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Darcy-Brinkman Convection in a Couple-Stress Fluid Saturated With Porous Layer Using Thermal Non-Equilibrium Model

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Abstract: *Stability of a couple-stress fluid-saturated porous layer when the fluid and solid phases are not in local thermalequilibrium is analysed. The Darcy -Brinkman model is used for the momentum equation and a two-field model that represents the fluid andsolid phase temperature fields separately is used for energy equation. The linearstability analysis is used to obtain the condition for bothstationary and oscillatory convection. The effect of thermal non-equilibrium on the onset of both stationary andoscillatory convection isdiscussed.*

Keywords: *local thermal non-equilibrium, couple-stress fluid, convection, Rayleigh number.*

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I. Introduction

Thermal convection in fluid-saturated porous media is of considerable interest due to their numerous applications in differentfields such as geothermal energy utilization, oil reservoir modeling, building thermal insulation, and nuclear waste disposals tomention a few. The problem of convective instability of a horizontal fluid-saturated porous layer heated from below has beenextensively investigated and the growing volume of work devoted to this area is well documented by Ingham and Pop (1998).

In modeling a fluid-saturated porous medium all the above investigations on double diffusive convection have assumed a state of local thermal equilibrium (LTE) between the fluid and the solid phase at any point in the medium. This is a common practice for most of the studies where the temperature gradient at any location between the two phases is assumed to be negligible. For many practical applications, involving high-speed flows or large temperature differences between the fluid and solid phases, the assumption of local thermal equilibrium is inadequate and it is important to take account of the thermal non-equilibrium effects. Due to applications of porous media theory in drying, freezing of foods and other mundane materials and applications in everyday technology such as microwave heating, rapid heat transfer from computer chips via use of porous metal foams and their use in heat pipes, it is believed that local thermal non-equilibrium (LTNE) theory will play a major role in future developments.

Recently, attention has been given to the LTNE model in the study of convection heat transfer in porous media. Much of this work has been reviewed in recent books by Ingham and Pop (1998, 2005) and Nield and Bejan (2006). Criteria for heat and mass transfer models in metal hydride packed beds has been investigated by Kuznetsov and Vafai (1995)and effects of non-equilibrium were suggested to be more significant at high Reynolds number and for high porosity. Kuznetsov (1996) studied a perturbation solution for a thermal non-equilibrium fluid flow through a three-dimensional sensible storage packed bed. Vafai and Amiri (1998) gave detailed information about the work on thermal non-equilibrium effects of fluid flow through a porous packed bed. The review of Kuznetsov (1998) gives detailed information about the most but very latest works on thermal non-equilibrium effects on internal forced convection flows. An excellent review of research on local thermal non-equilibrium phenomena in porous medium convection, primarily free and forced convection boundary layers and free convection within cavities, is given by Rees and Pop (2005).

With growing importance of non-Newtonian fluids with suspended particles in modern technology and industries, the investigations of such fluids are desirable. The study of such fluids have applications in a number of processes that occur in industry, such as the extrusion of polymer fluids, solidification of liquid crystals, cooling of metallic plate in a bath, exotic lubrication and colloidal and suspension solutions. In the category of non-Newtonian fluids couple stress fluids have distinct features, such as polar effects. The theory of polar fluids and related theories are models for fluids whose microstructure is mechanically significant. The constitutive equations for couple stress fluids were given by Stokes (1966). The theory proposed by Stokes is the simplest one for micro-fluids, which allows polar effects such as the presence of couple stress, body couple and non-symmetric tensors.

In this paper, we are intended to perform the linear stability analysis of the thermal convection in a couple-stress fluidsaturatedporous layer subjected to the assumption that the fluid and solid phases are not in local thermalequilibrium. Our objective in this paper is to study how the onset criterion is altered by the presence of couple stress parameter and thermal nonequilibriumon both the steady and oscillatory modes.

II. Mathematical formulation

We consider an infinite horizontal couple-stressl fluid-saturated rotating porous layer of depth 'd', with the verticallydownward gravity force acting on it. A uniform temperature gradient of $T_l - T_u$ where $T_l > T_u$ is maintained between thelower and upper surfaces. A Cartesian frame of reference is chosen with the origin in the lower boundary and the z-axis verticallyupwards. The porous layer is subjected to the rotation with an angular velocity Ω about the z-axis. The Darcy-Brinkman model isused for the momentum equation.
The basic governing equations are

$$\nabla \cdot \mathbf{q} = 0, \quad (2.1)$$

$$\frac{\rho_0}{\varepsilon}\frac{\partial \mathbf{q}}{\partial t} = -\nabla p - \frac{1}{K}\left(\mu - \mu_c \nabla^2\right)\mathbf{q} + \rho_f\, \mathbf{g}, \quad (2.2)$$

