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# A Modified Halpin-Tsai Model for Estimating the Modulus of Natural Fiber Reinforced Composites

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***Abstract:****The modulus of composites formed from three selected natural fiber sources (Empty Plantain Bunch Fiber, Empty Palm Bunch Fiber and Rattan Palm Fiber) - mercerized at optimum conditions - with two selected thermosetting resins (Polyester and Epoxy resins) has been studied. The modulus was obtained as the slope of the linear part of the stress-strain curve. Selected micromechanics models were used for composite modulus prediction and their closeness to the experimental value was studied using the $R^2$ obtained from regression of the two data points. The micromechanics models studied did not adequately model the Composite behavior, a new micromechanics model obtained by modifying the Halpin-tsai equation which gave a better fit was proposed in this study. The modified model presented in this work is recommended for prediction of composite modulus for natural fiber based composites especially after further improvement by adjusting the model parameters: A, $\alpha$ and $\beta$ to make them composite specific, using data from experiments.*
**Keywords:** *Micromechanics, composite, modulus, natural fiber, reinforce, multiaxial.*

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## I. Introduction

The elastic properties of a composite can be predicted by micromechanics models based on the properties of the individual constituent materials of the composite and their geometrical characteristics. Better prediction of the mechanical properties of natural fiber composites will help our understanding of the effect of the constituents on the final properties of the material. Using micromechanics models, the composite properties can be optimized for a given application by varying the composition of the composite. The simplest micromechanical model used to predict the composite elastic modulus parallel to the principal axis is Rule of Mixture ($RoM_P$). It is a parallel spring model based on the assumption that the fibers and matrix will experience equal strain during loading in fiber direction. The $RoM_P$ equation for the modulus of a continuous unidirectional fiber composite in the fiber direction can be generally represented as shown below;

$$E_1 = k\, E_f V_f + E_m V_m \quad (1)$$

Where $E_1$ is composite modulus in fiber direction, $E_f$ and $E_m$ are fiber and matrix modulus respectively and $V_f$ and $V_m$ are fiber and matrix volume fraction, while k is the fiber efficiency factor and has values as follows; for complete alignment and when stress is parallel to fibers (k = 1); for fibers laid in two directions at right angles (bi-directional or cross-laid fibers) and stress is in one of these directions (k = ½); for fibers in random and uniform distribution within a specific plane, and stress is in any direction in the plane of the fibers (k = 3/8) and for fibers in random and uniform distribution within three dimensions in space, and stress is in any direction (k = 1/5). $RoM_P$ provides the upper bound for the composite modulus when;

$$\nu_m = \nu_f \quad (2)$$

Where $\nu_m$ and $\nu_f$ are the matrix and fiber axial Poisson's ratios respectively. The composite modulus in the direction transverse to the fiber direction is given by $RoM_S$. This series spring model assumes that the fibers and matrix experience the same stress when the composite is loaded in the direction transverse to the fibers.

The $RoM_S$ equation is;

$$E_2 = \frac{E_f E_m}{E_f V_m + E_m V_f} \quad (3)$$

Where, $E_2$ is the composite modulus, in a direction transverse to the fibers. $RoM_S$ gives the lower bound for the composite modulus (Jones, 1998; Hyer and Waas, 2000; Katchy, 2008; Virk, 2010).

Modified models for composite modulus, in a direction transverse to the fibers, are also presented by Voyiadjis and Kattan (2005):

$$E_2 = \frac{V_f + \eta V_m}{\frac{V_f}{E_{f2}} + \frac{\eta V_m}{E_m}} \quad (4)$$

Where, $E_{f2}$ is fiber modulus in the transverse direction and $\eta$ is the stress-partitioning factor. The stress-partitioning factor satisfies the condition $0 < \eta < 1$, but usually taken between 0.4 and 0.6. Alternatively, we also have;

$$E_2 = \frac{E_{f2} E_m}{E_m \eta_f V_f + E_{f2} \eta_m V_m} \quad (5)$$

Where, $\eta_f$ and $\eta_m$ are stress-partitioning factors for the fiber and matrix respectively.

