

# MHD Free Convective Flow of a nanofluid over A Permeable Shrinking Sheet with Binary Chemical Reaction And Activation Energy

S.Anuradha<sup>1</sup>,K.Sasikala<sup>2</sup>

<sup>1</sup>Professor & Head in Department of Mathematics,Hindusthan College of Arts & Science,  
Coimbatore Tamilnadu,India

<sup>2</sup>Assistant Professor in Department of mathematics, Hindusthan College of Arts & Science,  
Coimbatore Tamilnadu,India

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**Abstract:** We have investigated a two dimensional unsteady free convective MHD flow and heat transfer of nanofluid over a permeable shrinking sheet with binary chemical reaction and activation energy. In this problem, mathematical formulation has designed for momentum, temperature and nanofluid solid volume profiles. Using similarity transformation, the coupled partial differential equations are non dimensionlized and then solved by using finite difference method. The results for the dimensionless velocity, temperature, and nanofluid solid volume profiles are discussed with the help of graphs.

**Keywords:** Nanofluid, MHD, permeable shrinking sheet, binary chemical reaction, activation energy

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## I. Introduction

A Nanofluid is a fluid containing nanometer sized particles called as nano particles. The nano particles used in nanofluids are typically made of metals, oxides, carbides or carbon nano tubes. Common base fluids include water, ethylene glycol and oil. Nanofluid is a two-phase mixture in which the solid phase consists of nano-seized particles. Nanofluid technology can help to develop better oils and lubricants. Magneto Hydrodynamics (MHD) is the academic discipline concerned with the dynamics of electrically conducting fluids include magnetic field. These fluids include salt water, liquid metals and ionized gases or plasmas. Nanofluid technology will help to develop better oils and lubricants. The use of nanofluids as coolants would allow for smaller size and better positioning of radiators. Nanofluids are used for cooling of microchips in computers and elsewhere. The boundary layer flow of an electrically conducting fluid in the presence of magnetic field has wide applications in many engineering problems such as MHD generator, plasma studies, nuclear reactors, geothermal energy extraction, and oil exploration.

The term nanofluid has been introduced by Choi[1].This novel fluid have been used potentially in numerous application in heat and mass transfer as well as micro electronics, fuel cells, Pharmaceutical sections. Sakisdís [2] analyzed the boundary layer flow on a continuous moving surface. Crane [3] found an exact solution of the boundary layer flow of the Newtonian fluid caused by the stretching of an elastic sheet moving in its own plane linearly.Buongiorno [4] was first who formulated the nanofluid model taking into account the effects of Brownian motion and thermophoresis. In his work he indicated that although there are some elements those affect nanofluid flow such as inertia, Brownian diffusion, thermophoresis, diffusiporesis, Magnus effect, fluid drainage and gravity only. Brownian diffusion and thermophoresis are important mechanisms in nanofluids. Wang [5] investigated the free convection flow on a vertical stretching surface.

Pop et al [6] investigated similarity solution by considering viscosity as an inverse function of temperature of the plate. There are several numerical studies on the modeling of natural convection heat transfer in nnaofluids.Gbadeyan et al [7] numerically studied natural convection flow of a nanofluid over a vertical plate with a uniform surface heat flux. Fang et al [8] used a second order a slip flow in their research. Choi and Eastman [9] discovered that the addition of less than 1% of nanoparticles into the base fluid doubles the heat conductivity of the fluid. Hamad et al [10] obtained the flow and mass transfer over a permeable sheet without fluid slip.Soundalgekar et al [11] studied the problem of flow of incompressible viscous fluid past a continuously moving semi-infinite plate by considering variable viscosity and variable temperature. Muthucumaraswamy [12] studied the effects of chemical reaction on vertical oscillating plate with variable temperature.Makinde [13] studied the effects of temperature dependent viscosity on free convective flow past a vertical porous plate in the presence of a magnetic field, thermal radiation and a first order homogeneous chemical reaction.

Carragher and Carane [14] examined heat transfer on a continuous stretching sheet. Magyari and Keller [15] studied both analytical and numerical solutions for boundary layer flow over an exponentially stretching surface with an exponential temperature distribution. The combined various effects of viscous dissipation and mixed convection on the flow of a viscous fluid over an exponentially stretching sheet were analyzed by Partha et al. [16], Elbashbeshy [17] investigated the flow and heat transfer over an exponentially stretching surface with wall mass suction numerically. Miklavcic and Wang [18] obtained the existence and uniqueness of steady viscous flow due to a shrinking sheet. Gireesha and Rudraswamy [19] studied the chemical reaction on MHD flow and heat transfer of a nanofluid near the stagnation point over a permeable stretching surface with non-uniform heat source/sink. Anuradha and Priyadarshini [20] analyzed MHD Free Convection Boundary Layer Flow of a nanofluid over a Permeable Shrinking Sheet in the Presence of Thermal Radiation and Chemical Reaction. We have investigated unsteady two dimensional MHD free convective boundary layer flow due to a permeable shrinking sheet with binary chemical reaction and activation energy. The results focused the binary chemical reaction parameter and activation energy of the flow.

