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On Intuitionist Fuzzy P-Ideals and H-Ideals in Bci-Algebras

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Abstract:*The Aim Of This Paper Is To Introduce The Notion Of Intuitionistic Fuzzy P-Ideals And H-Ideals In Bci- Algebras And To Investigate Some Of Their Properties..*
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I. Introduction

The Notion Of Bck-Algebras Was Introduced By Imai And Iseki In 1966.In The Same Year ,Iseki[3] Introduced The Notion Of A Bci-Algebra Which Is A Generalization Of A Bck-Algebra. . After The Introduction Of The Concept Of Fuzzy Sets By L.A. Zadeh [11] ,Several Researches Were Conducted On The Generalization Of The Fuzzy Sets. Y. B. Jun And J. Meng [6]Introduced Fuzzy P-Ideals In Bci-Algebras And Studied Several Properties. H.M.Khalid And B.Ahmad [8] Introduced Fuzzy H-Ideals In Bci-Algebras. The Idea Of Intuitionistic Fuzzy Set Was First Introduced By K.T.Atanassov [1,2], As A Generalization Of The Notion Of Fuzzy Set. In This Paper Using Atanassov's Idea ,We Establish The Intuitionistic Fuzzification Of The Concept Of
P-Ideals And H-Ideals In Bci-Algebras And Investigate Some Of Their Properties .

Preliminaries
In This Section We Include Some Elementary Definitions That Are Necessary For This Paper.

Definition 2.1 [3] An Algebra $(X , * ,0)$ Of Type (2,0) Is Called A Bci-Algebra If It Satisfies The Following Axioms:
(1) $((X*Y)*(X*Z))*(Z*Y) =0$,
(2)$(X*(X*Y))*Y=0$,
(3) $X * X = 0$,
(4) $X * Y = 0$ And $Y * X = 0$ Imply $X = Y$, For All $X ,Y ,Z\in X$.
In A Bci-Algebra X, We Can Define A Partial Ordering " ≤ " By Putting $X\le Y$ If And Only If $X*Y=0$.
In A Bci- Algebra X ,The Following Hold :
(5) $(X*Y)*Z = (X*Z)*Y$
(6) $X*0 = X$,
(7) $0*(X*Y) = (0*X)*(0*Y)$,
(8) $0 *(0* (X * Y)) = 0* (Y*X)$
(9) $(X * Z)* (Y * Z) \le X*Y$,
(10) $X \le Y$ Implies $X*Z \le Y*Z$ And $Z*Y \le Z*X$, For All $X ,Y, Z \in X$.

Example 2.2 The Set X = { 0, 1, 2, 3 } With The Following Cayley Table Is A Bci - Algebra .

*	0	1	2	3
0	0	0	0	3
1	1	0	0	3
2	2	2	0	3
3	3	3	3	0

Throughout This Paper X Always Means A Bci-Algebra Without Any Specification.
Definition 2.3 A Non- Empty Subset A Of X Is Called An Ideal Of X If
(1) $0 \in A$,
(2) $X * Y \in A$ And $Y \in A$ Imply $X \in A$.
Definition 2.4 [4] An Ideal A Of X Is Said To Be Closed If For All $X \in X$
$0 * X \in A$ Implies $X \in A$.
Definition 2.5[12] A Non- Empty Subset A Of X Is Called A P-Ideal Of X If
(1) $0 \in A$,
(2) If For All $X, Y, Z \in X$, $(X * Z)*(Y * Z) \in A$ And $Y \in A$ Imply $X \in A$.
If We Put Z=0, Then It Follows That A Is An Ideal. Thus Every P-Ideal Is An Ideal.
Proposition2.6[12] An Ideal Of A Bci-Algebra X Is A P-Ideal If And Only If $0*(0*X)\, Y \in A$

Implies $X \in A$,Where $X \in X$.

Definition 2.7 [8] A Non- Empty Subset A Of X Is Called An H-Ideal Of X If
(1) $0 \in A$,
(2) $X * (Y * Z) \in A$ And $Y \in A$ Imply $X * Z \in A$.
If We Put Z=0,Then It Follows That A Is An Ideal. Thus Every H-Ideal Is An Ideal.

