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Generation of Good Quality Direct Input Data for Mathematical Models of Pulsatile Flow of Blood in Animal CVS

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Abstract: *The aim of the paper is to investigate a good quality direct input data for mathematical models of pulsatile flow of blood in animal cardio-vascular system (CVS). It is first shown that the input data used in the literature of pulsatile flow of blood is either too ideal or incorrect. Then using Fourier series and numerical integration methods, four in-vivo experimental data (pressure gradient profiles for pulsatile flow of blood) for four different locations of animal CVS are converted into a mathematical form that could be readily used as input data for mathematical models. A comparison of experimental and mathematical profiles shows a good agreement between them (error about 10%). A computer program is developed using this method and is presented through appendix.*

Keywords: *Pulsatile, Pressure Gradient, Animal Cardio-Vascular System (CVS), Input Data, Blood, Fourier series.*

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I. Introduction

Womersley [1] considered oscillating flow and obtained the analytic expressions for axial velocity, flow rate, wall shear, etc. In terms of oscillatory pressure gradient of the form

$$\frac{p_1 - p_2}{l} = Ae^{i\omega t}, \qquad \ldots(1)$$

Blood flow in animal CVS is pulsatile which consists of two parts: oscillatory flow superimposed on steady state flow. Womersley [1] considered only oscillatory part of the pressure gradient. Steady part of the pressure gradient profile was neglected. Obviously this is too ideal input form.

Sud and Sekhon [2] considered pulsatile flow (oscillatory as well as steady part of the pressure gradient) as

$$\frac{\partial p}{\partial z} = A_0 + A_1 \cos \omega_p t, \qquad \ldots(2)$$

Where $\omega_p = 2\pi f_p$, f_p is pulse frequency, A_0 is a steady component of pressure gradient and is given by

$$A_0 = \frac{8\mu_f Q_{avg}}{\pi R^4} = \frac{4\mu_f u_{z,avg}}{R^2} \qquad \ldots(3)$$

R is the tube radius, $\mu_f = 4\times10^{-3}\ \mathrm{kg\,m^{-1}\,s^{-1}}$ and f_p = 1.2 hz.

Tsangaris and Drikakis [3] developed a mathematical model based on McDonald's [4] experimental data of pressure gradient profiles for the femoral artery of the dog. They discussed pulsating flow of a viscous incompressible fluid in an initially stressed elastic tube with anisotropic structure by taking pressure gradient of the following form

$$-\frac{\partial P(z,t)}{\partial z} = \left.\frac{\partial P_0}{\partial z}\right|_{z=0} + \sum_{1}^{\infty} \frac{i\beta_n}{R_i} P_n^* \exp\{in\omega(t - z/c_n)\}, \qquad \ldots(4)$$

Where $$P(z,t) = P_0 - \left.\frac{\partial P_0}{\partial z}\right|_{z=0} z + \sum_{n=1}^{\infty} P^*_n \exp\{in\omega(t - z/c_n)\} \qquad \ldots(5)$$

$$P_n = P_n^* J_0(i\beta_n y), \qquad \ldots(6)$$

ω is angular velocity and R_i (= r - h/2), r is average radius of the tube and c_n is the pulse wave velocity.

For pressure gradient profile in the femoral artery of a dog, Tsangaris and Drikakis [3] obtained Fourier series taking eleven terms. Abdalla [5] reduced number of terms in Fourier series. He represented pressure gradient with five terms as given below

$$-\frac{\partial p}{\partial z} = A_0 + \sum_{n=1}^{n=2} \left[A_n \cos n\omega_p t + B_n \sin n\omega_p t \right], \qquad t \geq 0; \qquad \ldots(7)$$

Where A_n & B_n are Fourier coefficients and are given in Table- I.

Table I: Fourier coefficients for femoral artery of a dog (n m^{-3})

A_0	*N*	A_n	B_n
4466.29	1	13652.17	3199.72
	2	-1679.85	17945.41
	3	10919.07	6359.46
	4	-4066.32	26.66
	5	-3466.37	-1919.84

Abdalla [5] considered the time period as 0.833 sec for the mathematical forms of some experimental pressure gradient profiles. But, for calculation of the Fourier coefficients, he took the period t = 2π, which appears to be incorrect. It is noticed that the period does not have fixed values in all cases. It may change from person to person, with surroundings and due to health conditions etc. Hence, one needs to be careful with the period of the pulsatile flow of blood.

