International Journal of Engineering Science Invention (IJESI)
ISSN (Online): 2319-6734, ISSN (Print): 2319-6726
www.ijesi.org ||Volume 11 Issue 2 Series I || February 2022 || PP 58-66

Droplet Model Used to Analyze the Early Universe

Hans-Otto Carmesin[1,2,3], Philipp Schöneberg[3]
[1]*(University Bremen, Germany)*
[2]*(Studienseminar Stade, Germany)*
[3]*(Observatory and Gymnasium Athenaeum Stade, Germany)*

ABSTRACT: *The continuous expansion of space since the Big Bang has been a great discovery of mankind. However, that continuous expansion is incomplete, as it fails to describe the physics at very high density and small light radii. In this paper, we provide a solution of that incompleteness problem by developing and analyzing a droplet model: Droplets of high dimensional vacuum form and grow, as soon as the density exceeds a corresponding critical density. At these dimensional phase transitions, the light horizon increases in an extremely rapid manner. As a consequence, the horizon problem is solved.*

KEYWORDS: cosmology, early universe, phase transitions, droplet model, formation of vacuum, horizon problem, incompleteness problem

Date of Submission: 05-02-2022 Date of Acceptance: 18-02-2022

I. INTRODUCTION

The expansion of the universe since the Big Bang has been observed by various methods (see e.g. [1-14]). Moreover, that expansion has been modelled on the basis of general relativity (see e.g. [15-24]). However, these models are not complete, see Fig. (1):

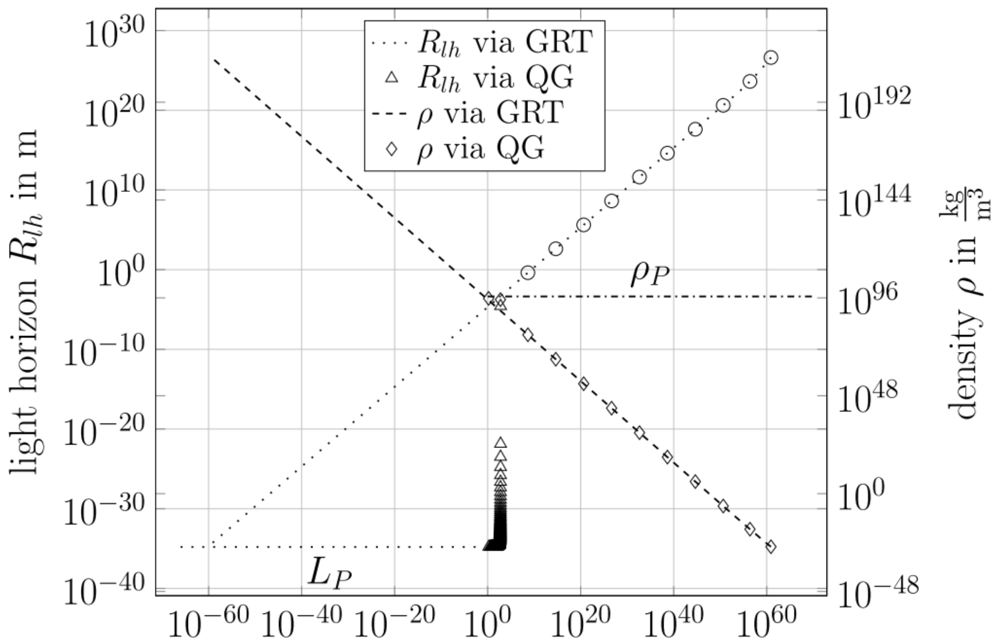

Fig. 1: Time evolution of the light horizon R_{lh} and of the density ρ as a function of time in Planck times t_P according to general relativity, GRT, and corresponding to quantum gravity, QG, (see [23], Fig. (5.7) or [22], Fig. (2.4)): At the Big Bang, the light horizon starts at approximately the Planck length, $R_{lh} \approx L_P$. Then the light horizon increases slightly by the formation of vacuum, and it increases extremely rapidly at a series of dimensional phase transitions, without the formation of vacuum (triangles, this process is described by QG). Later, the light horizon increases in three-dimensional space purely by the formation of vacuum (circles, only this process is described by GRT). Altogether, GRT is incomplete, as it describes the expansion of space only in the later universe.

Complete physics is characterized by the combination of gravity and quantum physics [25]. In that framework of quantum gravity, physical systems range from the Planck length $L_P = 1.616 \cdot 10^{-35}$ m to the present-day light horizon at $R_{lh} = 4.1 \cdot 10^{26}$ m. Moreover, the Planck density $\rho_P = 5.155 \cdot 10^{96}$ kg/m^3 cannot be exceeded in nature [21]. Naturally, in a complete physical system, the Planck length can be achieved. However, according to general relativity, the present-day light horizon would never have been smaller than 0.001 mm, at that radius, the density of the universe would be equal to the Planck density, so a further reduction of R_{lh} would be impossible, see figure (1), [23].

