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Golden Ratio Prime Numbers With Two Prime Digits

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ABSTRACT
After defining, the golden ratio prime numbers with two prime digits will be presented . How many golden ratio prime numbers with two prime digits are there in the interval (10^{n-1}, 10^n) (where n≥2 integer number)? On the one hand, it has been counted by computer among the prime numbers with two prime digits. On the other hand, the function (1), (2), (3), (4), (5) gives the approximate number of golden ratio prime numbers with two prime digits in the interval (10^{n-1},10^n). Near-proof reasonig has emerged from the conformity of Mills' prime numbers with golden ratio prime numbers with two prime digits. The sets of golden ratio prime numbers with two prime digits are probably infinite.

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I. INTRODUCTION

The sets of special prime numbers within the set of prime numbers are well-known. For instance, the Erdős-primes (the sum of the digits is prime) [8], Fibonacci-primes (F_0=0, F_1=1, F_n=F_{n-1}+F_{n-2}), Gauss-primes (in the form 4n+3), Leyland-primes (in the form x^y+y^x, where 1≤x≤y), Pell-primes (P_0=0, P_1=1, P_n=2P_{n-1}+P_{n-2}), Bölcsföldi-Birkás primes (all digits are prime, the number of digits is prime and the sum of digits is prime), etc. Question: Which further sets of special prime numbers are there within the set of prime numbers? We have found a further set of special prime numbers within the set of prime numbers. It is the set of golden ratio prime numbers with two prime digits…

The definition of the golden ratio:
the numbers m and n (where m>n) are in golden ratio, if (m+n)/m=m/n=c where,
$c = 1{,}618033988 \ldots \; irrational$.

1. Golden ratio prime numbers with prime digits 2, 3

Definition: the prime number p is golden ratio prime number with prime digits 2, 3 if
all digits are 2 or 3 and
$| p/c - q | < k$, where q is a positive integer number and
$| p/c^2 - r | < k$, where r is a positive integer number,
and k=0,05.
The positive integer numbers p, q, r are in golden ratio.

If k=0,05:

p	p/c	p/c²	Golden ratio
233	144	89	233, 144, 89
323233	199769	123464	323233,199769,123464

etc…

The golden ratio prime numbers with digits 2, 3 are as follows:

```
{}, {233}, {}, {}, {323233}, {}, {22223323, 33222223}, {223233323,
  232323323, 332323333, 333233333}, {2333223323,
  3232222333}, {22233332333, 22322222323, 22333332233, 23233223333,
  23322223333, 23332232323, 32223222233, 32332232333, 32332233223,
  32333322223, 33222232223, 33222323333}, {222323333333, 223222332233,
   223323223223, 232223322323, 233232332323, 233323332323,
  322322222323, 322323333223, 323232222223, 323332232323,
  323333232323, 332222322323, 332232232333, 333222322333,
  333232232233, 333323232233}}, etc.
```

G(n) is the factual frequency of golden ratio prime numbers with prime digits 2, 3 in the interval (10^{n-1}, 10^n), where n≥2 integer number.
G(2)=0, G(3)=1, G(4)=0, G(5)=0, G(6)=1 , G(7)=0, G(8)=2. G(9)=12, G(10)=16, G(11)=40, G(12)=74, G(13)=113, G(14)=102, G(15)=1025, G(16)=1709, G(17)=3228, G(18)=6104, etc.
H(n) function gives the number of golden ratio prime numbers with prime digits 2, 3 in the interval (10^{n-1}, 10^n), where n≥9 integer number.
The function H(n) is

$H(n)=0.43x1,817^{n-2}$ where n≥9… integer **(1)**

The factual number of golden ratio primes with prime digits 2,3 (G(n)) and the number of golden ratio prime numbers calculated according to function H(n) are as follows:

Number of digits	The factual number of golden ratio primes with digits 2,3 in the interval (10^{n-1}, 10^n)	The number of golden ratio primes with digits 2, 3 alculated according to function $H(n)=0,43x1,817^{n-2}$ in the interval (10^{n-1}, 10^n)	
n	G(n)	H(n)	G(n)/H(n)
9	12	28,12	0,43
10	16	51,09	0,31
11	40	92,82	0,43
12	74	168,66	0,44
13	113	306,46	0,37
14	102	556,84	0,18
15	1025	1011,18	1,01
16	1709	1838,39	0.93
17	3228	3340,35	0,97
18	6104	6069,42	1,01
19		11028,14	
20		20038,12	
21		36409,27	
22		66155.64	
23		120204,80	
		etc.	