$$\varepsilon\left(\rho_0\, c\right)_f \frac{\partial T_f}{\partial t} + \left(\rho_0\, c\right)_f \left(\mathbf{q}\cdot\nabla\right)T_f = \varepsilon\, k_f \nabla^2 T_f + h\left(T_s - T_f\right), \quad (2.3)$$

$$\left(1-\varepsilon\right)\left(\rho_0 c\right)_s \frac{\partial T_s}{\partial t} = \left(1-\varepsilon\right) k_s \nabla^2 T_s - h\left(T_s - T_f\right), \quad (2.4)$$

$$\rho = \rho_0\left[1 - \beta_t\left(T_f - T_l\right)\right], \quad (2.5)$$

where $\mathbf{q}(u, v, w)$is the velocity vector, p is the pressure, ε and K denote porosity and permeability, respectively, ρ and ν are density and kinematic viscosity, respectively.

In modeling energy equation for a fluid-saturated porous system, two kinds of theoretical approaches have been used. In the first model, the fluid and solid structures are assumed to be in local thermal equilibrium. This assumption is satisfactory for small-pore media such as geothermal reservoirs and fibrous insulations and small temperature differences between the phases. In the second kind of approach, the fluid and solid structures are assumed to be in thermal non-equilibrium. For many applications involving high-speed flows or large temperature difference between the fluid and solid phases, it is important to take account of the thermal non-equilibrium effects. If the temperatures difference between phases is a very important safety parameter (e.g., fixed bed nuclear propulsion systems and nuclear reactor modeling), the thermal non-equilibrium model in the porous media is an indispensable model.

The local thermal non-equilibrium, which account for the transfer of heat between the fluid and solid phases is considered. A two-field model that represents the fluid and solid phase temperature fields separately, is employed for the energy equation.

where $c$is the specific heat, k, the thermal conductivity and h being the inter-phase heat transfer coefficient and subscripts f and s denote fluid and solid phase, respectively. In two-field model the energy equations are coupled by means of the terms, which accounts for the heat lost to or gained from the other phase. The inter-phase heat transfer coefficient h depends on the nature of the porous matrix and the saturating fluid and the value of this coefficient has been the subject of intense experimental interest. Large values of h correspond to a rapid transfer of heat between the phases (LTE) and small values of h gives rise to relatively strong thermal non-equilibrium effects. In Eqs. (2)-(3) T_f and T_s are intrinsic average of the temperature fields and this allows one to set $T_f = T_s = T_w$ whenever the boundary of the porous medium is maintained at the temperature T_w.

The basic state of the fluid is quiescent and is given by,

$$\mathbf{q}_b = (0,0,0),\ p_f = p_{fb}(z),\ \rho = \rho_b(z),\ T_s = T_{sb}(z),\ h = 0. \quad (2.6)$$

The basic state temperatures of fluid phase and solid phase satisfy the equations

$$\varepsilon k_f \frac{d^2 T_{fb}}{dz^2} + h\left(T_{sb} - T_{fb}\right) = 0, \quad (2.7)$$

$$\left(1-\varepsilon\right)k_f\,\frac{d^2T_{sb}}{dz^2}+h\left(T_{fb}-T_{sb}\right)=0.$$
(2.8)

with boundary conditions

$T_{fb}=T_{sb}=T_l \qquad at \qquad z=0.$ (2.9)

$T_{fb}=T_{sb}=T_u \qquad at \qquad z=d,$ (2.10)

that the conduction state solutions are given by

$$T_{fb}=T_{sb}=\frac{T_l-T_u}{d}z+T_l. \tag{2.11}$$

Let us non dimensionalize using the following transformations,

$$\left(x,\ y,\ z\right)=\left(x^*,\ y^*,\ z^*\right)d,\ \ t=\frac{\left(\rho_0c\right)_f d^2}{k_f}t^*,\ \ \left(u',v',w'\right)=\frac{\varepsilon\,k_f}{\left(\rho_0c\right)_f d}\left(u^*,v^*,w^*\right),$$

$$T_f=(\Delta T)T_f^*,\ \ T'_f=(T_l-T_u)T_f^*,\quad T'_s=(T_l-T_u)T_s^*,.$$

(2.12)

to obtain non-dimensional equations in the form (on dropping the asterisks for simplicity),

$$\left[\left(\frac{1}{Va}\frac{\partial}{\partial t}+1\right)^2\nabla^2-C\left(\frac{1}{Va}\frac{\partial}{\partial t}+1\right)^2\nabla^4\right]w=Ra_T\left(\frac{1}{Va}\frac{\partial}{\partial t}+1\right)\nabla^2T_f$$