In view of the above-mentioned problems with input data, it is desirable to have proper input pressure gradient profile. So the aim of this paper is to obtain a reliable mathematical form of in-vivo experimental pressure gradient profile which can be readily used as input data for mathematical models of pulsatile flow of blood in animal CVS.

II. Method for Converting an Experimental Pressure Gradient Profile into Mathematical Form

There are many ways to convert geometrical profiles into mathematical form. Here Fourier series method is chosen for this purpose. Let the given geometrical profile be represented by Fourier series (Kreyszig [6]) as given by

$$-\frac{\partial p}{\partial z} = A_0 + \sum_{n=1}^{\infty} \left[A_n \cos n\omega_p t + B_n \sin n\omega_p t \right], \qquad \ldots(8)$$

Where

$$A_0 = \frac{1}{T}\int_0^T \left(-\partial p/\partial z\right) dt, \quad A_n = \frac{2}{T}\int_0^T \left(-\partial p/\partial z\right)\cos(n\omega_p t)\, dt$$

And $$B_n = \frac{2}{T}\int_0^T \left(-\partial p/\partial z\right)\sin(n\omega_p t)\, dt. \qquad \ldots(9)$$

$\omega_p = 2\pi f_p$, f_p is the frequency and t is time period of the pulsating pressure gradient.

Simpson's 1/3 rule is used to evaluate the integrals (9) as given by

$$\int_0^T f(t)dt = \frac{h}{3}[f(t_0) + f(t_{2m}) + 2\{f(t_2) + f(t_4) + \ldots\ldots\ldots + f(t_{2m-2})\} + 4\{f(t_1) + f(t_3) + \ldots\ldots\ldots + f(t_{2m-1})\}] \qquad \ldots(10)$$

Where the function f (t) is defined in the interval [0, t]. The interval [0, t] is divided into an even number of equal subintervals; say 2m, of length $h = \frac{T}{2m}$ with end points $t_0(=0)$, *and* $t_{2m}(=T)$.

The values of f(t). I.e. $(-\partial p/\partial z)$ for required values of t are obtained from the given experimental pressure gradient profiles. Experimental pressure gradient profiles in literature (Hwang et.al. [7], Pedley [8], Milnor [9] etc.] are given in graphical form. Time scale is presented on the x-axis, which ranges from 0-t seconds and $(-\partial p/\partial z)$ is presented on the y-axis. The x-axis is divided into number of equal intervals and the values of $(-\partial p/\partial z)$ at these points are measured manually from the given experimental profiles. Thus, the coefficients A_0, A_n and B_n can be determined and the given experimental geometrical forms are converted into mathematical forms.

Now, we shall use this method to obtain mathematical form of in-vivo experimental pressure gradient profiles of pulsatile flow of blood in animal CVS. A computer program in C++ language is developed for this method that takes very less time and presented through appendix.

III. Mathematical Forms of In-Vivo Experimental Pressure Gradient Profiles In Animal CVS

There are very few in-vivo experimental pressure gradient profiles in animal CVS available in literature. We could locate the following profiles:

1. Pressure gradient profile for descending thoracic aorta of a dog (Hwang et.al. [7]),
2. Pressure gradient profile for left circumflex coronary artery of a dog (Hwang et.al. [7]),
3. Pressure gradient profile for femoral coronary artery of a dog (Pedley [8]),
4. Pressure gradient profile for thoracic aorta of a dog (Milnor [9]).