Definition 2.8 [11] Let X Be A Non-Empty Set. A Fuzzy Set μ In X Is A Function
$\mu : X \to [0, 1]$.
Definition 2.9 [6] Let μ Be A Fuzzy Set In X. For $T \in [0,1]$,The Set $\mu_T = \{ X \in X \mid \mu(X) \geq T\}$ Is Called A Level Subset Of μ.
Definition 2.10 [6] A Fuzzy Set μ In X Is Called A Fuzzy Ideal Of X If For All $X, Y \in X$ We Have
(1) $\mu(0) \geq \mu(X)$,
(2)$\mu(X) \geq \text{Min}\{ \mu(X * Y), \mu(Y)\}$.
Definition 2.11 [6] A Fuzzy Ideal μ In X Is Said To Be Closed If For All $X \in X$,
$\mu(0 * X) \geq \mu(X)$,
Definition 2.12 [6] A Fuzzy Set μ In X Is Called A Fuzzy P-Ideal Of X If For All $X, Y, Z \in X$ We Have
(1) $\mu(0) \geq \mu(X)$,
(2)$\mu(X) \geq \text{Min}\{ \mu((X * Z) * (Y * Z)), \mu(Y)\}$.

Definition 2.13 [6] A Fuzzy Set μ In X Is Called A Fuzzy H-Ideal Of X If For All $X, Y, Z \in X$,
(1) $\mu(0) \geq \mu(X)$,
(2)$\mu(X * Z) \geq \text{Min}\{ \mu(X * (Y * Z)), \mu(Y)\}$.
Clearly Z=0 Gives μ Is A Fuzzy Ideal.
Definition 2.14 [2] An Intuitionistic Fuzzy Set (Ifs) A In A Non Empty Set X Is An Object Having The Form $A = \{ < X, \mu_A(X), \nu_A(X) > / X \in X \}$, Where The Functions $\mu_A : X \to [0,1]$ And $\nu_A : X \to [0,1]$ Denote The Degree Of Membership And The Degree Of Non Membership Of Each Element $X \in X$ To The Set A, Respectively, And $0 \leq \mu_A(X) + \nu_A(X) \leq 1$ For All $X \in X$.

Notation: For The Sake Of Simplicity, We Shall Use The Symbol $A = < \mu_A, \nu_A >$ For The Ifs $A = \{ < X, \mu_A(X), \nu_A(X) > / X \in X \}$.
Definition 2.15 [2] Let A Be An Intuitionistic Fuzzy Set Of A Set X. For Each Pair $< T, S > \in [0, 1]$, The Set $A_{<T,S>} = \{ X \in X : \mu_A(X) \geq T \text{ And } \nu_A(X) \leq S \}$ Is Called The Level Subset Of A .

Definition 2.16 [2] Let A Be An Ifs In X And Let $T \in [0, 1]$ .Then The Sets
$U(\mu_A; T) = \{ X \in X : \mu_A(X) \geq T \}$And $L(\nu_A, T) = \{ X \in X : \nu_A(X) \leq T \}$ Are Called
A μ -Level T -Cut And ν- Level T-Cut Of A , Respectively.

II. Intuitionistic Fuzzy P-Ideals

Definition 3.1 An Intuitionistic Fuzzy Set A In X Is Called An Intuitionistic Fuzzy P- Ideal Of X If For All $X, Y, Z \in X$ We Have
(1) $\mu_A(0) \geq \mu_A(X)$,
(2) $\nu_A(0) \leq \nu_A(X)$,
(3) $\mu_A(X) \geq \text{Min}\{ \mu_A((X * Z) * (Y * Z)), \mu_A(Y)\}$,

(4) $\nu_A(X) \leq \text{Max}\{\nu_A((X * Z) * (Y * Z)), \nu_A(Y)\}$.

Example 3.2. Let X = { 0,1,2,3 } With The Following Cayley Table Be A Bci Algebra.