According to an advanced analysis within quantum gravity, the above incompleteness of GRT is solved, thereby the light horizon has reached the Planck length, $R_{lh}(t) \approx L_P$. This is achieved by a series of dimensional phase transitions (see e.g. [20,26-28]).

So far, these phase transitions have been derived by four methods: a van der Waals type analysis of two objects (see e.g. [26,29]), a transition in a Bose gas (see e.g. [21, 30, 31]), a phase transition of connections (see e.g. [21]) and a theory of the dark energy (see e.g. [20,21,28]).

In this paper, we apply the droplet model in order to provide a fifth derivation of the phase transitions in the early universe. The droplet model is a very powerful physical tool, as it provides a very clear and convincing analysis and since it is very intuitive, in addition. Accordingly, droplet models have been used in various fields of physics very successfully: in nuclear physics [32-33], in fluid dynamics [34], in biology [35], in thermodynamics, and hydrodynamics [36], for instance.

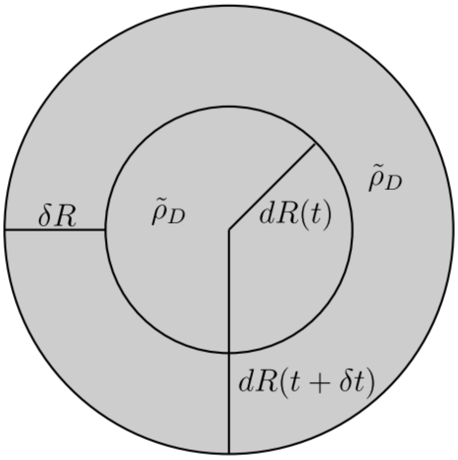

Fig. 2: Droplet model: A droplet with radius dR(t) forms additional vacuum according to quantum gravity and corresponding to the expansion of space in the universe. Simultaneously, the droplet loses vacuum, as the vacuum propagates at the velocity of light. We consider the droplet in possibly higher dimensional space or surroundings, so there is no flow of vacuum into the droplet that could compensate for the outflow. If the formation of vacuum in the droplet exceeds the outflow of vacuum from the droplet, then the droplet is stable and the corresponding dimension can form via the growth of such droplets.

II. DROPLET MODEL

In this section, we apply the droplet model to dimensional phase transitions in the early universe. Thereby, we often use Planck units, and we mark these by a tilde [20,21].

In our droplet model, a droplet represents a ball in D dimensional space, see Fig. (2). We analyze the change δR of the radius $dR(t)$ of the droplet during a time interval δt, see Fig. (2). Accordingly, there occurs a change δV of the volume dV of the droplet. The d in dR should mark that dR can be very small. Similarly, dV can be very small.

Thereby, the volume dV is the volume of the ball at a time t and with a radius dR. Hereby, we denote the volume of a ball with radius one in D dimensions by V_D. So, the volume of the ball is as follows:

$$dV(t) = V_D \cdot dR^D \quad (1)$$

During a time interval δt, the radius of the ball increases by δR. Thus, the new volume of the ball is as follows:

$$dV(t+\delta t) = V_D \cdot (dR+\delta R)^D \quad (2)$$

Hence, the volume of the formed vacuum is the volume of the shell in Fig. (2):

$$\delta V = \delta V_{shell} = dV(t+\delta t) - dV(t) = V_D \cdot [(dR+\delta R)^D - dR^D] \quad (3)$$

Thence, the formed vacuum gives rise to the following relative increase of the volume of the droplet in Fig. (2):

$$\delta V/dV = V_D \cdot [(dR+\delta R)^D - dR^D]/[V^D \cdot dR^D] = (1+\delta R/dR)^D - 1 = \varepsilon \quad (4)$$

Hereby and in general, we denote a **relative change of volume** by ε as follows:

$$\delta V/dV = \varepsilon \quad (5)$$

Accordingly, the **rate of change of the volume** in a time interval δt is the ratio of ε and δt:

$$\mathrm{Rate}(\varepsilon) = \varepsilon/\delta t \quad (6)$$

So, the formed vacuum gives rise to the following rate:

$$\mathrm{Rate}(\varepsilon) = [(1+\delta R/dR)^D - 1]/\delta t \quad (7)$$

It is well known, according to GRT, that the expansion of space since the Big Bang is caused by the density $\tilde{\rho}_{D=3}$ in three-dimensional space. Similarly, the rate at which vacuum forms is caused by the density $\tilde{\rho}_D$ in D dimensional space. The rate of that formation of vacuum has been derived in the framework of quantum gravity as follows, see equation (3.46) in [21]:

$$\mathrm{Rate}(\varepsilon_{formation}) = D^{1/2} \cdot (2\cdot\tilde{\rho}_D)^{(D-1)/4}/\delta t \quad (8)$$