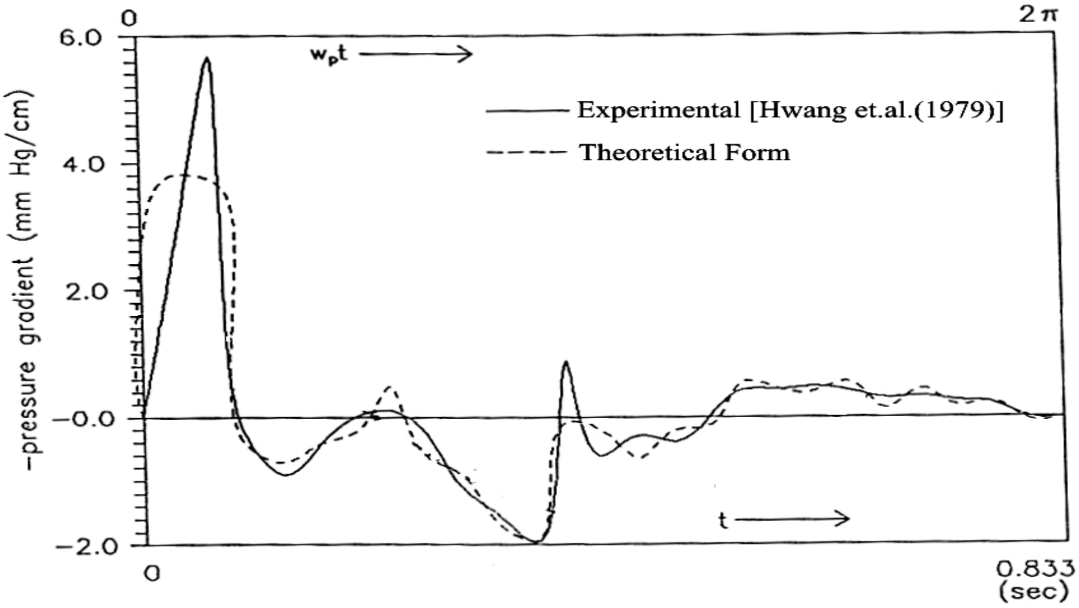

Figure 1. Comparison of experimental (Hwang et.al. [7]) and mathematical profiles of the pressure gradient in descending thoracic aorta (r=0.65 cm) of a dog.

Table II: Fourier coefficients for descending thoracic aorta (r= 0.65 cm) of a dog, (Hwang et.al. [7]).

A_0	*N*	A_n	B_n
1364.92	1	-11332.69	4043.53
	2	2683.66	7196.16
	3	-3851.00	-3736.13
	4	4488.65	7695.82
	5	4627.80	-8959.60
	6	-2627.71	3494.69
	7	5323.93	-4081.13
	8	-3601.25	128.80
	9	3718.95	-1020.85
	10	-3824.07	-3536.89

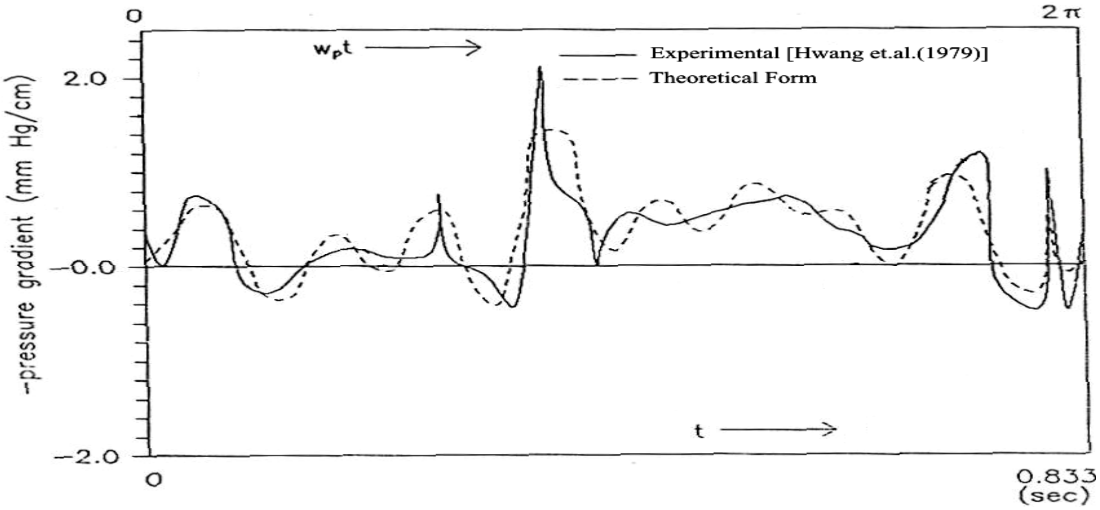

Figure 2. Comparison of experimental (Hwang et.al. [7]) and mathematical profiles of the pressure gradient in left circumflex coronary artery (r= 0.15 cm) of a dog.

Table III: Fourier coefficients for left circumflex coronary artery (r= 0.15 cm) of a dog (Hwang et.al. [7]).