(2.13),

$$\left(\frac{\partial}{\partial t}-\nabla^2\right)T_f+\left(\mathbf{q}\cdot\nabla\right)T_f=w+H\left(T_s-T_f\right), \tag{2.14}$$

$$\alpha\left(\frac{\partial}{\partial t}-\nabla^2\right)T_s=\gamma\,H\left(T_f-T_s\right), \tag{2.15}$$

where $Va=\varepsilon\,\mathrm{Pr}/Da$, the Vadasz number, $Ra_T=\beta_T gd(T_l-T_u)K/\varepsilon\,\nu\kappa_f$, the thermal Rayleigh number, $H=hd^2/\varepsilon\,k_f$, the inter-phase heat transfer coefficient, $\alpha=\kappa_f/\kappa_s$, the ratio of diffusivities, $\gamma=\varepsilon\,k_f/(1-\varepsilon)k_s$, the porosity modified conductivity ratio, $Pr=\nu/\kappa_f$, $Da=K/d^2$, $C=\mu_c/\mu_f d^2$ and $\kappa_f=\dfrac{k_f}{\left(\rho_0c\right)_f}$ being the effective Prandtl number, Darcy number, Couple stress parameter and thermal diffusivity of the fluid respectively.

Since the fluid and solid phases are not in thermal equilibrium, the use of appropriate thermal boundary conditions may pose a difficulty. However, the assumption that the solid and fluid phases share the same temperatures at that of the boundary temperatures helps in overcoming this difficulty. Accordingly, Eqs. (13)-(16) are solved for stress free isothermal isosolutal boundaries. Hence the boundary conditions for the perturbed variables are given by

$$w=\frac{\partial^2w}{\partial z^2}=T_f=T_s=0 \qquad \text{at } z=0,\ d\ . \tag{2.16}$$

Linear stability analysis

The linear stability of the static solution can be discussed by neglecting all quadratic terms and seeking solutions of the form exp(ωt), where ω is the complex quantity.

$$Ra_T = \frac{1}{a^2}\left(\frac{\left[\delta\left(\frac{\omega}{Va}+1\right)+C\delta^2\right]}{\alpha\omega+\delta+\gamma H}\right)\left[\delta(\delta+H\left(1+\gamma\right)+\omega\left[\alpha\omega+\delta\left(1+\alpha\right)+H\left(\alpha+\gamma\right)\right]\right]. \quad (2.17)$$

The growth rate ω is in general a complex quantity such that $\omega = \omega_r + i\omega_i$. The system with $\omega_r < 0$ is always stable, while for $\omega_r > 0$ it will become unstable. For neutral stability state $\omega_r = 0$. Therefore, we now set $\omega = i\,\omega_i$ in Eq. (2.17) and clear the complex quantities from the denominator, to obtain

$$Ra_T = \Delta_1 + i\,\omega_i\,\Delta_2\,, \quad (2.18)$$

where

$$\Delta_1 = A_0\left(A_1 + A_2\right) \quad (2.19)$$

$$\Delta_2 = A_0\left(A_3 + A_4\right), \quad (2.20)$$

Stationary convection

The direct bifurcation (steady onset) corresponds to $\omega_i = 0$ and the steady convection occurs at

$$Ra_T = \frac{1}{a^2}\left(\frac{\left[\delta+C\delta^2\right]}{\delta+\gamma H}\right)\left[\delta(\delta+H\left(1+\gamma\right)\right].\,, \quad (2.21)$$

It is worth mentioning that the stationary Rayleigh number is independent of the diffusivity ratio of the fluid and solid phases and also the Vadasz number. When $H \to \infty$, Eq. (2.21) gives

$$Ra_T^{St} = \left(\frac{1+\gamma}{\gamma}\right)\left(\delta(\delta+C\delta^2\right)\,,$$

Oscillatory convection

For oscillatory onset $\Delta_2 = 0$ ($\omega_i \neq 0$) and this gives a dispersion relation of the form (on dropping the subscript *i*)

$$A\left(\omega^2\right)^2 + B\left(\omega^2\right) + C = 0\,, \quad (2.22)$$

Now Eq. (2.18) with $\Delta_2 = 0$, gives

$$Ra_T^{Osc} = A_0\left(A_1 + A_2\right). \quad (2.23)$$

III. Results And Discussion

Linear stability analysis of a horizontal couple-stress fluid-saturated porous layer is carried out by considering athermal non-equilibrium model. The onset thresholds of both marginal and oscillatory convection are derived analytically as the functions of inter-phase heat transfer coefficient, porosity modified conductivity ratio, Vadasz number, Darcy Prandtl number, couple stress parameter and diffusivity ratio. We found that there is competition between the processes of thermal and solute diffusion that causes the convective instability to set in as oscillatory mode rather than stationary. It is found that for both large and small inter-phase heat transfer coefficient the system behaves like a LTE model while the intermediate values have strong influence on each of stationary, oscillatory modes. The effect of porosity modified conductivity ratio, Vadasz number is to enhance the instability of system. The diffusivity ratio strengthens the stabilizing effect of inter-phase heat transfer coefficient. For very small values of *H*, the critical values are independent of the porosity modified conductivity ratio γ as there is almost no transfer of heat between the fluid and solid phases. Further it is reported that increase in the couple-stress parameter enhances the stability of the system.

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