*	0	1	2	3
0	0	0	3	2
1	1	0	3	2
2	2	2	0	3
3	3	3	2	0

Let A= < μ_A, ν_A > Be An Ifs In X Defined By
$\mu_A(0) = \mu_A(1) = 0.9$, $\mu_A(2) = \mu_A(3) = 0.09$ And $\nu_A(0) = \nu_A(1) = 0.09$ And $\nu_A(2) = \nu_A(3) = 0.9$. Then A Is An Intuitionistic Fuzzy P-Ideal Of X.

Proposition 3.3 An Intuitionistic Fuzzy Ideal Of A Bci-Algebra Is An Intuitionistic Fuzzy P-Ideal If And Only If $\mu_A(X) \geq \mu_A(0 * (0 * X))$ And $\nu_A(X) \leq \nu_A(0 * (0 * X))$.
Notation. Let A And B Be Intuitionistic Fuzzy Sets In X . By A ≤ B We Mean That
$\mu_A(X) \leq \mu_B(X)$ And $\nu_A(X) \geq \nu_B(X)$

Theorem 3.4. Let A=(μ_A, ν_A) In X Be An Intuitionistic Fuzzy Ideal Of X. If X ≤Y ,Then
$\mu_A(X) \geq \mu_A(Y)$,$\nu_A(X) \leq \nu_A(Y)$, That Is , μ_A Is Order Reversing And ν_A Is Order Preserving.
Proof. Let X ,Y ∈X Such That X ≤ Y. Then
X∗Y =0 And Thus
$\mu_A(X) \geq \text{Min}\{ \mu_A(X * Y), \mu_A(Y) \}$
$= \text{Min}\{ \mu_A(0), \mu_A(Y)\}$
$= \mu_A(Y)$
And
$\nu_A(X) \leq \text{Max}\{ \nu_A(X * Y), \nu_A(Y) \}$
$= \text{Max}\{ \nu_A(0), \nu_A(Y)\}$
$= \nu_A(Y)$

Theorem3.5. Let A=(μ_A, ν_A) In X Be An Intuitionistic Fuzzy Ideal Of X. If X∗Y ≤Z ,Then
$\mu_A(X) \geq \text{Min}\{ \mu_A(Y), \mu_A(Z) \}$
$\nu_A(X) \leq \text{Max}\{ \nu_A(Y), \nu_A(Z) \}$
Proof. Let X ,Y , Z ∈X Such That X∗Y ≤ Z. Then
(X∗Y)∗Z =0 And Thus
$\mu_A(X) \geq \text{Min}\{ \mu_A(X * Y), \mu_A(Y) \}$
$\geq \text{Min}\{\text{Min}\{ \mu_A((X * Y) * Z), \mu_A(Z)\}, \mu_A(Y) \}$
$= \text{Min}\{\text{Min}\{ \mu_A(0), \mu_A(Z)\}, \mu_A(Y) \}$
$= \text{Min}\{ \mu_A(Y), \mu_A(Z)\} \}$
And
$\nu_A(X) \leq \text{Max}\{ \nu_A(X * Y), \nu_A(Y) \}$
$\geq \text{Max}\{\text{Max}\{ \nu_A((X * Y) * Z), \nu_A(Z)\}, \nu_A(Y) \}$
$= \text{Max}\{\text{Max}\{ \nu_A(0), \nu_A(Z)\}, \nu_A(Y) \}$
$= \text{Max}\{ \nu_A(Y), \nu_A(Z)\} \}$

