# A New Algorithm for initial basic feasible solution of Transportation Problem 

Madhavi.Malireddy<br>Department of MathematicsAMC Engineering College, Bangalore


#### Abstract

In literature, there are several algorithms for initial basic feasible solution of transportation problem. In this paper, a new algorithm is proposed for solving the transportation problem almost nearer to the optimal solution.To illustrate the proposed algorithm a numerical example is solved and the results are compared with the results of existing approaches.This approach is easy to understand and to apply on real life transportation problems for the decision makers.


Key words: Transportation problem, transportation cost, initial basic feasible solution.

## I. Introduction

The very interesting class of "allocation Methods" which is applied to a lot of very practical problems generally called 'Transportation Problems'. Transportation model was first introduced by F.L. Hitchcock I 1941[5]. The transportation problems can be modeled as standard linear programming problem,which can be solved by the simplex method. Later on, it was further improved by T.C. Koopmans in 1949 and G.B. Dantzig in 1951. Charnes and cooper[6] developed the stepping stone method which provides an alternative way of determining the simplex method information. An initial basic feasible solution (IBFS) for the transportation problem can be obtained by using the north-west corner rule, row minima,column minima,least cost entry methodor Vogel's approximation method [3]. The modified distribution method is useful for finding the optimal solution for the transportation problem.

Definition: The transportation problem is to transport various amounts of single homogeneous commodities that are initially stored at various sources, to different destinations in such a way that the total transportation cost is minimum.

## Requirements Assumption:

Each source has a fixed supply of units where this entire supply must be distributed to the destination (we let $\mathrm{a}_{\mathrm{i}}$ denote the number of units being supplied by the source $i, i=1$ to $m$ )
Similarly, each destination has a fixed demand for units, where this entire demand must be received from the source. (We let $b_{j}$ denote the number of units being received by destination $j, j=1$ to $n$ ).

## Cost Assumption:

The cost of distributing units from any particular source to any particular destination is directly proportional to the number of units distributed.
The Necessary and Sufficient condition for a Transportation problem to have feasible solution is $\sum_{i=1}^{m} a_{i}=\sum_{j=1}^{n} b_{j}$.
Mathematical Formation of Transportation problem:
Let $\mathrm{a}_{\mathrm{i}}=$ Quantity of product available at source i
$b_{j}=$ quantity of product required at destination $j$
$\mathrm{c}_{\mathrm{ij}}=$ Cost of transporting one unit of product from source i to destination j
$\mathrm{x}_{\mathrm{ij}} \quad=$ Quantity of product transported from source i to destination j
Then the transportation model would be in the form as follows:
Minimize $\mathrm{Z}=\sum_{\mathrm{j}=1}^{\mathrm{n}} \sum_{\mathrm{i}=1}^{\mathrm{m}} \mathrm{c}_{\mathrm{ij}} \mathrm{x}_{\mathrm{ij}}$
Subject to $\sum_{j=1}^{n} x_{i j}=a_{i} \quad i=1$ to $m$

$$
\begin{gathered}
\sum_{\mathrm{i}=1}^{\mathrm{m}} \mathrm{x}_{\mathrm{ij}}=\mathrm{b}_{\mathrm{j}} \quad \mathrm{j}=1 \text { to } \mathrm{n} \\
\forall \mathrm{i}, \mathrm{j} \mathrm{x}_{\mathrm{ij}} \geq 0
\end{gathered}
$$