The vacuum formed in the droplet propagates at the velocity of light c, as otherwise, it would be possible to measure a velocity v < c of an object relative to the vacuum. However, it is impossible to measure such a velocity v < c relative to the vacuum or relative to space, according to relativity [15, 20, 21, 27]. So the vacuum propagates outwards out of the droplet, see Fig. (2), at the velocity c. Thus, the vacuum propagating outwards fills a shell with the following thickness:

$$\delta R = c/\delta t \quad (9)$$

So, the droplet with radius dR becomes a droplet with radius dR – δR. Hence the volume moving outwards is as follows:

$$\delta V = \delta V_{shell} = V_D\cdot[dR^D - (dR - \delta R)^D]$$

Thence, the vacuum propagating outwards has the following relative volume:

$$\delta V/dV = V_D\cdot[dR^D - (dR - \delta R)^D]/[V^D\cdot dR^D] = 1 - (1 - \delta R/dR)^D = \varepsilon_{out} \quad (10)$$

Thus, the corresponding rate is as follows:

$$\mathrm{Rate}(\varepsilon_{out}) = [1 - (1 - \delta R/dR)^D]/\delta t \quad (11)$$

At the critical density $\tilde{\rho}_{D,c}$, the droplet just begins to grow. At this begin of growth of the droplet, the rate of outflowing vacuum Rate(ε_{out}) is equal to the rate of formation of vacuum Rate($\varepsilon_{formation}$):

$$\mathrm{Rate}(\varepsilon_{out}) = [1 - (1 - \delta R/dR)^D]/\delta t = D^{1/2} \cdot (2\cdot\tilde{\rho}_{D,c})^{(D-1)/4}/\delta t = \mathrm{Rate}(\varepsilon_{formation}) \quad (12)$$

That equation is solved for $(2\cdot\tilde{\rho}_{D,c})$ as follows:

$$2\cdot\tilde{\rho}_{D,c} = [1 - (1 - \delta R/dR)^D]^{4/(D-1)} \cdot D^{-2/(D-1)} \quad (13)$$

For simplicity, we apply the abbreviations q = δR/dR and x = $2\cdot\tilde{\rho}_{D,c}$. So the equation for the critical density is as follows:

$$x = [1 - (1 - q)^D]^{4/(D-1)} \cdot D^{-2/(D-1)} \quad (14)$$

Next, we analyze the time evolution of the density, see Fig. (1).

III. CALCULATION OF THE CRITICAL DENSITIES

In this section, we use formula (14) to calculate the critical densities for different droplet radii dR in which they become unstable. Therefore, we start by using the biggest droplet with the radius dR = R_{lh} of the light horizon radius so we can analyze the critical densities of the universe. Because we want to compare the results of this droplet with the whole spectrum of droplets, we also calculate the critical densities of the smallest droplet with the radius dR of one Planck length. Moreover, we examine some further examples with a relative thickness q of the shell of 0.5, 0.25, 10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}, and 10^{-5} to have a greater spectrum of droplets to compare with. So, we obtain the radius:

$$dR = \frac{\delta R}{q} \quad (15)$$

In particular, we may imagine the case $\delta R = L_P$. So, the radius is as follows:

$$dR = \frac{1}{q} \quad (16)$$

For calculating the different critical densities to every chosen droplet, for each dimension, we use a Microsoft Excel spreadsheet (Fig. 3), where we include the formula (14) and use the chosen droplets as a function of the variable q. Furthermore, we represent the scaled critical densities x as a function of the dimension D (Fig. 4).

constants:	
q	1
D	**x**
3	3.33E-01
4	3.97E-01
5	4.47E-01

Fig. 3: Part of the Spreadsheet to calculate the critical densities of each dimension for a droplet with the radius q.