Theorem 3.6 Let A And B Be Intuitionistic Fuzzy Ideals Of X Such That A ≤ B And
$\mu_A(0) = \mu_B(0)$ And $\nu_A(0) = \nu_B(0)$ .If A Is An Intuitionistic Fuzzy Ideal Of X ,Then So Is B.
Proof. By Proposition 3.3, It Is Enough To Show That
$\mu_B(X) \geq \mu_B(0*(0*X))$ And $\nu_B(X) \leq \nu_B(0*(0*X))$ For Each X ∈X
Putting $S=0*(0*X)$,Then $0*(0*(X*S)) = (0*(0*X))*(0*(0*S)) = (0*(0*X))*(0*(0*(0*(0*X))))$
$=(0*(0*X))*(0*(0*X))=0$ By Properties (7) And (8).Hence
$\mu_A(0*(0*(X*S))) = \mu_A(0) = \mu_B(0)$ And $\nu_A(0*(0*(X*S))) = \nu_A(0) = \nu_B(0)$ .Since A Is An Intuitionistic Fuzzy P-Ideal Of X And Using Proposition 3.3,We Get
$\mu_A(X*S) \geq \mu_A(0*(0*(X*S))) = \mu_B(0)$ And $\nu_A(X*S) \leq \nu_A(0*(0*(X*S))) = \nu_B(0)$
Thus $\mu_B(X*S) \geq \mu_A(X*S) \geq \mu_B(0) \geq \mu_B(S)$ And $\nu_B(X*S) \leq \nu_A(X*S) \leq \nu_B(0) \leq \nu_B(S)$.

Since B Is An Intuitionistic Fuzzy Ideal ,We Have
$\mu_B(X) \geq \text{Min}\{ \mu_B(X*S), \mu_B(S) \} = \mu_B(S) = \mu_B(0*(0*X))$ And
$\nu_B(X) \leq \text{Max}\{ \nu_B(X*S), \nu_B(S) \} = \nu_B(S) = \nu_B(0*(0*X))$
So B Is An Intuitionistic Fuzzy P-Ideal Of X.

Theorem 3.7 An Intuitionistic Fuzzy Set A Of A Bci-Algebra X Is An Intuitionistic Fuzzy P-Ideal Of X If And Only If For Each Pair $T, S \in [0,1]$,The Level Subset
$A_{<T,S>} = \{ X \in X : \mu_A(X) \geq T \text{ And } \nu_A(X) \leq S \}$ Is Either Empty Or A P-Ideal Of X.
Proof.($\Rightarrow$): Assume That A Is An Intuitionistic Fuzzy P-Ideal Of X And $A_{<T,S>} \neq \phi$ For Any $T, S \in [0,1]$. Then $\mu_A(0) \geq \mu_A(X)$ And $\nu_A(0) \leq \nu_A(X)$ .Therefore $\mu_A(0) \geq \mu_A(X) \geq T$ And $\nu_A(0) \leq \nu_A(X) \leq S$ For $T, S \in [0,1]$ Or $\mu_A(0) \geq T$ And $\nu_A(0) \leq S$ Imply $0 \in A_{<T,S>}$.Next ,Let $(X*Z)*(Y*Z) \in A_{<T,S>}$ And $Y \in A_{<T,S>}$.Then
$\mu_A((X*Z)*(Y*Z)) \geq T$, $\mu_A(Y) \geq T$ And $\nu_A((X*Z)*(Y*Z)) \leq S$, $\nu_A(Y) \leq S$ .Since A Is An Intuitionistic Fuzzy P-Ideal Of X, Therefore For All X ,Y ,Z In X ,
$\mu_A(X) \geq \text{Min}\{ \mu_A((X*Z)*(Y*Z)), \mu_A(Y) \} \geq T$ And $\nu_A(X) \leq \text{Max}\{ \nu_A((X*Z)*(Y*Z)), \nu_A(Y) \} \leq S$
Or $\mu_A(X) \geq T$ And $\nu_A(X) \leq S$ Imply $X \in A_{<T,S>}$.This Proves That The Level Set $A_{<T,S>}$ Is A P-Ideal Of X.
($\Leftarrow$): Suppose That For Each Pair $T, S \in [0,1]$, $A_{<T,S>}$ Is Either Empty Or A P-Ideal Of X.
For Any $X \in X$,Setting $\mu_A(X) = T$ And $\nu_A(X) = S$,Then $X \in A_{<T,S>}$ Since $A_{<T,S>} (\neq \phi)$ Is A P-Ideal Of X ,We Have $0 \in A_{<T,S>}$ And Hence $\mu_A(0) \geq T = \mu_A(X)$ And $\nu_A(0) \leq S = \nu_A(X)$.Thus
$\mu_A(0) \geq \mu_A(X)$ And $\nu_A(0) \leq \nu_A(X)$ For All $X \in X$ .Now We Prove That
$\mu_A(X) \geq \text{Min}\{ \mu_A((X*Z)*(Y*Z)), \mu_A(Y)\}$ And $\nu_A(X) \leq \text{Max}\{\nu_A((X*Z)*(Y*Z)), \nu_A(Y)\}$.
If Not ,Then ,There Exist $X_0.Y_0,Z_0 \in X$ Such That $\mu_A(X_0) < \text{Min}\{ \mu_A((X_0*Z_0)*(Y_0*Z_0)), \mu_A(Y_0)\}$ And $\nu_A(X_0) > \text{Max}\{\nu_A((X_0*Z_0)*(Y_0*Z_0)), \nu_A(Y_0)\}$.
Put $T_0 = ½[\mu_A(X_0) + \text{Min}\{ \mu_A((X_0*Z_0)*(Y_0*Z_0)), \mu_A(Y_0)\}]$ And
$S_0 = ½[\nu_A(X_0) + \text{Max}\{\nu_A((X_0*Z_0)*(Y_0*Z_0)), \nu_A(Y_0)\}]$.Then
$\mu_A(X_0) < T_0 < \text{Min}\{ \mu_A((X_0*Z_0)*(Y_0*Z_0)), \mu_A(Y_0)\}$ And
$\nu_A(X_0) > S_0 > \text{Max}\{\nu_A((X_0*Z_0)*(Y_0*Z_0)), \nu_A(Y_0)\}$.Hence $(X_0*Z_0)*(Y_0*Z_0) \in A_{<T,S>}$ And $Y_0 \in A_{<T,S>}$ .But $X_0 \notin A_{<T,S>}$,Thus $A_{<T,S>}$ Is Not A P-Ideal Of X. This Contradicts The Hypothesis.
Therefore A Is An Intuitionistic Fuzzy P-Ideal Of X.