## Formation of Transportation Table:

| $\mathrm{S}_{\mathrm{i}} / \mathrm{D}_{\mathrm{j}}$ | $\mathrm{D}_{1}$ | $\mathrm{D}_{2}$ | .......... | $\mathrm{D}_{\mathrm{n}}$ | Availability |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{S}_{1}$ | $\mathrm{c}_{11}\left(\mathrm{x}_{11}\right)$ | $\mathrm{c}_{12}\left(\mathrm{x}_{12}\right)$ | ....... | $\mathrm{c}_{1 \mathrm{n}}\left(\mathrm{x}_{1 \mathrm{n}}\right)$ | $\mathrm{a}_{1}$ |
| $\mathrm{S}_{2}$ | $\mathrm{c}_{21}\left(\mathrm{x}_{21}\right)$ | $\mathrm{c}_{22}\left(\mathrm{x}_{22}\right)$ | ...... | $\mathrm{c}_{2 \mathrm{n}}\left(\mathrm{x}_{2 \mathrm{n}}\right)$ | $\mathrm{a}_{2}$ |
| $\ldots$ | $\ldots \ldots \ldots$ | $\ldots \ldots \ldots$ | ........ | $\ldots \ldots \ldots$ | ......... |
| $\mathrm{S}_{\mathrm{m}}$ | $\mathrm{c}_{\mathrm{m} 1}\left(\mathrm{x}_{\mathrm{m} 1}\right)$ | $\mathrm{c}_{\mathrm{m} 2}\left(\mathrm{x}_{\mathrm{m} 2}\right)$ | $\ldots$ | $\mathrm{c}_{\mathrm{mn}}\left(\mathrm{x}_{\mathrm{mn}}\right)$ | $\mathrm{a}_{\mathrm{m}}$ |
| Demand | $\mathrm{b}_{1}$ | $\mathrm{b}_{2}$ | $\ldots \ldots$ | $b_{n}$ | $\sum_{i=1}^{m} a_{i}=\sum_{j=1}^{n} b_{j}$ |

## Algorithm for Transportation Method:

The Transportation Problem can be solved by the following steps:
Step1: Formulate the given problem in the matrix form.
Step 2: Obtain an Initial feasible solution.
Step3: Test the optimality of Initial solution.
Step4:Update the solution accordingly and repeat Step 3 until the most feasible solution is reached.

## Methodology for transportation problem:

The different methods for finding the initial basic feasible solution are:

1. Northwest Corner method
2. Least cost method
3. Vogel's approximation method
4. Row Minimum Method
5. Column Minimum Method

The Vogel's approximation method is an iterative method depends on penalty costs, which gives the initial basic feasible solution nearer to optimal solution.

## Advantages of proposed method:

The proposed method has the following advantages:
i) In the proposed method linear programming techniques are not used.
ii) The proposed method is not an iterative method.
iii) It is easy to understand and to apply.
iv) The proposed method takes less time to compute.

The proposed (ADS)method to find the initial feasible solution is:
Step 1: construct the transportation table from the given Transportation Problem.
Step2: Check whether the Transportation Problem is balanced or not, if not make itbalance.
Step3: Select the minimum and next to minimum cost in each row. If the minimumvalue repeated,then select it again as the next to minimum value. Sometimes the next to minimum value repeats, in such a case select the cell which hasmaximum allocation. As well as, check all the columns, if in any one of the columns the cells are not selected then select minimum and next tominimum of the corresponding column the column then select the minimum and next to minimum cost in theCorresponding column.
Step4: For each selected cell, Compute the sum of supply and demand of the of theselected cells in the corresponding row and column excluding that cell and then divide with total number of selected cells in the corresponding row andcolumn excluding that cell. Let it call as Average Demand and Supply (ADS).
Step5: Now choose the minimum average demand and supply (ADS) cell and Allocate the maximum possible to the ADS cell. If ADS is repeated twice then select the cell which has least cost in between them.
Step6: There may arise the following cases:Case i ): minimum ( $\mathrm{a}_{-} \mathrm{i}, \mathrm{b} \_\mathrm{j}$ ) $=\mathrm{a} \_\mathrm{i}$, then allocate $\mathrm{x}_{-} \mathrm{i} \mathrm{j}=\mathrm{a} \_\mathrm{i}$, and cross
 $x_{-} i j=b \_j$, and cross the $j^{\wedge}$ throw and reduce $a_{-} i \quad$ by ( $a_{-} i-b_{-} j$ ). Go to step 5.Case iii) minimum $a_{-} i=b_{-} j$, then allocatex_ij=b_j, and cross either the 【 j】^throw or $\mathrm{i}^{\wedge}$ th row but not both. Go to step 5
Step7: Repeat step 5 and step 6 until all the demands are satisfied and all the
Supplies areexhausted.
Step 8: Calculate the total cost of the transportation tableNumerical Example:
Table 1