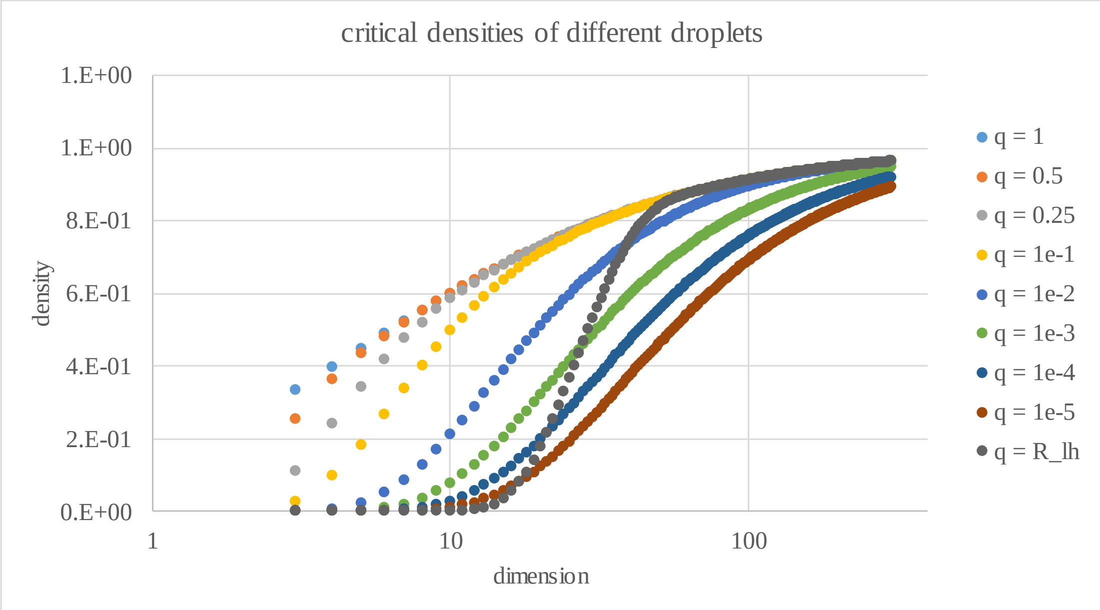

Fig. 4: Scaled critical densities x as a function of the dimension D for various relative thicknesses q.

Figure (4) shows that relatively small droplet radii $dR = \frac{1}{q}$ have relatively high critical densities for every dimension. In particular, the time evolution of the droplet with the size of the light horizon corresponds to the smallest droplets in the early universe, while it corresponds to the largest droplets in the late universe.

IV. SOLUTION OF THE HORIZON PROBLEM

In this section, we solve the horizon problem. For it, we need the path propagated by light and the radius of the light horizon as a function of time. The radius is related to the density. Accordingly, we derive the scaled density x as a function of time. For this purpose, we apply a generalized version of the Friedmann-Lemaitre equation, see equation (2.424) in [20]:

$$\frac{dx}{dt} = -(D+1) \cdot x^{\frac{D^2+5D+6}{4D+4}} \quad (17)$$

We abbreviate the exponent by b:

$$\frac{dx}{dt} = -(D+1) \cdot x^b \quad (18)$$

Now we separate the variables. Therefore, we multiply by dt and divide by x^b:

$$\frac{dx}{x^b} = -(D+1) \cdot dt \quad (19)$$

Next, we integrate:

$$\int \frac{dx}{x^b} = \int -(D+1) \cdot dt \quad (20)$$

We evaluate both integrals:

$$\frac{1}{-b+1} \cdot x^{-b+1} = -(D+1) \cdot t \quad (21)$$

Subsequently, we simplify:

$$x^{-b+1} = -(D+1) \cdot (b-1) \cdot t \quad (22)$$

Next, we abbreviate the exponent by a and $-(D+1) \cdot (b-1)$ by d. So we get the two invariants a and d at each dimension D. Thus, we derive the following equation:

$$x^a = d \times t \quad (23)$$

We divide by t:

$$\frac{x^a}{t} = d \quad (24)$$

We consider this equation for a time t_1 and a corresponding scaled density x_1 as well as for a time t_2 and a corresponding scaled density x_2. Using the invariant d, we derived the following equation:

$$\frac{x_{t_1}^a}{t_1} = d = \frac{x_{t_2}^a}{t_2} \quad (25)$$

Subsequently, we can use the reciprocal fractions, and multiply by x_1^a:

$$\frac{t_1}{x_{t_1}^a} = \frac{t_2}{x_{t_2}^a} \quad (26)$$

$$t_1 = \frac{t_2 \times x_{t_1}^a}{x_{t_2}^a} \quad (27)$$

Finally, we express the equation as follows:

$$t_1 = t_2 \cdot \left(\frac{x_{t_1}}{x_{t_2}}\right)^a \quad (28)$$

Next, we derive the path s_1 propagated by light of time dt:

$$ds = c \cdot dt \quad (29)$$

Thereby, the time interval dt is defined by the difference of the time of the last phase transition t_2 and the time of the actual dimensional phase transition t_1:

$$dt = t_2 + t_1 \quad (30)$$

Subsequently, we derive the expansion of the path s_1 as a consequence of the expansion of the universe. Accordingly, we multiply the distance s_1 by the ratio of the actual radius of the light horizon and the radius of the light horizon at that time t_1. So, we use the following equation:

$$ds_{1,expanded} = ds_1 \cdot \frac{R_{lh,0}}{R_{lh,1}} \quad (31)$$

We integrate equation (30):

$$\int ds_{1,expanded} = \int ds \cdot \frac{R_{lh,0}}{R_{lh.1}} \quad (32)$$

We evaluate both integrals. Because the interval dt between two phase transitions is relatively small, the ratio is approximately constant and we obtain the following equation:

$$ds_{2,expanded} - ds_{1,expanded} = c \cdot (t_2 - t_1) \cdot \frac{R_{lh,0}}{R_{lh,1}} \quad (33)$$