Theorem 3.8. Let I Be A P-Ideal Of X. Then There Exists An Intuitionistic Fuzzy P-Ideal A Of X Such That $A_{<T,S>} = I$,For Some $T, S \in [0,1]$
Proof. Define A Such That $\mu_A(X) = \begin{cases} T & X \in I, \\ 0 & X \notin I, \end{cases}$ And $\nu_A(X) = \begin{cases} S & X \in I, \\ 1 & X \notin I. \end{cases}$
Where T ,S Are Fixed Numbers In [0,1]. We Show That A Is An Intuitionistic P-Ideal Of X. Since I Is A P-Ideal Of X ,If $0*(0*X) \in I$,Then $X \in I$ .Hence $\mu_A(0*(0*X)) = \mu_A(X) = T$ And
$\nu_A(0*(0*X)) = \nu_A(X) = S$. If $0*(0*X) \notin I$, $X \in I$,Then $\mu_A(X) = T > 0 = \mu_A(0*(0*X))$And
$\nu_A(X) = S < 1 = \nu_A(0*(0*X))$. If $0*(0*X) \notin I$, $X \notin I$,Then $\mu_A(X) = 0 = \mu_A(0*(0*X))$And
$\nu_A(X) = 1 = \nu_A(0*(0*X))$. Therefore $\mu_A(X) \geq \mu_A(0*(0*X))$And $\nu_A(X) \leq \nu_A(0*(0*X))$.
This Means That A Satisfies $\mu_A(X) \geq \text{Min}\{ \mu_A((X*Z)*(Y*Z)), \mu_A(Y) \}$ And
$\nu_A(X) \leq \text{Max}\{ \nu_A((X*Z)*(Y*Z)), \nu_A(Y) \}$.Since $0 \in I$, $\mu_A(0) = T \geq \mu_A(X)$ And
$\nu_A(0) = S \leq \nu_A(X)$ For All $X \in X$ .So A Is An Intuitionistic Fuzzy Ideal Of X. It Is Clear That $A_{<T,S>} = I$ And So The Result Follows.