2. Golden ratio prime numbers with prime digits 2, 7

Definition: the prime number p is golden ratio prime number with prime digits 2, 7, if
all digits are 2 or 7 and
$| p/c - q | < k$, where q is a positive integer number and
$| p/c^2 - r | < k$, where r is a positive integer number,
and k=0,05.
The positive integer numbers p, q, r are in golden ratio.

If k=0,05:

p	p/c	p/c^2	Golden ratio
22777	14077	8700	22777, 14077, 8700
2227727	1376811	850916	2227727, 1376811, 850916
etc...			

The golden ratio prime numbers with prime digits 2, 7 are as follows:

{{}, {}, {}, {22777}, {}, {2227727, 2727727}, {}, {222272777, 772777727, 777777227}, {2227227227, 2272272727, 2277277777, 2277777277, 2722777777, 2727777727, 7722727277, 7777272227, 7777772777}, {22272272227, 22272277277, 22727277227, 27272222227, 27272722727, 27277722727, 27722227277, 27727727777, 72227277277, 72777722777, 72777777727, 77222722777, 77772727777}, {227272722727, 227772772277, 227777272727, 272227772227, 272777222777, 272777277727, 277772727227, 277777227727, 722272777277, 722272777777, 722727222277, 727272222227, 772227277277, 772227277777, 772772222777, 777227222227, 777722272277}},etc.

L(n) is the factual frequency of golden ratio prime numbers with prime digits 2, 7 in the interval $(10^{n-1}, 10^n)$, where n≥2 integer number.
L(2)=0, L(3)=0, L(4)=0 , L(5)=1, L(6)=0 , L(7)=2, L(8)=0, L(9)=3, L(10)=9. L(11)=13, L(12)=17, L(13)=33, L(14)=57, L(15)=81, L(16)=183, L(17)=969, L(18)=1641, L(19)=3305, etc.
M(n) function gives the number of golden ratio prime numbers with prime digits 2, 7 in the interval $(10^{n-1}, 10^n)$, where n≥14 integer number.
The function M(n) is

$M(n)=(n-12)^{4,1}$ where n≥14 integer **(2)**

The factual numbers of golden ratio primes with prime digits 2,7 (L(n)) and the number of golden ratio prime numbers with prime digits 2,7 calculated according to function M(n) are as follows:

Number of digits n	The factual number of golden ratio primes in the interval $(10^{n-1}, 10^n)$ L(n)	The number of golden ratio primes calculated according to function $M(n)=(n-12)^{4,1}$ in the interval $(10^{n-1}, 10^n)$ M(n)	L(n)/M(n)
14	57	17,15	3,32
15	81	90,41	0,90
16	183	294,07	0,62
17	969	734,14	1,32
18	1641	1550,32	1,06
19	3305	2916,77	1,13
20		5042,77	
21		8173,24	
22		12589,25	
23		18608,44	
		etc.	

3. **Golden ratio prime numbers with prime digits 3, 5**

Definition: the prime number p is golden ratio prime number with prime digits 3, 5, if all digits are 3 or 5 and
$| p/c - q | < k$, where q is a positive integer number and
$| p/c^2 - r | < k$, where r is a positive integer number,
and k=0,05.

If k=0,05:

p	p/c	p/c²	Golden ratio
5333	3296	2037	5333, 3296, 2037
53353	32974	20379	53353, 32974, 20379
etc…			

The golden ratio prime numbers with prime digits 3, 5 are as follows:

{{}, {}, {5333}, {53353}, {535333}, {3555353}, {55555553}, {335333533, 353533333, 355535333}, {3335553553, 3553535333, 3555535553, 5333553353, 5353553533, 5355533353, 5533533353}, {33353355353, 33353553533, 33555333353, 33555533533, 35335533533, 35535335533, 35553535553, 35555535553, 53353355353, 53533355353, 53533553533, 55333355533, 55335335353, 55355353553}, {333353555533, 335353335533, 335353353553, 335553555353, 353333555353, 353355555533, 355335553553, 355533335533, 535333535533, 535353353333, 535553555353}} etc.