## II. Mathematical Formulation

Consider a two dimensional unsteady MHD boundary layer flow of a nanofluid past a permeable shrinking sheet with the uniform velocity  $U$ . In the presence of magnetic field, strength  $B_0$  is applied parallel to the  $y$  axis and thermal radiation is considered in the flow region, it is assumed that the induced magnetic field, the external electric field are negligible due to polarization of charges. Let us consider  $x$  axis to be focused along the shrinking sheet and  $y$  axis normal to the surface.

The governing equation of conservation of mass, momentum, energy and nano particle volume in the presence of magnetic field towards a permeable shrinking sheet can be written in Cartesian coordinates  $x$  &  $y$ .

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\mu}{k_p} u - \frac{\sigma_e B_0^2 u}{\rho} + (1 - C_\infty) \rho f_\infty \beta g (T - T_\infty) - (\rho_f - \rho f_\infty) g (C - C_\infty) \tag{2}$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\partial}{\partial y} \left( \alpha \frac{\partial T}{\partial y} \right) + \tau \left\{ D_B \left( \frac{\partial T}{\partial y} \frac{\partial C}{\partial y} \right) + \frac{D_T}{T_\infty} \left( \frac{\partial T}{\partial y} \right)^2 \right\} + \frac{Q_0}{(\rho c)_f} (T - T_\infty) - \frac{1}{(\rho c)_f} \left( \frac{\partial q_r}{\partial y} \right) \tag{3}$$

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty} \frac{\partial^2 T}{\partial y^2} - K_r^2 (\phi - \phi_\infty) \left( \frac{T}{T_\infty} \right)^n e^{\left( \frac{-E_a}{\kappa T} \right)} \tag{4}$$

And the Boundary conditions are:

$$u = u_w(x, t) = -\frac{cx}{(1 - \lambda t)}, v = v_w(x, t), T = T_w(x, t), C = C_w(x, t) \text{ at } y = 0$$

$$u = 0, T \rightarrow T_\infty, C \rightarrow C_\infty \text{ as } y \rightarrow \infty$$

(5)

The relevant wall mass transfer velocity becomes

$$v_w(x, t) = -\sqrt{\frac{c}{(1 - \lambda t)}} s \tag{6}$$

Where  $s$  = constant wall mass transfer parameter

$s > 0$  for suction

and  $s < 0$  for injection, respectively.

$u, v$  = velocity components along  $x$  and  $y$  directions

$\alpha$  = thermal diffusivity

$Q_0$  = heat generation coefficient

$\rho$  = density of nanofluid,  $\rho_p$  = nanoparticle density

$\rho_c$  = specific heat of nanofluid at constant pressure

$\tau$  = the ratio of nanoparticle heat capacity,

$\sigma_e$  = the electrical conductivity

$C_p$  = the specific heat and constant pressure

$\beta$  = volumetric thermal expansion coefficient

$\mu$  = the thermal viscosity

$D_B$  = the Brownian diffusion coefficient

$D_T$  = the thermophoresis diffusion coefficient

$K_r^2(\phi - \phi_\infty) \left(\frac{T}{T_\infty}\right)^n e^{\left(\frac{E_a}{\kappa T}\right)}$  = the modified Arrhenius equation in which  $K_r^2$  is the reaction rate,  $E_a$  the

activation energy,  $\kappa = 8.61 \times 10^{-5} \text{eV/K}$  the Boltzmann constant and  $n$  fitted rate constant which generally lies in the range  $-1 < n < 1$ .

The Radiative heat flux term by using The Rosseland approximation is given by

$$q = \frac{4\sigma^*}{3k_1^*} \frac{\partial T^{*4}}{\partial y^*} \tag{7}$$

Where  $\sigma^*$  and  $k_1^*$  are the Stefan-Boltzmann constant and the mean absorption coefficient respectively. We assume that the temperature difference within the flow are sufficiently small such that  $T^{*4}$  may be expressed as a linear function of the temperature. This is accomplished by expanding in a Taylor Series about  $T_\infty^*$  and neglecting higher order terms. Thus,

$$T^{*4} \cong 4T_\infty^{*4} - 3T_\infty^{*4} \tag{8}$$

By using equation (6) and (7), into equation (3) is reduced to

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\partial}{\partial y} \left( \alpha \frac{\partial T}{\partial y} \right) + \tau \left\{ D_B \left( \frac{\partial T}{\partial y} \frac{\partial C}{\partial y} \right) + \frac{D_T}{T_\infty} \left( \frac{\partial T}{\partial y} \right)^2 \right\} + \frac{Q_0}{(\rho c)_f} (T - T_\infty) - \frac{1}{(\rho c)_f} \left( -\frac{16\sigma^* T_\infty^{*3}}{3k_1^*} \right) \frac{\partial^2 T^*}{\partial y^2} \tag{9}$$