In order to calculate the distance $ds_{total,2}$ at a time t_2, we add the additional distance $ds_{2,expanded}$-$ds_{1,expanded}$ to $ds_{total,1}$:

$$ds_{total,2} = ds_{total,1} + ds_{2,expanded} - ds_{1,expanded} \quad (34)$$

In order to calculate the fraction in the above equation, we analyse the present day light horizon and the values that it had at earlier times according to the expansion of the universe as follows: For it we calculate such a value $R_{lh,1}$ as a function of the scaled density x_1. For instance, for the case of a constant mass, the following relation holds:

$$R_{lh.1}^D \cdot x_1 = R_{lh.2}^D \cdot x_2 \quad (35)$$

However, in the early universe, the space was filled with radiation. Accordingly, the density x_1 is proportional to the radius $R_{lh,1}^{-(D+1)}$, corresponding to the redshift. So the following equation holes

$$R_{lh.1}^{D+1} \cdot x_1 = R_{lh.2}^{D+1} \cdot x_2 \quad (36)$$

Subsequently, we solve for $R_{lh,1}$:

$$R_{lh,1} = R_{lh,2} \cdot \left(\frac{x_2}{x_1}\right)^{\frac{1}{(D+1)}} \quad (37)$$

Since the calculations depend on an earlier value of the light horizon radius, it is necessary to know the first value. Observable objects are limited by the Planck scale. So the light horizon radius cannot be smaller than two observable objects, see figure (5) [37]. This small light horizon is achieved at the Planck scale [37].

Fig. 5: Cubic model of the light horizon at the Dimension D = 3. A ball has the radius L_P. The light horizon is the distance between the centres of balls of both ends. So the light horizon corresponds to two L_P, see [37].

So, for the first phase transition we apply the following equation:

$$R_{lh,300} = 2 \cdot \left(\frac{x_{301}}{x_{300}}\right)^{\frac{1}{(300+1)}} \quad (38)$$

However, because of the speed of light, the length of two Planck lengths can only be reached in two Planck times. So accordingly the time of the first phase transition has to be t_{301} = 2 Planck lengths. For example, t_{300} is calculated as follows, see equation (28):

$$t_{300} = 2 \cdot \left(\frac{x_{t_{300}}}{x_{t_{301}}}\right)^{a} \quad (39)$$

The possible radii of the droplets range from the Planck length to the light horizon. In the following we perform two investigations: (1) The time evolution of the distance covered by light for the case of the smallest droplet, and (2) the time evolution of the distance covered by light for the case of the largest droplet. We develop spreadsheet in order to perform investigation (1) and investigation (2), see figure (6 until 9).

D	x	t	R_lh	ds_expanded	ds_total
3		2.51E+56	2.56E+58	7.53E+64	3.39E+69
3		4.41E+50	2.56E+55	1.32E+62	3.39E+69
3		4.41E+44	2.56E+52	1.32E+59	3.39E+69
			break in table		
299	9.62E-01	2.03E+00	2.00E+00	6.14E+64	7.81E+66
300	9.63E-01	2.02E+00	2.00E+00	6.08E+64	7.75E+66
301	9.63E-01	2.00E+00	2.00E+00	7.69E+66	7.69E+66

Fig. 6: Investigation (1) of the smallest droplet: Part of the Spreadsheet to calculate the light horizon radius R_{lh} and the distance covered by light ds_{total} based on the scaled critical densities x, the time t and dimension D for the smallest droplet radius.

In this table we calculate recursively beginning at the highest dimension D = 301 and leading to the lowest dimension D = 3. We present the dimensions in column 1, see figure (6). Secondly, for each dimension D, we calculate the scaled critical density x according to equation (14) and present the result in column 2, see figure (6). Thirdly, for dimension D = 301, we apply two Planck times according to equation (39). The times are presented in column 3, and the unit is the Planck time. Fourthly, for dimension D = 300, we calculate the time according to equation (28) by using the time of dimension D = 301. Fifthly, and for each dimension D < 300, we calculate the time t recursively according to equation (28). Sixthly, for dimension D = 301, we apply the light horizon at the Planck scale $R_{lh,301}$ = 2 Planck lengths according to equation (38). The radii are presented in column 4, and the unit is the Planck length. Seventhly, for dimension D = 300, we calculate the length according to equation (37) by using the value of the light horizon radius of dimension D = 301. Eighthly, and for each dimension D < 300, we calculate the radius R_{lh} recursively according to equation (37). Ninthly, we determine the distance covered by light in two steps. First we evaluate all distances between two phase transitions by using equation (33), see column 5 in figure (6). Thereby, we use the values of column 4 according to equation (33). Next we determine the complete path covered by light by calculating the total of all of these paths, according to equation (34). The results are presented in column 6, see figure (6).