Halpin and Tsai (1969) developed a semi-empirical method to predict the composite properties. Halpin-Tsai method tries to make a sensible interpolation between upper and lower bounds of composite properties. Halpin-Tsai equation is;

$$E^* = E_m \left(\frac{1+\xi\eta V_f}{1-\eta V_f}\right) \quad (6)$$

Where $$\eta = \frac{E_f - E_m}{E_f + \xi E_m} \quad (7)$$

E* is composite modulus, $E_f$ and $E_m$ are fiber and matrix modulus respectively, $V_f$ is fiber volume fraction and $\xi$ is reinforcing efficiency (which depends on fiber geometry, packing arrangement and loading condition). A variety of empirical equations for $\xi$ are available in literature and they depend on the shape of the particle and the modulus that is being predicted. For circular or rectangular fiber, assuming tensile modulus on the principal fiber direction is desired;

$$\xi = 2\left(\frac{L}{T}\right) \text{ or } 2\left(\frac{L}{D}\right) \quad (8)$$

Where L is the length of the fiber in the one direction and T or D is thickness or diameter respectively. In some cases, for the reinforcing efficiency a constant value $\xi = 2$ has been used (Katchy, 2008).

The reinforcing efficiency $\xi$ can be calculated from experimental test result, where composite modulus, E* and fiber volume fraction, $V_f$ are known and $V_m$ is matrix volume fraction which is equal to $1-V_f$ assuming a zero void fraction, using the equation below;

$$\xi = \frac{E_f(E^*-E_m) - V_f E^*(E_f - E_m)}{E_m\{(E_f - E^*) - V_m(E_f - E_m)\}} \quad (9)$$

Values of reinforcement efficiency, $\xi$, can vary from 0 to $\infty$. When $\xi = \infty$, Halpin-Tsai equation becomes $RoM_P$ and for $\xi = 0$, Halpin-Tsai equation is reduced to $RoM_S$. The higher reinforcing efficiency signifies that fibers are contributing to the composite stiffness. Halpin-Tsai method offers the advantage of being simple (easy to use in design process) and offers more exact prediction but normally requires empirical data to determine the reinforcing efficiency, $\xi$.

Though Halpin-Tsai equation is basically used for modulus in the transverse direction, it has been modified for randomly oriented fiber reinforced composites by using the relation below;

$$E = \frac{3}{8}E_1 + \frac{5}{8}E_2 \quad (10)$$

Where $E_1$ and $E_2$ (composite modulus in fiber direction and in transverse direction respectively) are obtained from Halpin-Tsai equation by using $\xi = 2(l_f/d_f)$ and $\xi = 0.5$ respectively ($l_f$ and $d_f$ are fiber length and diameter respectively) (Jones, 1998; Daniel and Ishai, 2005; Ku et al, 2011).

Similar to the Halpin-Tsai equation is the Bintrup equation for composite modulus in transverse direction is given as;

$$E_2 = \frac{(E'_m E_f)}{[E_f V_m + V_f E'_m]} \quad (11)$$

Where $E'_m = E_m/(1-\nu_m^2)$ and $\nu_m$ is the Poisson ratio of the matrix.

The modulus for discontinuous fiber composite can be estimated using Cox Shear-Lag model. The $RoM_P$ is modified by including a length factor, which is a function of fiber length, fiber and matrix properties, fiber geometry and placement. The modified $RoM_P$ equation is;

$$E = \eta_l E_f V_f + E_m V_m \quad (12)$$

$$\eta_l = 1 - \frac{\tanh(\beta_{cox}\, l/2)}{\beta_{cox}\, l/2} \quad (13)$$

$$\beta_{cox} = \sqrt{\left(\frac{2\pi G_m}{E_f A_f \ln\left(\frac{R}{r_0}\right)}\right)} \quad (14)$$

Where $\eta_l$ is fiber length distribution factor, l is fiber length, $G_m$ is matrix shear modulus, $A_f$ is fiber cross sectional area, $r_0$ and R are the fiber radius and half of inter-fiber spacing respectively. For square and hexagonal fiber arrangement and fiber of circular cross section the fiber volume fraction is given by equations 15 and 16 respectively.

$$V_f = \frac{\pi r_0^2}{4R^2} \quad (15)$$

$$V_f = \frac{2\pi r_0^2}{\sqrt{3}R^2} \quad (16)$$

This model assumes that the interface between fiber and matrix is perfect, fiber and matrix response is elastic and no axial force is transmitted through the fiber ends (Cox, 1952; Piggot, 1980; Folkes, 1982; Virk, 2010).
Facca et al (2006) also presented modified equations for the shear-lag parameter;

$$\beta_{cox} = \frac{1}{r}\sqrt{\left(\frac{2E_m}{E_f(1+\nu_m)In(\frac{P_f}{V_f})}\right)} \quad (17)$$