By using the following set of similarity variables, the partial differential equations (2), (4) and (9) can be converted into the ordinary differential equation

$$\eta = y \sqrt{\frac{c}{v(1-\lambda t)}}, \psi = \sqrt{\frac{cv}{v(1-\lambda t)}} x f(\eta), \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \varphi(\eta) = \frac{C - C_\infty}{C_w - C_\infty} \tag{10}$$

Define  $\psi(x, y)$  as

$$u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x} \tag{11}$$

From the above transformations the non dimensional, non linear, coupled differential equations are obtained as:

$$f''' + ff'' - f'^2 - A \left( f' + \frac{\eta}{2} f'' \right) - Mf' - \delta f + Ra_x (\theta - Nr\varphi) = 0 \tag{12}$$

$$\frac{1}{Pr_{eff}} \theta'' + f\theta' - A \frac{\eta}{2} \theta' + Nb\varphi'\theta' + Nt\theta'^2 = 0 \tag{13}$$

$$\varphi'' + Le \left( f' - \frac{\eta}{2} f'' \right) \varphi' + \left( \frac{Nt}{Nb} \right) \theta'' - Le\sigma(1 + \delta\theta)^n \varphi e^{\left(\frac{E}{1+\delta\theta}\right)} = 0 \tag{14}$$

Where  $Pr_{eff} = \frac{Pr}{\left(1 + \frac{4R}{3}\right)}$  is Prandtl Number,  $A = \frac{\lambda}{c}$  is Heat Source Parameter,  $M = \frac{\sigma_e B_0^2}{(\rho c)_f}$  is Magnetic

Parameter,  $\lambda = \frac{Q_0 x^{1/2}}{(\rho c)_f \sqrt{(1-C_\infty)g\beta(T_w - T_\infty)}}$  is Heat source parameter,  $\delta = \frac{\mu}{k_\rho} u$  is Permeable Parameter,  $v = \frac{\mu}{\rho_f}$

is the Kinematic Viscosity,  $R = \frac{4\sigma^* T_\infty^3}{k\alpha_m \rho_f c}$  is Radiation Parameter,  $Le = \frac{\alpha}{D_B}$  is Lewis Number,

$Ra_x = \frac{(1-C_\infty)\beta g f_\infty (T_w - T_\infty)}{c^2 x}$  is the Local Rayleigh Number,  $Re_x = \frac{ax^2}{v_\infty}$  is the Local Reynolds

Number,  $Nt = \frac{\tau D_T (T_w - T_\infty)}{\nu T_\infty}$  is the Thermophoresis Parameter,  $E = \frac{E_a}{kT}$  is the activation energy,

$Nr = \frac{(\rho_p - \rho_{f_\infty}) \beta g f_\infty (C_w - C_\infty)}{(1 - C_\infty) f_\infty \beta (T_w - T_\infty)}$  is Buoyancy Ratio Parameter,  $Nb = \frac{\tau D_B (C_w - C_\infty)}{\nu}$  is Brownian

motion Parameter,  $\sigma = \frac{K_r^2}{c}$  is the reaction rate parameter.