III. Intuitionistic Fuzzy H-Ideals

Definition 4.1 An Intuitionistic Fuzzy Set A In X Is Called An Intuitionistic Fuzzy H- Ideal Of X If For All X ,Y, $Z \in X$ We Have
(3) $\mu_A(0) \geq \mu_A(X)$,
(4) $\nu_A(0) \leq \nu_A(X)$,
(3) $\mu_A(X*Z) \geq \text{Min}\{ \mu_A(X*(Y*Z)), \mu_A(Y)\}$,
(4) $\nu_A(X*Z) \leq \text{Max}\{\nu_A(X*(Y*Z)), \nu_A(Y)\}$
Clearly Z=0 Gives A An Intuitionistic Fuzzy Ideal.
Example 4.2. Let X = { 0,L,M,N,P,Q} With The Following Cayley Table Be A Bci Algebra.

$*$	0	L	M	N	P	Q
0	0	0	0	N	N	N
L	L	0	L	P	N	P
M	M	M	0	Q	Q	N
N	N	N	N	0	0	0
P	P	N	P	L	0	L
Q	Q	Q	N	M	M	0

Let A= < $\mu_{A,}$ ν_A > Be An Ifs In X Defined By
$\mu_A(0) = 0.9$,$\mu_A(L) = 0.5$, $\mu_A(M) = \mu_A(Q) = \mu_A(P) = \mu_A(N) = 0.09$ And $\nu_A(0) = 0.09$,
$\nu_A(L) = 0.5$, $\nu_A(M) = \nu_A(Q) = \nu_A(P) = \mu_A(N) = 0.09$ Then A Is An Intuitionistic Fuzzy H-Ideal Of X.

Proposition 4.3. An Intuitionistic Fuzzy Set A Of A Bci-Algebra X Is A H-Ideal Of X If And Only If For Each Pair T ,S $\in$ [0,1] , $A_{<T,S>}$ Is Either Empty Or A H-Ideal Of X.

Theorem 4.4. Let A Be An Intuitionistic Fuzzy Ideal Of X .Then The Following Are Equivalent
(I). A Is An Intuitionistic Fuzzy H- Ideal ,
(Ii). $\mu_A((X * Y) * Z) \geq \mu_A(X * (Y * Z))$ And $\nu_A((X * Y) * Z) \leq \nu_A(X * (Y * Z))$
For All X ,Y ,Z $\in$ X,
(Iii). $\mu_A(X * Y) \geq \mu_A(X * (0 * Y))$ And $\nu_A(X * Y) \leq \nu_A(X * (0 * Y))$
Proof. (I) $\Rightarrow$(Ii) Since A Is An Intuitionistic Fuzzy H-Ideal Of X ,We Have
$\mu_A((X * Y) * Z) \geq$ Min { $\mu_A((X * Y) *(0* Z))$, $\mu_A(0)$}= $\mu_A((X * Y) *(0* Z))$And
$\nu_A((X * Y) * Z) \leq$ Max { $\nu_A((X * Y) *(0* Z))$, $\nu_A(0)$}= $\nu_A((X * Y) *(0* Z))$
On The Other Hand $(X * Y) *(0* Z) =(X * Y) *((Y* Z)*Y) \leq X * (Y * Z)$,Thus
$\mu_A(X *(Y * Z)) \leq \mu_A((X * Y) *(0* Z))$ And $\nu_A(X *(Y * Z)) \geq \nu_A((X * Y) *(0* Z))$
(Ii) $\Rightarrow$(Iii) Letting Y=0 And Z=Y In $\mu_A(X *Z) \geq$ Min{ $\mu_A((X * (Y * Z))$, $\mu_A(Y)$},
$\nu_A(X*Z) \leq$ Max{$\nu_A((X * (Y * Z))$, $\nu_A(Y)$}
(Iii)$\Rightarrow$ (I) Since $(X * (0 * Y))* (X * (Z * Y)) \leq (X * Y) *(0* Y) \leq Z$,By Proposition3.3 We Have
$\mu_A(X *(0 * Y)) \geq$Min { $\mu_A(X *(Z * Y))$, $\mu_A(Z)$}And
$\nu_A(X *(0 * Y)) \leq$Max { $\nu_A(X *(Z * Y))$, $\nu_A(Z)$}
Therefore A Is An Intuitionistic Fuzzy H-Ideal Of X.