Where $\nu_m$, $P_f$ and r are poisson ratio of matrix, packing factor of fibers ($\pi$ for square packing and $2\pi/\sqrt{3}$ for hexagonal packing) and radius of fiber respectively.
For axisymmetric cases, the shear lag parameter below gives more accurate results:

$$\beta_{cox} = \left[\frac{2}{r^2 E_f E_m}\left[\frac{E_f V_f + E_m V_m}{\frac{V_m}{4G_f} + \frac{1}{2G_m}\left[\frac{1}{V_m} In\left(\frac{1}{V_f}\right) - 1 - \frac{V_m}{2}\right]}\right]\right]^{1/2} \quad (18)$$

Where $G_f$ and $G_m$ are shear modulus of fiber and matrix respectively. A generalized form of equation 18 is given below:

$$\beta_{cox} = \left[\frac{2}{r^2 E_f E_m}\left[\frac{E_f V_f + E_m V_m}{\frac{V_m}{4G_f} + \frac{1}{2G_m}\left[\frac{1}{V_m} In\left(\frac{1}{\chi + V_f}\right) - 1 - \frac{V_m}{2}\right] + \frac{1}{rD_s}}\right]\right]^{1/2} \quad (19)$$

The parameter $\chi$ is generally taken as 0.009, while $D_s$ is an interface parameter. A value of $D_s = \infty$ indicates perfect adhesion. The above equation can therefore be used to characterize improvement in interfacial adhesion.
The modulus of partially oriented composite can be estimated by including the fiber orientation distribution factor by Krenchel (1964) in the $RoM_P$ equation. The resulting equation is;

$$E = \eta_O E_f V_f + E_m V_m \quad (20)$$

$$\eta_O = \sum_n a_n cos^4\theta_n \quad (21)$$

Where $\eta_O$ is fiber orientation distribution factor, $a_n$ is the proportion of the fiber making $\theta_n$ angle to the applied load.
The modulus (stiffness) of discontinuous fiber composite with partially orientated fibers can be predicted by combining equations 12 and 20.

$$E = \eta_l \eta_O E_f V_f + E_m V_m \quad (22)$$

However, if $\eta_l$ is unity (for long fibers) this returns the same results as Krenchel (Equation (20)).
The modulus of natural fibers has been reported to decrease with increasing fiber diameter (Lamy and Baley, 2000;Bodros and Baley, 2008). The modulus of composite reinforced with natural fiber can be estimated by equation proposed by Summerscales et al (2010). The $RoM_P$ equation is extended to include a fiber "diameter" distribution factor, $\eta_d$ as in equation 23:

$$E = \eta_d \eta_l \eta_O E_f V_f + E_m V_m \quad (23)$$

When the fibers used in the composite are well characterized $\eta_d$ can be taken as 1 i.e. the modulus of the batch of fiber used has been measured independently.
These modifications are because the rule of mixtures cannot be directly applied to short fiber composites because the assumption of uniform strain does not hold. The critical fiber length (fiber length at which the maximum stress in fiber equals the tensile strength of the fiber ($l_c$)) or critical fiber aspect ratio ($\lambda_{crit}$) is the basis for further modification. There are three special cases (Katchy, 2008):

1. Fiber length is less than critical length ($l < l_c$)

$$E_c = (\tau_i l/d) E_f V_f + E_m V_m \quad (24)$$

2. Fiber length is equal to critical length ($l = l_c$)

$$E_c = (\tau_i l_c/d) E_f V_f + E_m V_m \quad (25)$$

3. Fiber length is greater than critical length ($l > l_c$)

$$E_c = (1 - l_c/2l) E_f V_f + E_m V_m \quad (26)$$

$E_c$ and $\tau_i$ are the composite moduli and mean shear stress at the fiber/matrix interface respectively.
Strength of the unidirectional (continuous fiber) composite can be predicted by assuming all the reinforcing fibers have identical strength and the strain in the fibers and the matrix is equal during loading. If the fiber failure strain is less than the matrix failure strain then the composite longitudinal tensile strength (parallel to the fibers) can be estimated using Kelly-Tyson equation (27) (Kelly and Tyson, 1965);

$$\sigma_c = \sigma_f V_f + (\sigma_m)_{\varepsilon f}(1 - V_f) \quad (27)$$

Where $\sigma_c$ is unidirectional composite tensile strength, $\sigma_f$ is fiber tensile strength and $(\sigma_m)_{\varepsilon f}$ is matrix stress at the strain equal to failure strain in the fibers. Equation (27) is not true for low fiber volume fraction (2% or less -

which is below the critical value for effective load transfer between fiber and matrix), therefore for low fiber volume fraction the composite strength is approximated by;

$$\sigma_c \cong \sigma_{m_{max}} \left(1 - V_f\right) \quad (28)$$

Where $\sigma_{m_{max}}$ is the maximum matrix tensile strength.