A_0	*N*	A_n	B_n
4500.51	1	2938.26	2103.13
	2	649.06	18.96
	3	396.62	-1124.90
	4	-342.60	-43.53
	5	699.35	-4098.68
	6	-763.11	956.39
	7	-1950.11	-1579.31
	8	-2031.38	-61.45
	9	-1359.08	3097.95
	10	-663.84	503.37

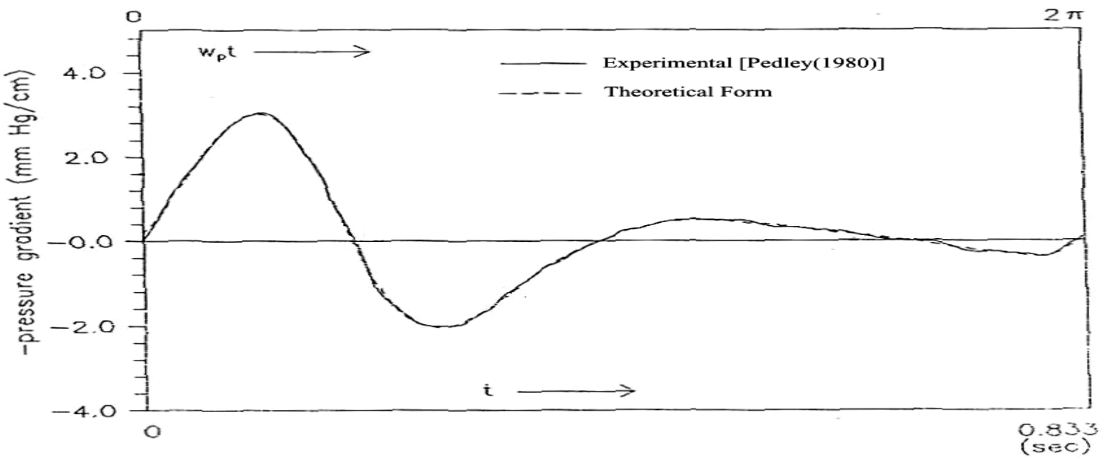

Figure 3. Comparison of experimental (Pedley [8]) and mathematical profiles of the pressure gradient in femoral artery (r=0.2 cm) of a dog.

Table IV: Fourier coefficients for femoral artery (r= 0.2 cm) of a dog (Pedley [8]).

A_0	*N*	A_n	B_n
2372.35	1	-9263.54	867.58
	2	3903.00	17190.46
	3	8510.28	-6161.66
	4	-4128.19	-1143.66
	5	333.10	985.21
	6	-12.75	65.50
	7	52.60	-292.88
	8	-296.11	116.34

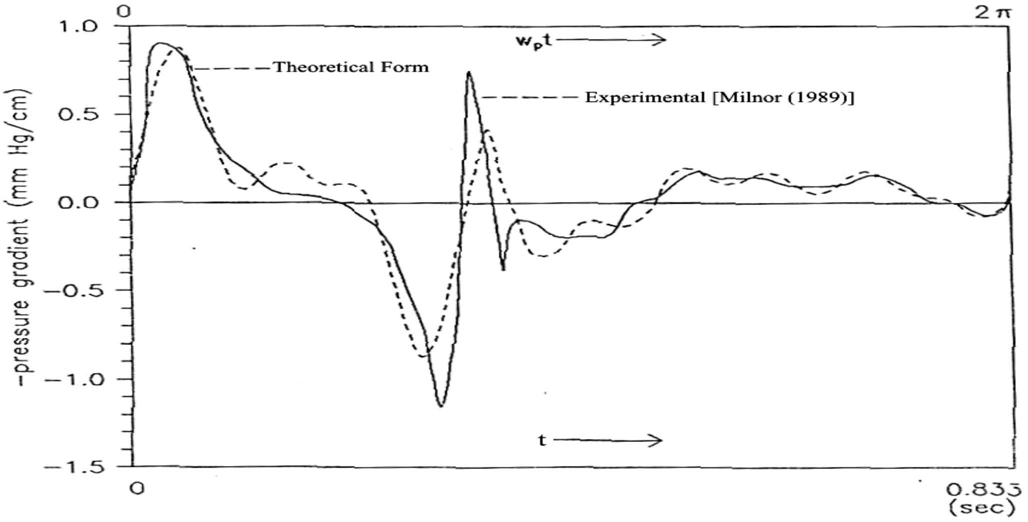

Figure 4. Comparison of experimental (Milnor [9]) and mathematical profiles of the pressure gradient in thoracic aorta (r=0.47 cm) of a dog.

Table V: Fourier coefficients for thoracic aorta (r=0.47 cm) of a dog (Milnor [9]).

A_0	***N***	A_n	B_n
631.73	1	-2912.33	952.42
	2	548.01	2588.90
	3	419.24	-1436.03
	4	65.03	-165.66
	5	-1032.77	-2352.53
	6	-1539.84	1282.44
	7	9.67	343.48
	8	-93.11	1237.42
	9	1433.55	367.90

IV. Results And Discussion

Using method given in previous section, four experimental pressure gradient profiles have been converted into mathematical forms i.e. Fourier coefficients are calculated and given through Table II to V. Thus, obtained mathematical forms are compared with experimental profiles as shown in figures 1 to 4. It is observed that they have good agreement with experimental profiles in most part of the interval, except at sharp points where error is more; on an average error is about 10%.