Boundary conditions are reduced to

$$f = s, f' = -1, \theta = 1, \varphi = 1 \text{ at } \eta = 0$$

$$f = 0, \theta = 0, \varphi = 0 \text{ as } \eta \rightarrow \infty \tag{15}$$

### III. Numerical Analysis

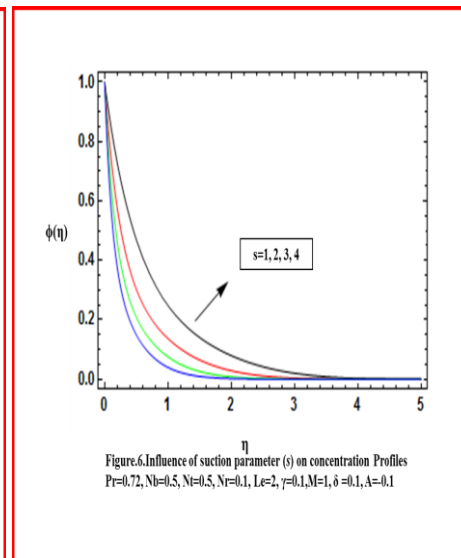
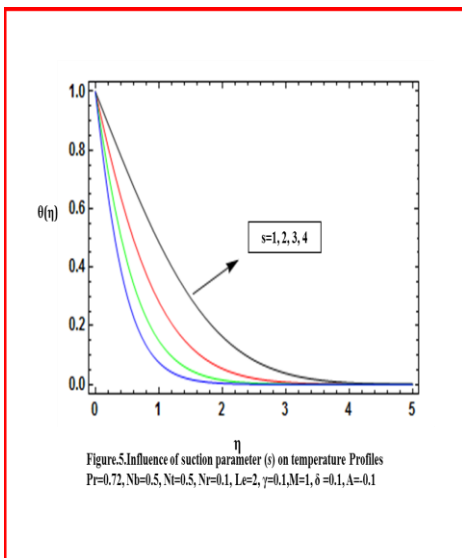
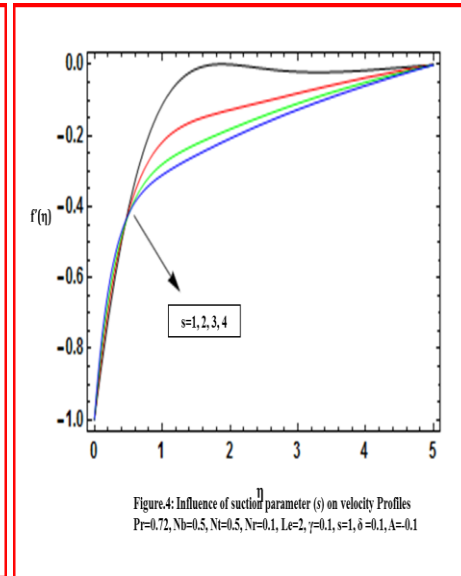
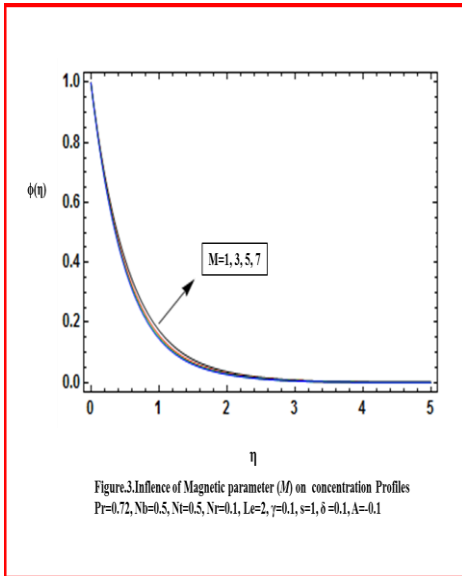
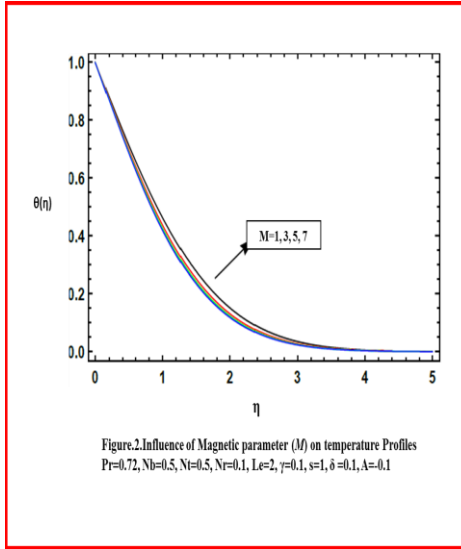
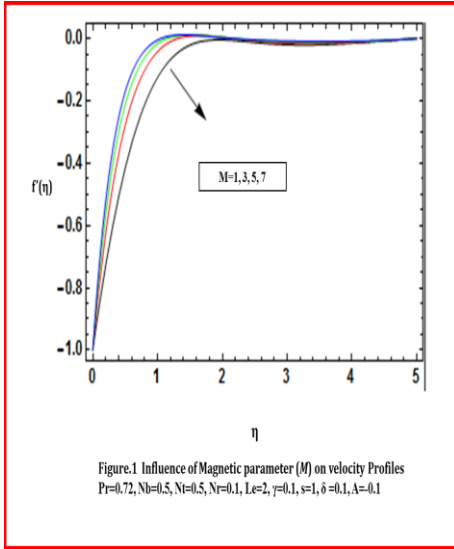
Using similarity transformation, the coupled partial differential equations are non dimensionized and then equations (12) – (14) subject to boundary conditions (15) are non linear and solved numerically by using finite difference method. The results for the dimensionless velocity, temperature, and nanofluid solid volume profiles are discussed with the help of graphs.