R(n) is the factual frequency of golden ratio prime numbers with prime digits 3, 5 in the interval $(10^{n-1}, 10^n)$, where n≥2 integer.
R)2)=0, R(3)=0, R(4)=1 , R(5)=1, R(6)=1 , R(7)=1, R(8)=1, R(9)=3, R(10)=7, R(11)=14, R(12)=51, R(13)=61, R(14)=92, R(15)=140, R(16)=948, R(17)=1743, R(18)=3709, etc.
S(n) function gives the number of golden ratio prime numbers with prime digits 3, 5 in the interval $(10^{n-1}, 10^n)$, where n≥11 integer number.
The function S(n) is **$S(n)=(n-9)^{3,75}$** where n≥11 integer **(5)**

The factual number of golden ratio primes with prime digits 3, 5 (R(n)) and the number of golden ratio prime numbers with prime digits 3, 5 calculated according to function S(n) are as follows:

Number of digits n	The factual number of golden ratio primes in the interval $(10^{n-1}, 10^n)$ R(n)	The number of golden ratio primes calculated according to function $S(n)=(n-9)^{3,75}$ in the interval $(10^{n-1}, 10^n)$ S(n)	R(n)/S(n)
11	14	13,45	1,04
12	51	61,55	0,83
13	61	181,02	0,34
14	92	417,96	0,22
15	140	828,07	0,17
16	948	1476,11	0,64
17	1743	2435,50	0,46
18	3709	3787,99	0,98
19		5623,41	
20		8039,38	
21		11141,14	
22		15041,38	
23		19860,04	
etc.			

4. Golden ratio prime numbers with prime digits 3, 7

Definition: the prime number p is golden ratio prime number with prime digits 3, 7 if all digits are 3 or 7 and
$| p/c - q | < k$, where q is a positive integer number and
$| p/c^2 – r | < k$, where r is a positive integer number,
and k=0,05.
The positive integer numbers p, q, r are in golden ratio.

If k=0,05:

p	p/c	p/c^2	Golden ratio
7333	**4532**	**2801**	**7333, 4532, 2801**
3333773	**2060385**	**1273388**	**3333773, 2060385, 1273388**

etc.

The golden ratio prime numbers with prime digits 3, 7 are as follows:

{{}, {733}, {7333}, {}, {}, {3333773, 3773773, 7737733}, {37373773, 73377377, 73773373, 77733377}, {333733333, 373777333, 377337773, 377737777, 733777333, 773733773}, {3337333777, 3337737373,etc.

J(n) is the factual frequency of golden ratio prime numbers with prime digits 3, 7 in the interval (10^{n-1}, 10^n), where n≥2 integer number.
J(2)=0, J(3)=1, J(4)=1, J(5)=0, J(6)=0, J(7)=3, J(8)=4, J(9)=6, J(10)=15, J(11)=18, J(12)=22, J(13)=54, J(14)=95, J(15)=134, J(16)=289, J(17)=1633, J(18)=3800, J(19)=5102, etc.
H(n) function gives the number of golden ratio prime numbers with prime digits 5, 7 in the interval (10^{n-1}, 10^n), where n≥11 integer number.
The function H(n) is $\mathbf{H(n)=(n-9)^{3,751}}$ where n≥11 integer (4)

The factual numbers of golden ratio prime numbers with prime digits 3, 7 is P(n) and the number of golden ratio prime numbers with prime digits 3, 7 calculated according to function Q(n) are as follows:

Number of digits n	The factual number of golden ratio primes in the interval (10^{n-1}, 10^n) J(n)	The number of golden ratio primes calculated according to function $H(n)=(n-9)^{3,751}$ in the interval (10^{n-1}, 10^n) H(n)	J(n)/H(n)
11	18	13,46	1,34
12	22	61,61	0,36
13	54	181,27	0,30
14	95	418,64	0,23
15	134	829,55	0,16
16	289	1478,98	0,20
17	1633	2440,57	0,67
18	3800	3796,33	1,00
19		5636,38	
20		8058,68	
		etc.	

5. Golden ratio prime numbers with prime digits 5, 7

Definition: the prime number p is golden ratio prime number with prime digits 5, 7 if all digits are 5 or 7 and
$| p/c - q | < k$, where q is a positive integer number and

$| p/c^2 – r | < k$, where r is a positive integer number,
and k=0,05.
The positive integer numbers p, q, r are in golden ratio.

If k=0,05:

p	**p/c**	**p/c^2**	**Golden ratio**
5555777	**3433659**	**2122118**	**5555777, 3433659, 2122118**
57777557	**35708494**	**22069063**	**57777557, 35708494, 22069063**
etc.			