The composite strength is given by the higher of the two values calculated using equations (27) and (28).

The tensile strength of quasi-unidirectional composite loaded slightly off axis to the fiber direction is given by Potter (1994);

$$\sigma_{cu} = \sigma_c sec^2\theta \quad (29)$$

Where $\sigma_{cu}$ is ultimate composite strength, $\sigma_c$ is unidirectional composite tensile strength and $\theta$ is angle between the fiber axes and the composite loading axes.

Katchy (2008) presented some equations for the case when the fiber failure strain is greater than the matrix failure strain, but this case is hardly ever seen in practical applications.

Katchy (2008) presented three model equations for application in strength of short fiber reinforced composites, based on the critical fiber length, as follows:

1. Fiber length is less than critical length ($l < l_c$)

$$\sigma_c = (\tau_i l/d)V_f + \sigma_m V_m \quad (30)$$

2. Fiber length is equal to critical length ($l = l_c$)

$$\sigma_c = (\tau_i l_c/d)V_f + \sigma_m V_m \quad (31)$$

3. Fiber length is greater than critical length ($l > l_c$)

$$\sigma_c = \left(1 - \frac{l_c}{2l}\right) \sigma_{f,max} V_f + \sigma_m V_m \quad (32)$$

$\sigma_c$ and $\sigma_{f,max}$ are the composite tensile strength and maximum fiber tensile strength respectively.

Facca et al (2007) used a micromechanical model which was a semi-empirical modification of the rule of mixture to model composite behavior for several natural fibers and E-glass, with good prediction:

$$\sigma_c = \left(1 - \frac{l_c}{2l}\right) \sigma_{f,max} V_f + \sigma_m^* (1 - V_f) \quad (l \geq l_c) \quad (33)$$

Modified equation for cylindrical fibers, ($l \leq l_c$)

$$\sigma_c = \alpha \left(\frac{\tau_i l}{d}\right) V_f + \sigma_m^* (1 - V_f) \quad (34)$$

Modified equation for rectangular fibers, ($l \leq l_c$)

$$\sigma_c = \alpha \left(\frac{\tau_i l}{2}\right) V_f \left(\frac{W+T}{WT}\right) + \sigma_m^* (1 - V_f) \quad (35)$$

Where $\alpha, \sigma_m^*$, d, W and T are the clustering parameter, matrix stress evaluated at the peak composite strength, cylindrical fiber diameter, rectangular fiber width and rectangular fiber thickness respectively.

The mechanical properties predicted by the appropriate micromechanics model were compared to the experimental results to assess the error in the prediction. Knowing that the micromechanics models have inbuilt limitations and assumptions (i.e. they often assume perfect bond between fibers and matrix, fibers are homogenous, linear elastic and regularly spaced in the composite and the matrix is also homogenous, linear elastic and void free), the micromechanics model which most closely predicts the experimental data will be deemed more appropriate for natural fiber composites.

## II. Methodology

Empty plantain bunch fiber, Oil palm empty fruit fiber and rattan palm fibers were mercerized at their respective optimum NaOH concentration and treatment times (4wt % NaOH for 120mins, 6wt% NaOH for 90mins and 4wt% NaOH for 120mins respectively) and were chopped into lengths of 10mm, 30mm and 50mm and the aspect ratios corresponding to the lengths were obtained as a ration of length to average fiber diameter. These fibers were used to produce randomly oriented fiber composites by the hand lay-up method, using a stainless steel sheet female mould with a marble tile male mould having dimensions $300x300x3mm^3$ for fiber volume fractions 10%, 30% and 50% respectively using polyester and epoxy as resin.

Prior to the composite preparation, the mould surface was polished well and a mould-releasing agent (mirror-glaze) was applied on the surface of the mould. General unsaturated polyester resin (HSR 8113M) was mixed well with 1 wt. % cobalt naphtenate accelerator and 1wt. % by MEKP catalyst, while the epoxy resin was mixed with amine hardener in a ratio of 2:1. All chemicals were supplied by Nycil Industrial Chemicals, Ota, Ogun State, Nigeria. The fiber mat was placed in the mould and the resin mixture was poured evenly on it. Using a metallic roller, the air bubbles were carefully removed and the mat was allowed to wet completely. The mould was closed and the excess resin was allowed to flow out as 'flash' by pressing in a hydraulic press. The pressure was held constant during the curing process at room temperature for 24 hours. The composite sheet was post cured at 80°C for 4 hours. Test specimens, according to ASTM standards, were cut from the sheet.