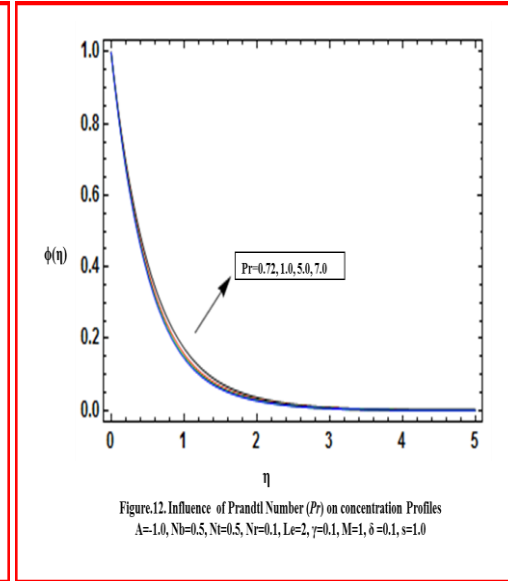
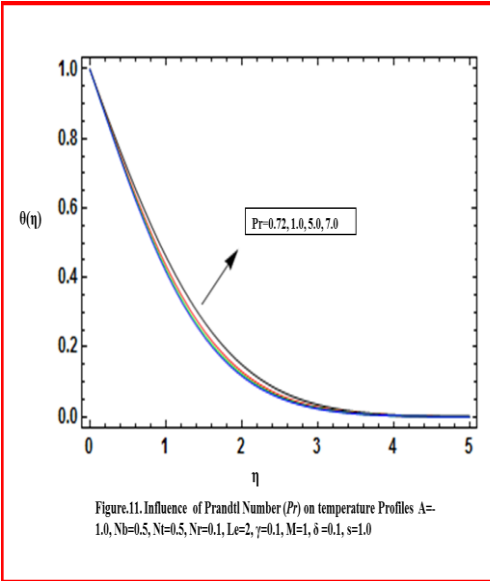
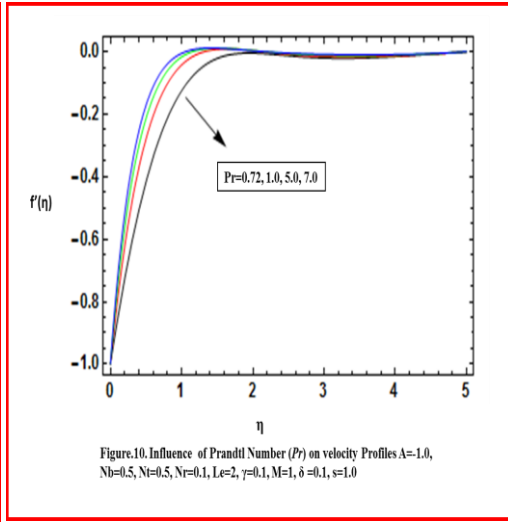
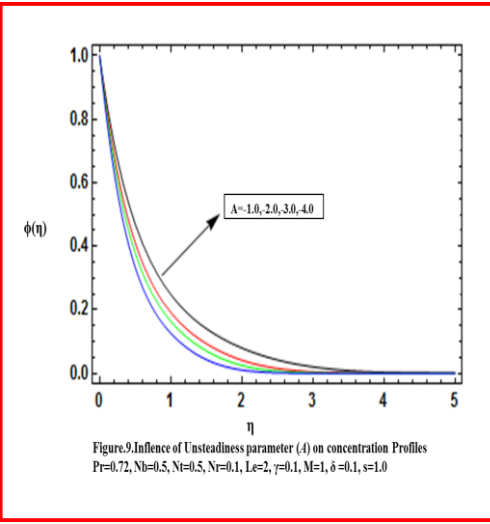
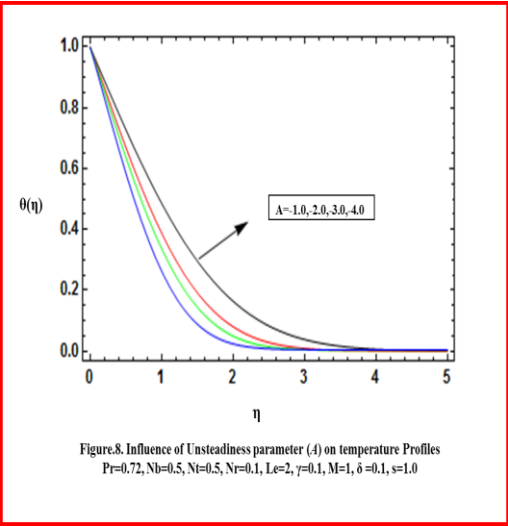
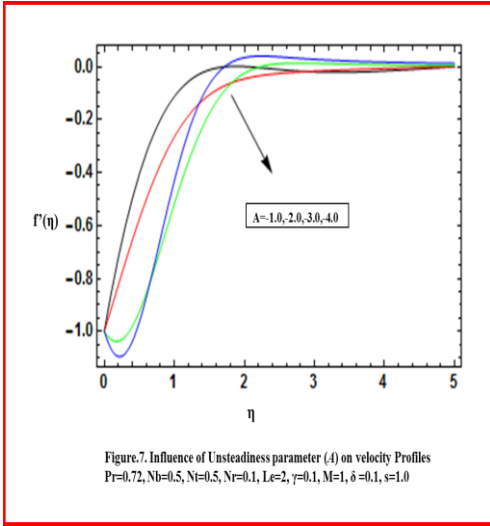
### IV. Results And Discussions