Tenthly, for the expansion of the universe in three-dimensional space we proceed as follows:

(10a) We take the time values corresponding to figure (2.23) in [20].

(10b) We take the values of the light horizon radius corresponding to figure (2.23) in [20].

(10c) We calculate ds_{total} according to equation (34).

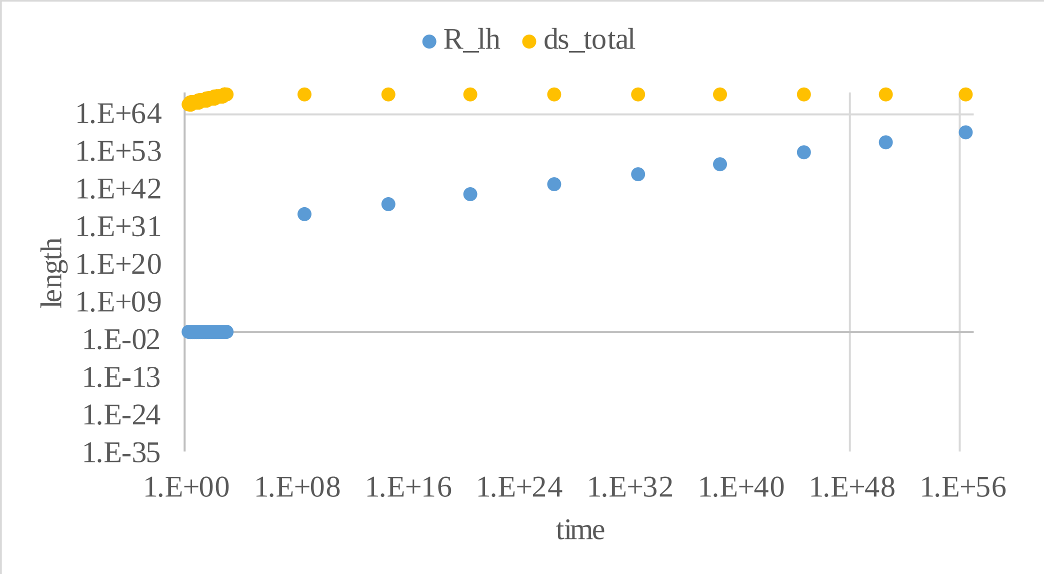

Fig. 7: Investigation (1) of the smallest droplet: Radius R_{Llh} and the distance covered by light ds_{total} as a function of time t.

D	x	t	R_lh	ds_expanded	ds_total
3	2.26E-60	4.91E+77	3.04E+12	1.24E+132	1.24E+132
4	3.98E-30	1.68E+51	8.34E+04	1.55E+113	1.55E+113
5	3.38E-18	1.27E+38	3.43E+02	2.85E+102	2.85E+102
			break in table		
299	9.62E-01	2.03E+00	2.00E+00	6.14E+64	7.81E+66
300	9.63E-01	2.02E+00	2.00E+00	6.08E+64	7.75E+66
301	9.63E-01	2.00E+00	2.00E+00	7.69E+66	7.69E+66

Fig. 8: Investigation (2) of the largest droplet: Part of the Spreadsheet to calculate the light horizon radius R_{lh} and the distance covered by light ds_{total} based on the scaled critical densities x, the time t and dimension D for the droplet with the radius of the light horizon radius.