The tensile properties were determined using Hounsfield Monsanto Universal Tensometer Machine, based on ASTM D 638-99.

The slope of the linear part of the stress-strain curve obtained from the tensile test was used as the Young's modulus for the analysis in this work.

## III. Results And Analysis

The general equation for micromechanics modeling of composite modulus is given in equation (12). Four equations are presented [equations (14), (17), (18) and (19)] for computation of the shear lag parameter which is applied in equation 13 to obtain the fiber length distribution factor and then equation (12) can be used to estimate the composite modulus, based on the modulus and volume fractions of the fiber and matrix.

For this work, equation (18) was used to compute the shear lag parameter. This is because equation (14) requires inter-fiber spacing, which is not available for our random fiber orientation, while equation (17) requires packing factor, for which values are not available for a random arrangement. Equations (24) to (26) could not be used because of challenges in obtaining or measuring the critical fiber length and mean shear stress at fiber-matrix interface. Two multiaxial models, the Bintrup equation (equation 11) and the modified Halpin-tsai equation (equations 6, 7 and 10) for different constant reinforcing efficiencies in the transverse direction, were also used and compared to equation 12 to know the model that best fits the data.

In this study, it was observed that composite modulus increased with increase in fiber volume fraction to a maximum before a decline. A simple model which is a modification of Halpin-tsai equation is presented below based on the above observation.

$$E = A\sin(\alpha V_f^{\beta})\left(\frac{3}{8}E_1 + \frac{5}{8}E_2\right) \quad (36)$$

Where A, $\alpha$ and $\beta$ are a model constants and $V_f$ is fiber volume fraction. $E_1$ and $E_2$ are composite modulus in lateral and transverse directions based on Halpin-tsai equation using $\xi = 2$ for the transverse direction. Equation 36 was used to simulate composite modulus for A=1, $\alpha = 3\pi/2$ and $\beta = 11/16$ and the simulated results are presented in Table 1 to Table 6 alongside other micromechanics models.

Micromechanics Study for Empty Plantain Bunch-Polyester Reinforced Composite

It can be observed from Table 1 that the experimental modulus increases to a maximum, with increase in volume fraction, after which it declines. The micromechanics models do not follow this trend, instead their predicted modulus increase continually with increase in fiber volume fraction, except the modified model proposed in this work. The model predicted modulus in each case was compared to the experimental modulus using an $R^2$ value obtained by regression of the experimental modulus against the model predicted modulus, for each model. The most accurate model would be one with the highest $R^2$ value. All model predicted modulus values were significantly different from the experimental, though the modified model proposed in this work was the closest to the experimental modulus with $R^2$ of 0.4712. The shear-lag model is the least accurate model for empty plantain bunch-polyester composite with an $R^2$ value of 0.1155 and this can be explained based on the fact that the model is a uniaxial model and thus is likely to be less accurate. The Bintrup model is the least accurate of the multiaxial models.

**Table 1**: Comparison of Predicted Modulus for Empty Plantain Bunch-Polyester

| Fiber Aspect Ratio (m/m) | Volume Fraction (%) | Modulus Expt. (GPa) | Shear-lag model | Halpin-Tsai ($\xi$=2) | Halpin-Tsai ($\xi$=0.5) | Bintrup model | Osoka-Onukwuli Model |
|---|---|---|---|---|---|---|---|
| 23.6183 | 10 | 4.4744 | 4.9559 | 2.0400 | 2.1339 | 3.8737 | 1.7575 |
| 23.6183 | 30 | 5.6100 | 13.8464 | 4.5323 | 4.8844 | 6.7356 | 4.3126 |
| 23.6183 | 50 | 3.7312 | 23.1306 | 7.8186 | 8.6000 | 10.6260 | 1.8393 |
| 70.8550 | 10 | 3.9542 | 5.4607 | 2.4628 | 2.5567 | 4.2965 | 2.1056 |
| 70.8550 | 30 | 4.6690 | 14.7097 | 5.6925 | 6.0445 | 7.8958 | 5.3370 |
| 70.8550 | 50 | 3.5700 | 24.0899 | 9.4701 | 10.2514 | 12.2774 | 2.1925 |
| 118.0916 | 10 | 3.6803 | 5.5616 | 2.6000 | 2.6938 | 4.4336 | 2.2186 |
| 118.0916 | 30 | 4.7661 | 14.8823 | 6.0415 | 6.3935 | 8.2448 | 5.6451 |
| 118.0916 | 50 | 3.1617 | 24.2817 | 9.9251 | 10.7064 | 12.7324 | 2.2898 |
| R-squared | | | 0.1155 | 0.1852 | 0.1859 | 0.1857 | 0.4712 |