Theorem 4.5 Let A Be An Intuitionistic Fuzzy Ideal Of Bci-Algebra X. If $\mu_A(X * Y) \geq \mu_A(X)$ And $\nu_A(X * Y) \leq \nu_A(X)$ For All X ,Y $\in$X ,Then A Is A H-Ideal Of X.
Proof. Min { $\mu_A(X *(Y * Z))$, $\mu_A(Y)$} $\leq$ Min{ $\mu_A((X *Z)*(Y * Z))$, $\mu_A(Y*Z)$} $\leq \mu_A(X*Z)$ And
Max { $\nu_A(X *(Y * Z))$, $\nu_A(Y)$} $\geq$ Max{ $\nu_A((X *Z)*(Y * Z))$, $\nu_A(Y*Z)$} $\geq \nu_A(X*Z)$ For All
X ,Y, Z $\in$X

Definition4.6[9] A Fuzzy Set μ In X Is Said To Be A Fuzzy Subalgebra Of X If
$\mu_A(X * Y) \geq$Min { $\mu_A(X)$, $\mu_A(Y)$} For All X , Y $\in$ X.

Definition4.7 A Intuitionistic Fuzzy Set A In X Is Said To Be A Intuitionistic Fuzzy Subalgebra Of X If
$\mu_A(X * Y) \geq$Min { $\mu_A(X)$, $\mu_A(Y)$}And
$\nu_A(X * Y) \leq$ Max{ $\nu_A(X)$, $\nu_A(Y)$} For All X , Y $\in$ X.

Theorem 4.8 An Intuitionistic Fuzzy H-Ideal Of X Is An Intuitionistic Fuzzy Subalgebra Of X.
Proof. If A Is An Intuitionistic Fuzzy H-Ideal ,Then
$\mu_A(X *Z) \geq$ Min{ $\mu_A(X * (Y * Z))$, $\mu_A(Y)$}And
$\nu_A(X*Z) \leq$ Max{$\nu_A(X * (Y * Z))$, $\nu_A(Y)$}
Putting Z=Y ,Then $\mu_A(X *Y) \geq$ Min{ $\mu_A(X)$, $\mu_A(Y)$}And
$\nu_A(X*Y) \leq$ Max{$\nu_A(X)$, $\nu_A(Y)$}
This Shows That A Is An Intuitionistic Fuzzy Subalgebra.

Theorem 4.9. Let A And B Be Intuitionistic Fuzzy Ideals Of A Bci-Algebra X Such That
A$\leq$ B And $\mu_A(0) = \mu_B(0)$ And $\nu_A(0) = \nu_B(0)$ .If A Is An Intuitionistic Fuzzy H-Ideal Of X ,Then So Is B.
Proof. By Theorem 4.4 (Iii) It Is Enough To Show That $\mu_B(X * Y) \geq \mu_B(X * (0 * Y))$ And
$\nu_B(X * Y) \leq \nu_B(X * (0 * Y))$ For Each X ,Y $\in$ X .Putting S=X*(0*Y),We Have (X*S)*(0*Y)=0

Hence $\mu_A((X*S)*(0*Y)) = \mu_A(0) = \mu_B(0)$ And $\nu_A((X*S)*(0*Y)) = \nu_A(0) = \nu_B(0)$. By Theorem 4.4 (Iii) ,Since A Is An Intuitionistic Fuzzy H-Ideal Of X, $\mu_A((X*S)*Y) \geq \mu_A((X*S)*(0*Y)) = \mu_B(0)$ And $\nu_A((X*S)*Y) \leq \nu_A((X*S)*(0*Y)) = \nu_B(0)$.Thus $\mu_B((X*S)*Y) \geq \mu_A((X*S)*(0*Y)) \geq \mu_B(0) \geq \mu_B(S)$
And $\nu_B((X*S)*Y) \leq \nu_A((X*S)*(0*Y)) \leq \nu_B(0) \leq \nu_B(S)$.Since B Is An Intuitionistic Fuzzy Ideal ,We Have $\mu_B(X*Y) \geq \text{Min}\{\mu_B((X*Y)*S), \mu_B(S)\} = \mu_B(S) = \mu_B(X*(0*Y))$ And
$\nu_B(X*Y) \leq \text{Max}\{\nu_B((X*Y)*S), \nu_B(S)\} = \nu_B(S) = \nu_B(X*(0*Y))$ And The Result Follows.