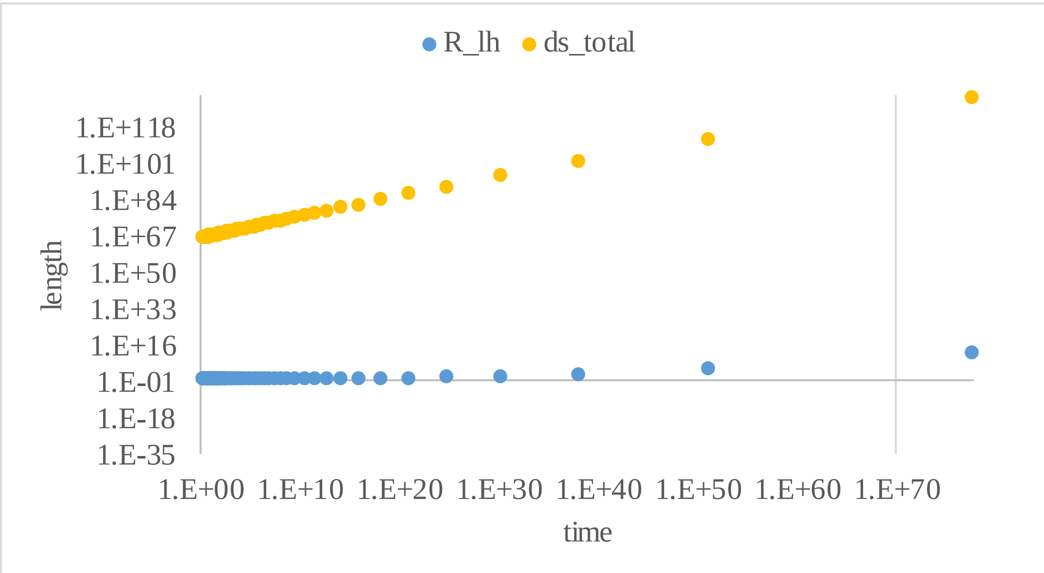

Fig. 9: Investigation (2) of the largest droplet: Radius R_{Llh} and the distance covered by light ds_{total} as a function of time t.

Investigation (1) of the smallest droplet as well as investigation (2) of the largest droplet, see figures (6, 7, 8, 9) show that the total distance achieved by light is essentially larger than the light horizon radius. Moreover, we performed additional investigations with intermediate droplet sizes, whereby we obtained the same result:
The total distance achieved by light is essentially larger than the light horizon radius. Altogether, our numerical studies show in a convincing manner that the horizon problem is solved in the framework of our droplet model.

V. CONCLUSION

In this paper, we address the incompleteness problem that the continuous expansion of space since the Big Bang, according to general relativity theory, does not describe the complete time evolution of the light horizon ranging from the Planck scale to the present day light horizon, see figure (1).

That problem has been solved by a series of discontinuous phase transitions in the early universe [20, 21]. While these phase transitions have been modeled by four methods so far, we present a fifth model: the droplet model. This model has essential advantages: it is intuitive, robust, can be solved exactly, and it has already been applied successfully in various fields of science.

We base our analysis on the basic dynamics of the vacuum [20, 21, 25]. With it, we derive the critical densities for phase transitions as a function of the radius of the droplets in equation (14) and figure (4). Using these critical densities, we derive the time evolution of distances achieved in the universe by propagating light. We show that these distances are large compared to the light horizon, so that the light was able to thermalize the universe. In this manner we solve the horizon problem in addition to our solution of the incompleteness problem.

ACKNOWLEDGEMENTS

We thank Paul Sawitzki, Jonas Lieber, Kimberly Böttcher and Matthias Carmesin for interesting discussions and I. Carmesin and for helpful discussions and proofreading the manuscript.