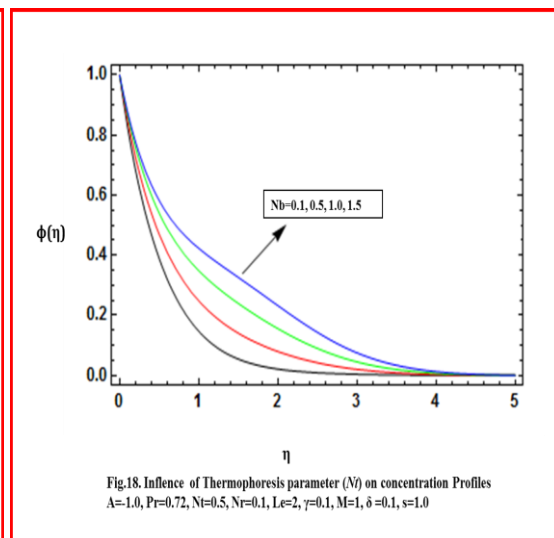
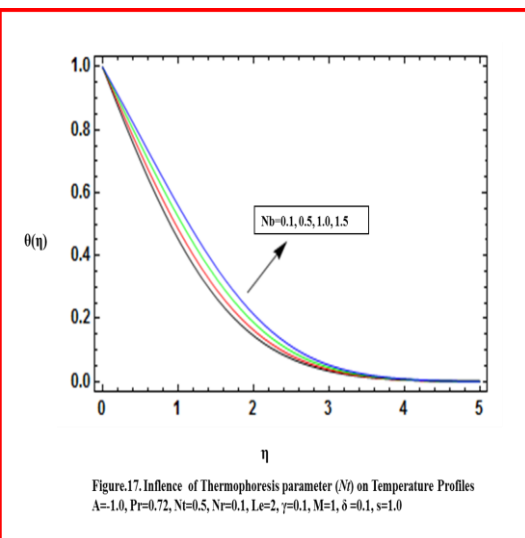
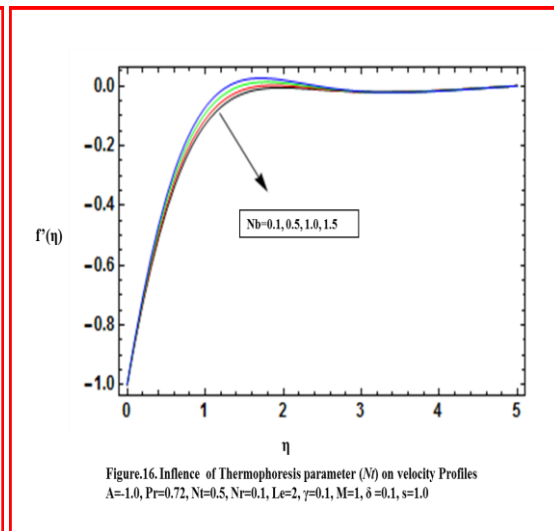
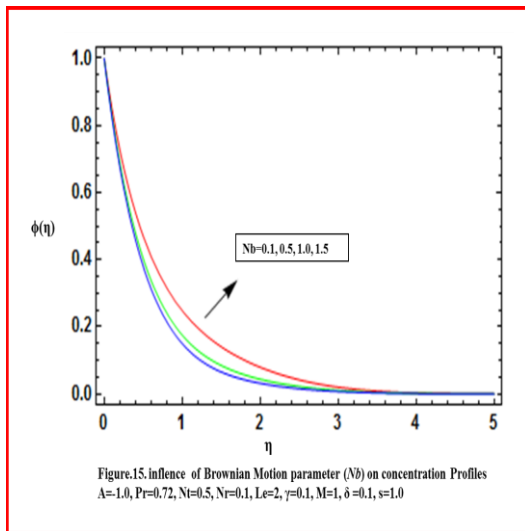
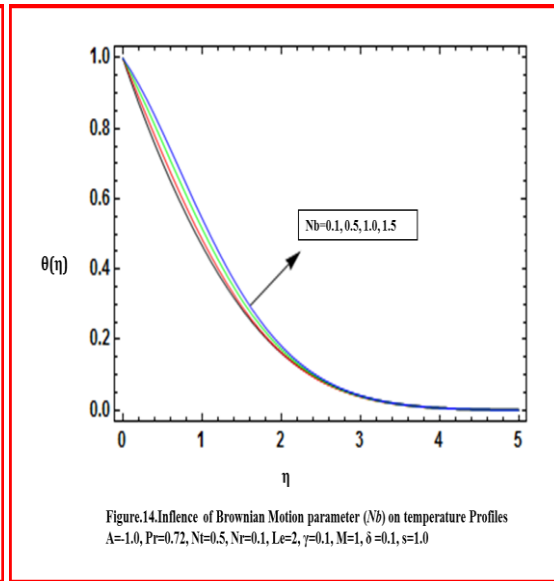
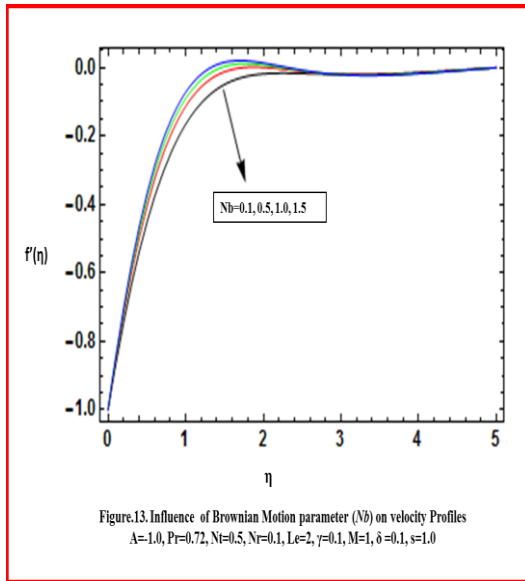
In this present study, MHD flow of the nanofluids over a permeable shrinking sheet in the presence of binary chemical reaction and activation has been studied. Consider that  $Pr=0.72$ ,  $Nb=0.5$ ,  $Nt=0.5$ ,  $Nr=0.1$ ,  $Le=2$ ,  $\gamma=0.1$ ,  $s=1$ ,  $\delta =0.1$ ,  $A=-0.1$ ,  $M=1$ . Graphs are displayed to explain the behavior of the flow fields with different non- dimensional parameters.