Micromechanics Study for Empty Plantain Bunch-Epoxy Reinforced Composite

It can be observed from Table 2 that the experimental modulus increases to a maximum, with increase in volume fraction, after which it declines. The micromechanics models do not follow this trend, instead their predicted modulus increase continually with increase in fiber volume fraction, except the modified model proposed in this work. The model predicted modulus in each case was compared to the experimental modulus using an $R^2$ value obtained by regression of the experimental modulus against the model predicted modulus, for each model. The most accurate model would be one with the highest $R^2$ value. All model predicted modulus values were significantly different from the experimental, though the modified model proposed in this work was the closest to the experimental modulus with $R^2$ value of 0.8479.

**Table 2:** Comparison of Predicted Modulus for Empty Plantain Bunch-Epoxy

| Fiber Aspect Ratio (m/m) | Volume Fraction (%) | Modulus Expt. (GPa) | Shear-lag model | Halpin-Tsai (ξ=2) | Halpin-Tsai (ξ=0.5) | Bintrup model | Osoka-Onukwuli Model |
|---|---|---|---|---|---|---|---|
| 23.6183 | 10 | 1.1027 | 5.1994 | 2.7035 | 2.5794 | 4.9708 | 2.2266 |
| 23.6183 | 30 | 2.2720 | 13.9007 | 5.8711 | 5.4106 | 8.2515 | 5.1839 |
| 23.6183 | 50 | 1.7210 | 23.0768 | 10.0399 | 9.0350 | 12.5860 | 2.1472 |
| 70.8550 | 10 | 1.6760 | 5.7651 | 3.0756 | 2.9515 | 5.3429 | 2.5330 |
| 70.8550 | 30 | 2.9710 | 14.9014 | 6.8604 | 6.3999 | 9.2408 | 6.0574 |
| 70.8550 | 50 | 1.7710 | 24.1959 | 11.3964 | 10.3915 | 13.9425 | 2.4373 |
| 118.0916 | 10 | 1.6320 | 5.8782 | 3.1852 | 3.0612 | 5.4526 | 2.6233 |
| 118.0916 | 30 | 2.5850 | 15.1015 | 7.1338 | 6.6733 | 9.5141 | 6.2987 |
| 118.0916 | 50 | 1.6430 | 24.4198 | 11.7448 | 10.7399 | 14.2909 | 2.5119 |
| R-squared | | | 0.0362 | 0.0246 | 0.0283 | 0.0239 | 0.8479 |

Micromechanics Study for Empty Palm Bunch-Polyester Reinforced Composite

It can be observed from Table 3 that the experimental modulus increases to a maximum, with increase in volume fraction, after which it declines. The micromechanics models do not follow this trend, instead their predicted modulus increase continually with increase in fiber volume fraction, except the modified model proposed in this work. The model predicted modulus in each case was compared to the experimental modulus using an $R^2$ value obtained by regression of the experimental modulus against the model predicted modulus, for each model. The most accurate model would be one with the highest $R^2$ value. All model predicted modulus values were significantly different from the experimental, though the modified model proposed in this work was the closest to the experimental modulus with an $R^2$ value of 0.8477. The shear-lag model is the least accurate model for empty palm bunch-polyester composite with an $R^2$ value of 0.0836 and this can be explained based on the fact that the model is a uniaxial model.