REFERENCES

[1]. C. Wirtz, Aus der Statistik der Spiralnebel, *Astronomische Nachrichten, 222*, 1924, 33-48.
[2]. E. Hubble, A relation between distance and radial velocity among extra-galactic nebulae, *Proc. of National Acad. of Sciences, 15*, 1929, 168-173.
[3]. A. Penzias, R. W. Wilson, A measurement of excess antenna temperature at 4080 Mc/s, *Astrophysical Journal Letters, 142*, 1965, 419-421.
[4]. G. Efstathiou et al. (Planck Collaboration), Planck 2018 results VI. Cosmological parameters, *Astronomy and Astrophysics, 641(A6)*, 2020, 1-67.
[5]. D. W. Pesce et al., The megamaser cosmology project: XIII. Combined Hubble constant constraints, *Astrophysical Journal Letters, 891*, 2020, L1.
[6]. A. G. Riess, et al., Cosmic Distances Calibrated at 1 % Precision with Gaia EDR3 Parallaxes and Hubble Space Telescope Photometry of 75 Milky Way Cepheids Confirm Tension with ΛCDM, *The Astrophysical Journal Letters, 908(L6)*, 2021, 1-11.
[7]. O. Philcox, M. Ivanov, M. Simonovic and M. Zaldarriaga, Combining Full-Shape and BAO Analyses of Galaxy Power Spectra: A 1.6 % CMB-Independent Constraint on H_0, *arXiv, 2002.04035v3*, 2020, 1-42.
[8]. G. E. Addison, D. J. Watts, C. L. Bennett, M. Halperin, G. Hinshaw, J. L. Weiland, Elucidating ΛCDM: Impact of Baryon Acoustic Oscillation Measurements on the Hubble Constant Discrepancy, *ApJ, 853(2)*, 2018, 1-12.
[9]. T. M. C. Abbott, et al., Dark Energy Survey Year 1 Results: Cosmological Constraints from Galaxy Clustering and Weak Lensing, *Phys. Rev. D, 102*, 2020, 1-34.
[10]. S. Birrer, et al. TDCOSMO: IV. Hierarchical time-delay cosmography - joint inference of the Hubble constant and galaxy density profiles. *Astronomy and Astrophysics, 643*, 2020, 1-40.
[11]. C. Escamilla-Rivera and A. Najera, Dynamical dark energy models in the light of gravitational-wave transient catalogues, *arXiv, 2103.02097v1*, 2021, 1-25.
[12]. J. P. Blakeslee et al., The Hubble Constant from Infrared Surface Brightness Fluctuation Distances, *The Astrophysical Journal, 911(65)*, 2021, 1-12.
[13]. N. Suzuki, et al., The Hubble Space Telescope Cluster Supernova Survey: V. Improving the Dark Energy Constraints above z > 1 and Building an Early-Type-Hosted Supernova Sample, *ApJ, 746*, 2011, 85-105.
[14]. A. G. Riess et al., A 2.4 % Determination of the Local Value of the Hubble Constant, *The Astrophysical Journal, 826(1)*, 2016, 1-65.
[15]. A. Einstein, Zur Elektrodynamik bewegter Körper, *Annalen der Physik, 17*, 1905, 891-921.
[16]. A. Einstein, Die Feldgleichungen der Gravitation, *Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften*, 1915, 844-847.
[17]. A. Einstein, Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie, *Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften*, 1917, 142-152.
[18]. A. Friedmann, Über die Krümmung des Raumes, *Z. f. Physik, 10*, 1922, 377-386.
[19]. G. Lemaitre, Un Univers homogene de masse constante et de rayon croissant rendant compte de la vitesse radiale des nebuleuses extra-galactiques, Annales de la Societe Scientifique de Bruxelles, A47, 1927, 49-59.
[20]. H.-O. Carmesin, *Die Grundschwingungen des Universums – The Cosmic Unification* (Berlin: Verlag Dr. Köster, 2019).
[21]. H.-O. Carmesin, *Quanta of Spacetime Explain Observations, Dark Energy, Gravitation and Nonlocality* (Berlin: Verlag Dr. Köster, 2021a).
[22]. H.-O. Carmesin, *Cosmological and Elementary Particles Explained by Quantum Gravity* (Berlin: Verlag Dr. Köster, 2021b).
[23]. H.-O. Carmesin, *The Universe Developing from Zero-Point Energy Discovered by Making Photos, Experiments and Calculations* (Berlin: Verlag Dr. Köster, 2021b).

[24]. H.-O. Carmesin, Physical Explanation of the H_0 – Tension, *International Journal of Engineering and Science Invention (IJESI), 10(8)II*, 2021, 34-38.
[25]. H.-O. Carmesin, *Quantum Physics Explained by Gravity and Relativity* (Berlin: Verlag Dr. Köster, 2022).
[26]. H.-O. Carmesin, *Vom Big Bang bis heute mit Gravitation – Model for the Dynamics of Space* (Berlin: Verlag Dr. Köster, 2017).
[27]. H.-O. Carmesin, *Entstehung dunkler Energie durch Quantengravitation – Universal Model fort he Dynamics of Space, Dark Matter and Dark Energy* (Berlin: Verlag Dr. Köster, 2018).
[28]. H.-O. Carmesin, A Model for the Dynamics of Space - Expedition to the Early Universe, *PhyDid B, FU Berlin, hal-02077596*, 2018, 1-9.
[29]. J. D. van der Waals, *Over de Continuiteit van den gas- en vloeistoftoestand*, (Leiden: Sijthoff, 1873).
[30]. P. Sawitzki, H.-O. Carmesin, Dimensional transitions in a Bose gas, *PhyDid B, FU Berlin*, 2021, 53-59.
[31]. S. Bose, Plancks Gesetz und Lichtquantenhypothese, Z. f. Physik, 26, 1924, 178-181.
[32]. G. Gamov, Mass Defect Curve and Nuclear Constitution, *Proceedings of The Royal Society A: Mathematical, Physical and Engineering Sciences, 126*, 1930, 632-644.
[33]. C. F. v. Weizsäcker, Zur Theorie der Kernmassen, *Zeitschrift f. Physik A, 96*, 1935, 431-458.
[34]. W. A. Sirignano, *Fluid Dynamics and Transport of Droplets and Sprays* (Cambridge: Cambridge University Press, 2009).
[35]. D. Zwicker et al. Growth and division of active droplets provides a model for protocells, Nature Physics, 13, 2016, 408-414.
[36]. R. S. R. Sidin, *Droplet size distribution in condensing flow* (Enschede: University of Twente, 2009).
[37]. P. Schöneberg, H.-O. Carmesin Solution of a Density Problem in the Early Universe. *PhyDid B*, 2020, 43-46.

Hans-Otto Carmesin, et. al. "Droplet Model Used to Analyze the Early Universe." *International Journal of Engineering Science Invention (IJESI)*, Vol. 11(02), 2022, PP 58-66. Journal DOI- 10.35629/6734