Proposition 4.10 An Intuitionistic Fuzzy Set A Of A Bci-Algebra X Is A Closed Intuitionistic Fuzzy Ideal Of If And Only If For Every T ,S $\in [0,1]$,$A_{<T,S>}$ Is Either Empty Or A Closed Ideal Of X.

Theorem 4.11 Let I Be A H-Ideal Of A Bci-Algebra X.Then There Exists An Intuitionistic Fuzzy H-Ideal A Of X Such That $A_{<T,S>} = I$ For Some T ,S $\in [0,1]$.
Proof. Define An Intuitionistic Fuzzy Set A By

$$\mu_A(X) = \begin{cases} T & X \in I \\ 0 & X \notin I \end{cases} \quad \text{And} \quad \nu_A(X) = \begin{cases} S & X \in I \\ 1 & X \notin I \end{cases}$$

With $T + S \leq 1$
We Will Show That A Is Intuitionistic Fuzzy H-Ideal Of X .Since I Is A H-Ideal Of X ,If
$X*(Y*Z) \in I$,$Y \in I$,Then $X*Z \in I$.. Hence $\mu_A(X*(Y*Z)) = \mu_A(Y) = \mu_A(X*Z) = T$ And
$\nu_A(X*(Y*Z)) = \nu_A(Y) = \nu_A(X*Z) = S$ .If $X*(Y*Z) \notin I$,$Y \notin I$, $X*Z \in I$,Then
$\mu_A(X*Z) = T > 0 = \mu_A(X*(Y*Z)) = \mu_A(Y)$ And $\nu_A(X*Z) = S < 1 = \nu_A(X*(Y*Z)) = \nu_A(Y)$
If $X*(Y*Z) \notin I$,$Y \notin I$, $X*Z \notin I$,Then $\mu_A(X*Z) = 0 = \mu_A(X*(Y*Z)) = \mu_A(Y)$ And
$\nu_A(X*Z) = 0 = \nu_A(X*(Y*Z)) = \nu_A(Y)$.Therefore $\mu_A(X*Z) \geq \text{Min}\{\mu_A(X*(Y*Z)), \mu_A(Y)\}$And
$\nu_A(X*Z) \leq \text{Max}\{\nu_A(X*(Y*Z)), \nu_A(Y)\}$
Since $0 \in I$, $\mu_A(0) = T \geq \mu_A(X)$ And $\nu_A(0) = S \leq \nu_A(X)$ For All $X \in X$. So A Is An Intuitionistic Fuzzy H-Ideal. It Is Clear That $A_{<T,S>} = I$.

Theorem 4.12 Let X Be A Bci-Algebra .Then The Following Are Equivalent :
(i) Every H-Ideal Of X Is Closed.
(ii) **E**very Intuitionistic Fuzzy H-Ideal Of X Is A Closed Fuzzy Ideal Of X.
Proof. Let A Be An Intuitionistic Fuzzy H-Ideal Of X. Then By Proposition 4.3
$A_{<T,S>}$ Is A H-Ideal Of X. Thus $A_{<T,S>}$ Is A Closed Ideal Of X. Hence By Proposition4.10
$A_{<T,S>}$ Is A Closed Intuitionistic Fuzzy Ideal Of X. Conversely Assume That I Is A H-Ideal
Of X. By Theorem 4.11 There Exists An Intuitionistic Fuzzy H-Ideal A Of X Such That
$A_{<T,S>} = I$ For Some T ,S $\in [0,1]$.Since A Is An Intuitionistic Fuzzy H-Ideal Of X ,A Is A Closed Intuitionistic Fuzzy Ideal Of X.Hence By Proposition 4.8 , $A_{<T,S>}$ Is A Closed Ideal Of X.So I Is A Closed Ideal Of X.

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