Number of terms in Fourier series is determined by trial and error method. For the value of n, when error is about 10%, the computation is stopped at that n. That is why different profiles have different number of Fourier coefficients. Accuracy can be improved by increasing the number of terms in the Fourier series.

V. Conclusion

Pressure gradient profiles are required as input data in mathematical models of pulsatile flow of blood. But it is observed that forms of pressure gradient profiles, considered in literature, are either too ideal (womersley [1]) or incorrect (Sud aand Sekhon [2] etc.). Obviously the obtained results from such models will not be relevant and useful to the physical situations, which they correspond to. Efforts have been made to provide relevant input data for mathematical models of pulsatile flow of blood in animal CVS. Four in-vivo experimental pressure gradient profiles for four different locations of animal CVS are converted into mathematical form (Fourier series) and shown through figures 1 to 4. The values of Fourier coefficients are given through Table II to V. A comparison of mathematical profiles with experimental profiles shows a good agreement between them (error about 10%). A computer program is developed for this procedure in C++ language and presented through appendix.

References

[1] Womersley, J.R., 1955. Method for the calculation of velocity, rate of flow and viscous drag in arteries when the pressure gradient is known. J. Physiol., 127, 553.

[2] Sud, V.K. And Sekhon, G.S., 1985. Arterial flow under periodic body acceleration. Bull. Math. Biol., 47(1), 35

[3] Tsangaris, S. And Drikakis, D., 1989. Pulsating blood flow in an initially stressed, anisotropic elastic tube: linear approximation of pressure waves. Med. And biol. Engng. And comput., 27, 82.

[4] Mcdonald, D.A., Blood flow in arteries (edward arnold, london 1974).
[5] Abdalla, W.I.A.S., Study of pulsatile flow of blood in the presence of external periodic excitations. Ph.d. Thesis, iit powai, mumbai, 1998..
[6] Kreyszig, E., Advanced engineering mathematics (8th ed., john wiley and sons, inc., new york, 1999).
[7] Hwang, N.H.C.; Gross, D.R. And Patel, D.J., Quantitative cardiovascular studies (university park press, baltimore, 1979).
[8] Pedley, T.J., The fluid mechanics of large blood vessels (cambridge university press, london, 1980).
[9] Milnor, W.R., Hemodynamics (williams and wilkins, london, 1989).

Appendix

```
 /* Program To Experimental Data Into Theoretical Data (Mathematical Form)*/
# Include <Stdio.H>
# Include <Math.H>
# Include <Conio.H>
# Include <Iostream.H>
Void Main ()
{
Int I, N;
Float A0, J, F ;
Float A[20], B[20], Pge[52], Pgm[52];

Clrscr();
For (I=0; I<=50; I=I+1){
Cout<<"Enter Value Of Pressure Gradient For J="<<I<<"\T";;
Cin>>Pge[I];
}
A0 = Pge[0] + Pge[50];
For (I=0; I<=46; I=I+2){
A0 = A0 + 2*Pge[I+2] + 4*Pge[I+1];}
A0 = 0.00667*(A0+4*Pge[49]);
For (I=0; I<=50; I=I+1){
Pgm[I] =A0;}

Again :
N=N+1;
For (I=1; I<=46; I=I+2){
J=J+0.02;
A[N] = A[N] + 2*Pge[I+2]*Cos(N*44*F*(J+0.04)/7) + 4*Pge[I+1]* Cos(N*44*F*(J+0.02)/7);
 B[N] = B[N] + 2*Pge[I+2]*Sin(N*44*F*(J+0.04)/7) + 4*Pge[I+1]* Sin(N*44*F*(J+0.02)/7);
}
For (I=0; I<=50; I=I+1){
Pgm[I] = Pgm[I] + A[N]*Cos(N*44*F*I*0.02) +B[N]*Sin(N*44*F*I*0.02);}
For (I=0; I<=50; I=I+1){
X[I]=Abs(Pgm[I]-Pge[I]);
If (X[I]>0.001){
Goto Again;}}
Clrscr();
Cout<<"\Nj\Tpressure Gradient\N";
For(J=0; J<=50; J=J+1){
Cout<<J<<"\T"<<Pgm[J]<<"\N";}
Getch ();
}
```

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