Fig.1, Fig.2 and Fig.3 are plotted to study the actions of different dimensionless parameters on the flow velocity, temperature and concentration profiles. It is conspicuous that elevating the values of Magnetic parameter ( $M$ ) decreases the fluid velocity but intensifies the both thermal and concentration profiles. Fig.4, Fig.5 and Fig.6 are portrayed the flow velocity, temperature and concentration distribution for different values of Suction parameter(s). It is noted that higher values of suction parameter(s) accelerating the velocity profile but reverse effect on temperature and concentration boundary layer. Fig.7, Fig.8 and Fig.9 visualized the variations of velocity, temperature and concentration distribution for different values of Unsteadiness parameter ( $A$ ). It is observed that inflating the values of Unsteadiness parameter ( $A$ ) show an enhancement in the flow velocity field but declined the temperature and concentration flow field.

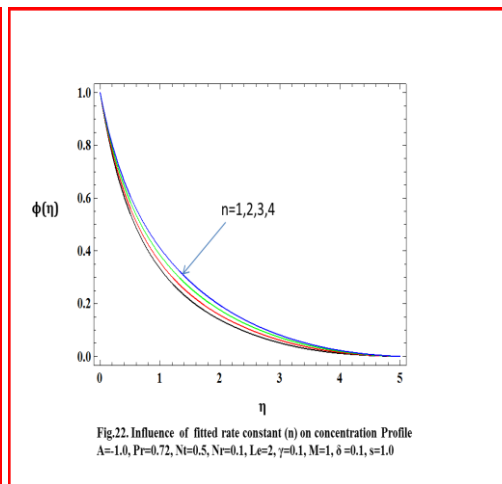
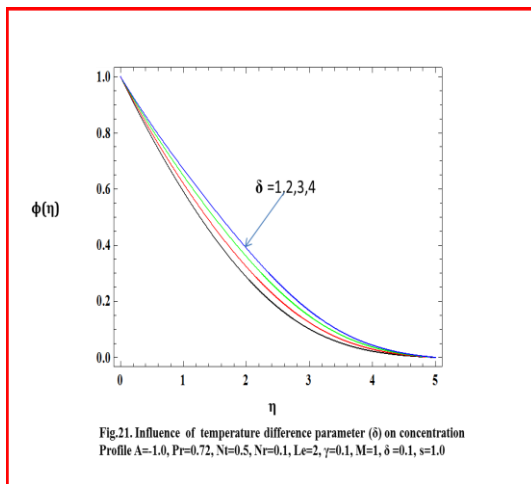
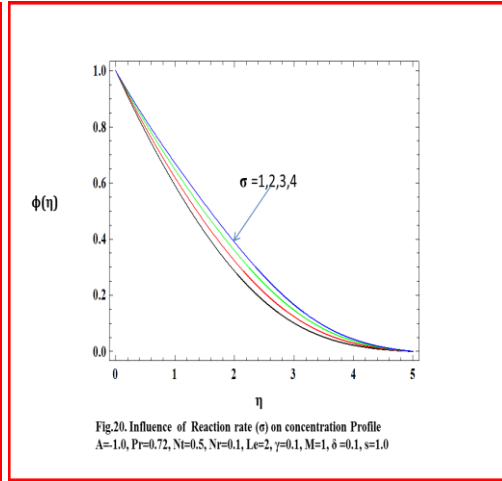
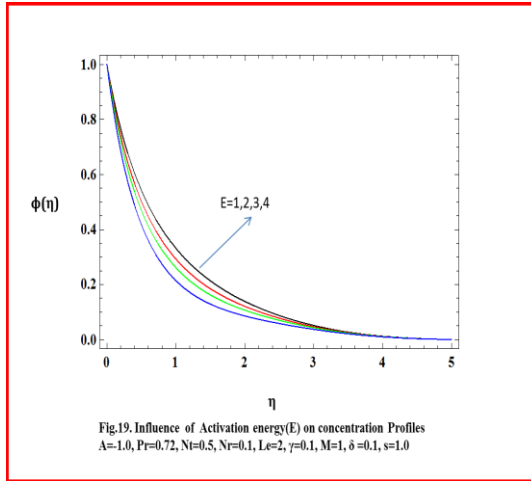
Fig.10, Fig.11 and Fig.12 exhibit the velocity, temperature and concentration distribution for various values of Prandtl Number ( $Pr$ ). It reveals that the higher value of  $Pr$  inflating the velocity profile and reduced the temperature and concentration flow fields. Fig.13, Fig.14 and Fig.15 plotted the effect of Brownian Motion parameter ( $Nb$ ) on the flow field velocity, temperature and concentration. It is established that there is a reduction in the temperature and concentration profile but reverse effect on velocity field with the hike in Brownian Motion parameter ( $Nb$ ). Fig.16, Fig.17 and Fig.18 displayed the impact of Thermophoresis parameter ( $Nt$ ) on the flow field velocity, temperature and concentration. It is noticed that the temperature and concentration fields are fall but velocity field rises with the improvement in Thermophoresis parameter ( $Nt$ ). Fig.19 and Fig.20 plotted the effect of Activation energy( $E$ ) and Reaction rate ( $\sigma$ ) on concentration profile. It is clear that larger values