kyungpook Math. J., 13, 211-215, 1973.
[12] Pokhariyal G. P. and Mishra R. S, Curvature tensor and their relativistic significance II, YokohamaMathematical Journal 19, 97-103, 1971.
[13] D. G. Prakasha, On Ricci η-recurrent $(LCS)_n$-manifolds, Acta Universitatis Apulensis, 24, 109-118,2010.
[14] A. Selberg, Harmonic analysis and discontinuous groups in weakly symmetric Riemannian spaces with applications to Dirichlet series, Indian Math. Soc., 20, 47-87, 1956.
[15] A. A. Shaikh, On Lorentzian almost paracontact manifolds with a structure of the concircular type, KyungpookMath.J., 43, 2, 305-314, 2003.
[16] A. A. Shaikh, Some results on $(LCS)_n$-manifolds, Journal of the Korean Mathematical Society, vol.46, 3, 449-461, 2009.
[17] A. A. Shaikh and S. K. Hui, On generalized ϕ-recurrent $(LCS)_n$-manifolds, AIP Conference Proceedings,1309, 419-429, 2010.
[18] A. A. Shaikh, T. Basu and S. Eyasmin, On the existence of ϕ-recurrent $(LCS)_n$-manifolds, Extracta-Mathematicae, 23, 1, 71-83, 2008.
[19] A. A. Shaikh and T. Q. Binh, On weakly symmetric $(LCS)_n$-manifolds, Journal of Advanced Mathematical Studies, 2, 2, 103-118, 2009.
[20] Schouten J.A, Ricci Calculas(second Edition), Springer-Verlag., 322, 1954.

[21] Z. I. Szabo, Structure theorems on Riemannian spaces satisfying $R(X, Y) \cdot R = 0$. I. The local version,J. Differential Geom. 17, no. 4, 531-582, 1982.

[22] T. Takahashi, Sasakian ϕ-symmetric space, Tohoku Math. J., 29, 91-113, 1977.

[23] L. Tamassy and T. Q. Binh, On weakly symmetric and weakly projective sym- metric Riemannianmanifold, Colloquia Math. Soc., 50, 663-667, 1989.

[24] Venkatesha and B. Sumangala, On m-projective curvature tensor of a generalized Sasakian space form,Acta Math. Univ. Comenianae, Vol. LXXXII, 2, 209-217, 2013.

[25] A. A. Walker, On Ruses spaces of recurrent curvature, Proc. London. Math.Soc., 52, 36-64, 1950.

[26] S. K. Yadav, P. K. Dwivedi and D. Suthar, On $(LCS)_{2n+1}$-manifolds satisfying certain conditions onthe concircular curvature tensor, Thai Jounal of Mathematics, 9, 3, 597-603, 2011.

[27] A. Yildiz, U. C. De and Erhan Ata, On a type of Lorentzian α-Sasakian manifolds, MATH. REPORTS,16(66), 1, 61-67, (2014).

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