| $\mathrm{S}_{\mathrm{i}} / \mathrm{D}_{\mathrm{j}}$ | $\mathrm{D}_{1}$ | $\mathrm{D}_{2}$ | $\mathrm{D}_{3}$ | $\mathrm{D}_{4}$ | source |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{S}_{1}$ | 3 | 1 | 7 | 4 | 300 |
| $\mathrm{~S}_{2}$ | 2 | 6 | 5 | 9 | 400 |
| $\mathrm{~S}_{3}$ | 8 | 3 | 3 | 2 | 500 |
| Destination | 250 | 350 | 400 | 200 |  |

## Solution:

Step2: the Given Transportation table is balanced.
Step3: the minimum and next to minimum cost in each row are shown in the following table in each row.
Table 1

| $\mathrm{S}_{\mathrm{i}} / \mathrm{D}_{\mathrm{j}}$ | $\mathrm{D}_{1}$ | $\mathrm{D}_{2}$ | $\mathrm{D}_{3}$ | $\mathrm{D}_{4}$ | source |
| :---: | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{S}_{1}$ | 3 | 1 | 7 | 4 | 300 |
| $\mathrm{~S}_{2}$ | 2 | 6 | 5 | 9 | 400 |
| $\mathrm{~S}_{3}$ | 8 | 3 | 3 | 2 | 500 |
| Destination | 250 | 350 | 400 | 200 |  |

Step4:Calculate the ADS for each selected cell. The corresponding ADS are 650,700,675,650,675 and850 respectively. Among these 650 is the minimum ADS, which occur for two cells, select the minimum cost cell, in the corresponding row. 1 is minimumCost. Allocate $\min (300,350)=300$ to the corresponding cell. Delete the Row and reduce the corresponding column demand by 50 .Repeat the allocation procedure for ADS cells until all the demands are satisfied and supplies are exhausted. The allocation transportation table is presented in table 2.

Table 2

| $\mathrm{S}_{\mathrm{i}} / \mathrm{D}_{\mathrm{j}}$ | $\mathrm{D}_{1}$ | $\mathrm{D}_{2}$ | $\mathrm{D}_{3}$ | $\mathrm{D}_{4}$ |
| :---: | :--- | :--- | :--- | :--- |
| $\mathrm{~S}_{1}$ | 3 | $1(300)$ | 7 | 4 |
| $\mathrm{~S}_{1}$ | $2(250)$ | 6 | $5(150)$ | 9 |
| $\mathrm{~S}_{1}$ | 8 | $3(50)$ | $3(250)$ | $2(200)$ |

The total Transportation cost is
$1 \times 300+2 \times 250+5 \times 150+3 \times 50+3 \times 250+2 \times 200=2850$
The optimal solution $=2850$.

## II. Result Analysis:

Table 3

| Method | Total Transportation cost |
| :--- | :--- |
| North-West corner Rule | 4400 |
| Row Minimum Method | 2850 |
| Column Minimum Method | 3600 |
| Least Cost Method | 2900 |
| Vogel's approximation Method | 2850 |
| Proposed MCDT Method | 2850 |
| Optimum solution | 2850 |

As observed from Table 3, The proposed ADS method provides comparatively a better initial basic feasible solution than the result obtained by the traditional algorithms which are either optimal or near to optimal.

## III. Conclusion:

The proposed method is very easy to understand and to apply to for transportation problems with large data. Since it is not an iterative method like Vogel's approximation, computational cost is very less and also it is easy to write computer program for this proposed method.

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