of  $E$  develops the concentration field. Increasing reaction rate ( $\sigma$ ) decreases concentration profile. Fig.21 and Fig.22 depicted the effect of temperature difference parameter( $\delta$ ) and fitted rate constant(n) on concentration profile. It observed that temperature difference parameter( $\delta$ ) and fitted rate constant(n) diminish the concentration field.











## V. Conclusion

In this present analysis, we have studied two dimensional unsteady free convection MHD flow and heat transfer of nanofluid over a permeable shrinking sheet with binary chemical reaction and activation energy. The validity of the present computations has been confirmed via benchmarking based on several earlier studies. The entire computation procedure is implemented using a program written in Mathematica software.

- The increase in magnetic parameter is to increase in velocity and decrease in concentration and temperature profiles.
- The increasing values of suction parameter accelerate the velocity and decrease in concentration and temperature profiles.
- As Prandtl number Pr decreases the thickness of thermal boundary layer becomes thickness of the velocity boundary layer. So the thickness of thermal boundary layer increases as Prandtl number Pr decreases and hence temperature profiles decreases with the increase of Prandtl number Pr.
- The increase of Unsteadiness parameter is to increases in velocity. the heat transfer rate increases with the increase of unsteadiness parameter A which in turn reduces the temperature of fluid.
- The thermophoresis parameter Nt phenomenon describes the fact that small micron size particular suspended in non-isothermal fluid will acquire a velocity in the direction of decreases temperature.
- Brownian motion parameters increase both the velocity and temperature profiles and decrease the concentration profile.
- Increasing values of E develops the concentration field. Increasing reaction rate (σ) decreases concentration profile.
- Higher values of temperature difference parameter(δ) and fitted rate constant(n) diminish the concentration field.



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