**Table 3:** Comparison of Predicted Modulus for Empty Palm Bunch-Polyester

| Fiber Aspect Ratio (m/m) | Volume Fraction (%) | Modulus Expt. (GPa) | Shear-lag model | Halpin-Tsai (ξ=2) | Halpin-Tsai (ξ=0.5) | Bintrup model | Osoka-Onukwuli Model |
|---|---|---|---|---|---|---|---|
| 22.2222 | 10 | 2.9470 | 2.4075 | 1.6152 | 1.5378 | 3.3370 | 1.3303 |
| 22.2222 | 30 | 4.4500 | 5.3889 | 3.0363 | 2.7599 | 4.8087 | 2.6809 |
| 22.2222 | 50 | 2.4989 | 8.4551 | 4.8340 | 4.2679 | 6.6501 | 1.0338 |
| 66.6667 | 10 | 3.6756 | 2.5026 | 1.6972 | 1.6198 | 3.4190 | 1.3978 |
| 66.6667 | 30 | 4.4643 | 5.5645 | 3.2425 | 2.9661 | 5.0149 | 2.8629 |
| 66.6667 | 50 | 3.1069 | 8.6547 | 5.0995 | 4.5338 | 6.9156 | 1.0906 |
| 111.1111 | 10 | 3.5699 | 2.5216 | 1.7177 | 1.6404 | 3.4396 | 1.4147 |
| 111.1111 | 30 | 4.3021 | 5.5996 | 3.2921 | 3.0157 | 5.0645 | 2.9067 |
| 111.1111 | 50 | 3.1357 | 8.6946 | 5.1606 | 4.5945 | 6.9767 | 1.1037 |
| R-squared | | | 0.0836 | 0.1049 | 0.0980 | 0.1037 | 0.8477 |

Micromechanics Study for Empty Palm Bunch-Epoxy Reinforced Composite

It can be observed from Table 4 that the experimental modulus increases to a maximum, with increase in volume fraction, after which it declines. The micromechanics models do not follow this trend, instead their predicted modulus increase continually with increase in fiber volume fraction, except the modified model proposed in this work. The model predicted modulus in each case was compared to the experimental modulus using an $R^2$ value obtained by regression of the experimental modulus against the model predicted modulus, for each model. The most accurate model would be one with the highest $R^2$ value. There is significant difference between the experimental modulus and predicted for most models with the modified model proposed in this work having the best fit with $R^2$ value 0.7076. All other micromechanics models gave very poor fit to the experimental modulus for empty palm bunch-epoxy composite.

**Table 4**: Comparison of Predicted Modulus for Empty Palm Bunch-Epoxy

| Fiber Aspect Ratio (m/m) | Volume Fraction (%) | Modulus Expt. (GPa) | Shear-lag model | Halpin-Tsai (ξ=2) | Halpin-Tsai (ξ=0.5) | Bintrup model | Osoka-Onukwuli Model |
|---|---|---|---|---|---|---|---|
| 22.2222 | 10 | 1.1572 | 2.7292 | 2.0562 | 1.9609 | 4.2930 | 1.6935 |
| 22.2222 | 30 | 2.0740 | 5.6116 | 3.6202 | 3.2879 | 5.8714 | 3.1964 |
| 22.2222 | 50 | 1.4303 | 8.5935 | 5.5677 | 4.9128 | 7.7899 | 1.1908 |
| 66.6667 | 10 | 1.6508 | 2.8330 | 2.1190 | 2.0237 | 4.3557 | 1.7452 |
| 66.6667 | 30 | 2.4607 | 5.8123 | 3.7751 | 3.4428 | 6.0263 | 3.3332 |
| 66.6667 | 50 | 1.7133 | 8.8248 | 5.7631 | 5.1082 | 7.9853 | 1.2325 |
| 111.1111 | 10 | 1.5134 | 2.8538 | 2.1339 | 2.0386 | 4.3706 | 1.7575 |
| 111.1111 | 30 | 2.1628 | 5.8525 | 3.8108 | 3.4785 | 6.0619 | 3.3647 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 111.1111 | 50 | 1.5882 | 8.8711 | 5.8066 | 5.1518 | 8.0289 | 1.2419 |
| R-squared | | | 0.0232 | 0.0121 | 0.0139 | 0.0134 | 0.7076 |

Micromechanics Study for Rattan Palm Fiber-Polyester Reinforced Composite

It can be observed from Table 5 that the experimental modulus increases to a maximum, with increase in volume fraction, after which it declines. The micromechanics models do not follow this trend, instead their predicted modulus increase continually with increase in fiber volume fraction, except the modified model proposed in this work. The model predicted modulus in each case was compared to the experimental modulus using an $R^2$ value obtained by regression of the experimental modulus against the model predicted modulus, for each model. The most accurate model would be one with the highest $R^2$ value. All the micromechanics models presented failed to effectively predict the experimental modulus for rattan palm fiber-polyester composite, with the shear lag model having the highest $R^2$ value of 0.2880. This is a deviation from previous observations.