The golden ratio prime numbers with prime digits 5, 7 are as follows:

{}, {}, {}, {}, {}, {5555777}, {57777557, 77577557}, {575577757, 577555577, 577755757}, {5557775777, 5755575557, 5757575557, 5757575777, 5775777577, 7555777577, 7575557557, 7577577577}, {55575777757, 55577755577, 55777755757, 57577555757, 57755575777, 75755577577, 75775555577, 77555557577, 77555755757, 77577575557}, {555777757757, 557557775557, 557575577777, 557577575557, 557755575557, 575575755577, 575757757577, 575777557577, 575777757757, 575777775557, 577755575557, 755775757757, 757575575557, 757575575777, 757755575557, 775557555557, 775557557777}}

P(n) is the factual frequency of golden ratio prime numbers with prime digits 5, 7 in the interval $(10^{n-1}, 10^n)$, where n≥2 integer number.
P(2)=0,P(3)=0, P(4)=0, P(5)=0, P(6)=0, P(7)=1, P(8)=2, P(9)=3, P(10)=8, P(11)=10, P(12)=17, P(13)=32, P(14)=61, P(15)=42, P(16)=169, P(17)=782, P(18)=1433, P(19)=3412, etc.
Q(n) function gives the number of golden ratio prime numbers with prime digits 5, 7 in the interval $(10^{n-1}, 10^n)$, where n≥10 integer number.
The function Q(n) is $\mathbf{Q(n)=(n-9)^{3,5}}$ where n≥10 integer

(5) The factual numbers of golden ratio prime numbers with prime digits 5, 7 is P(n) and the number of golden ratio prime numbers with prime digits 5, 7 calculated according to function Q(n) are as follows:

Number of digits n	The factual number of golden ratio primes in the interval $(10^{n-1}, 10^n)$ P(n)	The number of golden ratio primes calculated according to function $Q(n)=(n-9)^{3,5}$ in the interval $(10^{n-1}, 10^n)$ Q(n)	P(n)/Q(n)
11	10	11,31	0,88
12	17	46,77	0,36
13	32	128,00	0,25
14	61	279,51	0,22
15	42	529,09	0,08
16	169	907,49	0,19
17	782	1448,15	0.54
18	1433	2187,00	0,66
19	3412	3162,28	1,08
20		4414,43	
21		5985,68	
22		7921,40	
23		10267,11 etc.	

6. If k=0,001 is, are the golden ratio prime numbers with two prime digits

2, 3:

{}, {22223323, 33222223}, {223233323,
232323323, 332323333, 333233333}, etc.

2, 7:
{{}, {}, {}, {22777}, {}, {2227727, 2727727}, {}, {222272777,
772777727, 777777227}, {2227227227, 2272272727, 2277277777, etc.

3,5:

{{}, {}, {5333}, {53353}, {535333}, {3555353}, {55555553}, {335333533,
353533333, 355535333}, {3335553553, 3553535333, 3555535553,etc.

3,7:

{{}, {733}, {7333}, {}, {}, {3333773, 3773773, 7737733}, {37373773,
73377377, 73773373, 77733377}, {333733333, 373777333, 377337773,etc.
5, 7:
{}, {}, {}, {}, {}, {5555777}, {57777557, 77577557}, {575577757,
577555577, 577755757}, {5557775777, 5755575557, 5757575557,
5757575777, 5775777577, 7555777577, 7575557557,etc.

5. Number of the elements of the set of golden ratio prime numbers with prime digits 3, 7 [3], [9],[10], [11], [12].

Let's take the set of Mills' prime numbers!
Definition: The number m=[M ad 3^n] is a prime number, where M=1,3063778838630806904686144926O2 is the Mills' constant, and n=1,2,3,… is an arbitrary positive integer number. It is already known that the number of the elements of the set of Mills' prime numbers is infinite. The Mills' prime numbers are the following: m=2, 11, 1361, 2521008887,…
The connection n→m is the following:1→2, 2→11, 3→1361, 4→2521008887,… The Mills' prime number m=[M ad 3^n] corresponds with the interval (10^{m-1},10^m) and vice versa. For instance:2→(10, 10^2), 11→(10^{10},10^{11}), 1361→(10^{1360},10^{1361}), etc. and vice versa. The number of the elements of the set of Mills' prime numbers is infinite. As a consequence, the number of the intervals (10^{m-1}, 10^m) that contain at least one Mills' prime number is infinite. The number of golden ratio primes with prime digits 3, 7 in the interval (10^{m-1},10^m) is H(m)=$(m-9)^{3,751}$. The number of golden ratio prime numbers with prime digits 3, 7 is probably infinite:
lim J(n)=∞ and lim H(n)=∞ are probably where n is positive integer and n→∞.

II. CONCLUSION

Countless different sets of special prime numbers have been known. We have found the following set of special prime numbers within the set of prime numbers. There may be further sets of special prime numbers that we do not know yet. Finding them will be task of researchers of the future.

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