**Table 5:** Comparison of Predicted Modulus for Rattan Palm Fiber-Polyester

| Fiber Aspect Ratio (m/m) | Volume Fraction (%) | Modulus Expt. (GPa) | Shear-lag model | Halpin-Tsai (ξ=2) | Halpin-Tsai (ξ=0.5) | Bintrup model | Osoka-Onukwuli Model |
|---|---|---|---|---|---|---|---|
| 8.1733 | 10 | 1.4148 | 1.2157 | 1.1668 | 1.1409 | 2.7569 | 0.9610 |
| 8.1733 | 30 | 2.8296 | 1.6668 | 1.5335 | 1.4589 | 2.8837 | 1.3558 |
| 8.1733 | 50 | 2.1576 | 2.1354 | 1.9593 | 1.8395 | 3.0205 | 0.4190 |
| 24.5198 | 10 | 1.2981 | 1.2320 | 1.1731 | 1.1473 | 2.7632 | 0.9662 |
| 24.5198 | 30 | 3.0956 | 1.7040 | 1.5508 | 1.4741 | 2.8989 | 1.3692 |
| 24.5198 | 50 | 2.8016 | 2.1818 | 1.9781 | 1.8582 | 3.0393 | 0.4230 |
| 40.8664 | 10 | 1.9681 | 1.2353 | 1.1746 | 1.1487 | 2.7646 | 0.9673 |
| 40.8664 | 30 | 3.2375 | 1.7114 | 1.5542 | 1.4776 | 2.9023 | 1.3723 |
| 40.8664 | 50 | 2.3033 | 2.1911 | 1.9822 | 1.8624 | 3.0435 | 0.4239 |
| R-squared | | | 0.2880 | 0.2571 | 0.2496 | 0.2859 | 0.1018 |

Micromechanics Study for Rattan Palm Fiber-Epoxy Reinforced Composite

It can be observed from Table 6 that the experimental modulus increases to a maximum, with increase in volume fraction, after which it declines. The micromechanics models do not follow this trend, instead their predicted modulus increase continually with increase in fiber volume fraction, except the modified model proposed in this work. The model predicted modulus in each case was compared to the experimental modulus using an $R^2$ value obtained by regression of the experimental modulus against the model predicted modulus, for each model. The most accurate model would be one with the highest $R^2$ value. The modified model proposed in this work predicted the modulus more accurately that other micromechanics models with an $R^2$ value of 0.5621 while the Bintrup model is the least accurate of all models studied for rattan palm fiber-epoxy composite modulus prediction.

**Table 6:** Comparison of Predicted Modulus for Rattan Palm Fiber-Epoxy

| Fiber Aspect Ratio (m/m) | Volume Fraction (%) | Modulus Expt. (GPa) | Shear-lag model | Halpin-Tsai (ξ=2) | Halpin-Tsai (ξ=0.5) | Bintrup model | Osoka-Onukwuli model |
|---|---|---|---|---|---|---|---|
| 8.1733 | 10 | 1.9649 | 1.5496 | 1.5302 | 1.5094 | 3.5345 | 1.2602 |
| 8.1733 | 30 | 3.4090 | 1.9222 | 1.8714 | 1.8132 | 3.4484 | 1.6524 |
| 8.1733 | 50 | 1.0260 | 2.3127 | 2.2486 | 2.1639 | 3.4008 | 0.4809 |
| 24.5198 | 10 | 1.6321 | 1.5665 | 1.5337 | 1.5129 | 3.5380 | 1.2631 |
| 24.5198 | 30 | 2.7131 | 1.9627 | 1.8797 | 1.8216 | 3.4567 | 1.6597 |
| 24.5198 | 50 | 1.0182 | 2.3649 | 2.2587 | 2.1740 | 3.4109 | 0.4831 |
| 40.8664 | 10 | 1.1422 | 1.5699 | 1.5344 | 1.5136 | 3.5388 | 1.2637 |
| 40.8664 | 30 | 1.5762 | 1.9708 | 1.8816 | 1.8234 | 3.4585 | 1.6613 |
| 40.8664 | 50 | 1.0315 | 2.3753 | 2.2609 | 2.1762 | 3.4131 | 0.4835 |
| R-squared | | | 0.0963 | 0.0956 | 0.1015 | 0.0176 | 0.5621 |

## Conclusion

The existing micromechanics models fail to effectively predict the modulus of natural fiber reinforced composites. The modification of the Halpin-tsai equation as proposed in this work improved its effectiveness in modeling composites from natural (plant) fibers as opposed to synthetic fibers. The new modified model gave modulus predictions that followed the profile of the experimental modulus for all samples and closest to the experimental based on the $R^2$ value. Further improvement can be made on the modified Halpin-tsai model presented in this work by adjusting the model parameters: A, $\alpha$ and $\beta$ to make them composite specific, using data from experiments.

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