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Proposed Nonparametric Tests for the Simple Tree Alternative for Location and Scale Testing in a Mixed Design

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ABSTRACT *Six nonparametric tests are proposed for the mixed design consisting of a randomized complete block design (RCBD) and a completely randomized design (CRD). The proposed tests are designed to test for differences in location and/or scale for a simple tree alternative. The tests are a combination of the Fligner-Wolf test, modified Page's test, and modified Ansari-Bradley test. A simulation study is conducted to determine how well the proposed tests maintain their significance levels. Powers of the six proposed tests are estimated under a variety of cases: changing the underlying distribution, changing the number of treatments, increasing the variance between the CRD and RCBD portion, changing the proportions of the number of blocks in the RCBD to the sample size for each treatment in the CRD, and changing the parameter arrangements. A recommendation for which test has higher power is given.*

KEYWORDS *Completely randomized design, location-scale problem, power, randomized complete block design*

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I. Introduction

The situation in which the researchers want to compare control with several treatments has been used in many scientific fields, particularly in medical experiments. For example, some researchers are interested in comparing different treatments (therapies) with control (standard therapy) or placebo in clinical trials to determine whether at least one of the treatments is better than the control. The simple tree alternative test is the most appropriate hypothesis in such situations. The simple tree alternative is given by:

$$\mathrm{H_0}: \mu_1 = \mu_2 = \cdots = \mu_\mathrm{k} \qquad (1)$$
$$\mathrm{H_a}: \mu_1 \leq [\mu_2, \mu_3, \ldots \mu_k] \text{ (at least one strict inequality)}$$

where μ_i is the location parameter of population i.

Recently, many test statistics have been developed for testing the difference in location parameters when the data are a mixture of randomized complete block design (RCBD) and a completely randomized design (CRD). This mixed design occurs when researchers start with a randomized complete block design, but they might have problems because they cannot get enough homogenous experimental units or there is some reason beyond their control such as subjects dropping out or moving away. Since researchers do not want to discard data, they end up evaluating the leftover observations as a CRD. In this case, this design will be known as a mixed design of RCBD and CRD. Dubnicka, Blair, and Hettmansperger [1] proposed a test statistic for a mixed design consisting of paired and independent two-sample data. Dubnicka [1] combined the Wilcoxon-signed rank test statistic for paired samples and the Mann-Whitney test statistic for two independent samples. Dubnicka [1] research was extended to other cases; see [2]; [3]; [4]; [5].

In some cases, researchers want to make inferences about the difference in the means (location) and variances (scale) of populations simultaneously. Lepage [6] initiated the most commonly used nonparametric test for the location-scale problem by combining the Wilcoxon rank-sum and Ansari-Bradley's test statistics. Duran [7] developed a test based on Lepage's test but using Mood's test for the scale parameter. Other researchers have proposed a similar type of Lapage test, for example, [8]; [9]; [10]; [11].

This research presents new nonparametric tests for a simple tree alternative for location and scale testing in a mixed design of RCBD and CRD. The null and alternative hypotheses for this research are given below:

$$\mathrm{H_0}: \mu_1 = \mu_2 = \cdots = \mu_\mathrm{k} \quad , and \quad \mathrm{H_0}: \sigma_1 = \sigma_2 = \cdots = \sigma_\mathrm{k} \qquad (2)$$
$$\mathrm{H_a}: \mu_1 \leq [\mu_2, \mu_3, \ldots \mu_k] \quad and/or \; \mathrm{H_a}: \sigma_1 \leq [\sigma_2, \ldots, \sigma_\mathrm{k}] \text{ with at least one strict inequality.}$$

where μ_i is the location parameter of population i and σ_i is the scale parameter of population i with $i = 1,2,\dots,k$ and k is the total number of populations. Population one $i = 1$ is indicated as the control population, while populations 2 through k are the treatment populations.

1.1. Background

1.1.1. Fligner-Wolfe

The Fligner-Wolfe test is a nonparametric test for testing the differences among the control population and other treatment populations [12]. The null and alternative hypotheses are given in (1) with 1 denoting the control population and 2,3,…,k denoting the treatment population.

To compute the Fligner-Wolfe test statistic (T_{L1}), arrange all observations from smallest to largest, then assign a rank to each observation. Sum the ranks that belong to the combined treatment sample. Let r_{ij} be the rank of the observation x_{ij}, and n_i be the number of observations in each treatment, and k be the number of treatments. The Fligner-Wolfe test statistic is:

$$T_{L1} = \sum_{i=2}^{k} \sum_{j=1}^{n_i} r_{ij}. \tag{3}$$

The expected value and variance of the Fligner-Wolfe test (T_{L1}) when the null hypothesis is true are, respectively,

$$E(T_{L1}) = \frac{n_t(N+1)}{2} \text{ and } Var(T_{L1}) = \frac{n_c n_t(N+1)}{12} \tag{4}$$

where n_c and n_t are the numbers of observations in the control sample and combined treatment sample respectively and $N = n_c + n_t$.

The standardized version of the Fligner-Wolf test is:

$$Z_{L1} = \frac{T_{L1} - E(T_{L1})}{\sqrt{Var(T_{L1})}} \tag{5}$$

Under H_0, the standardized version of the Fligner-Wolf test (Z_{L1}) has an asymptotic standard normal distribution. The null hypothesis is rejected when (Z_{L1}) exceeds Z_α at the α level of significance where Z_α is the $(1-\alpha)100\%$ of the standard normal distribution.

1.1.2. Modified Page's Test

The modified Page's test was designed for a simple tree alternative in a randomized complete block design for location problems by [4]. The null and alternative hypotheses for modified Page's test for simple tree alternative are given in (1).

To compute the modified Page's test (T_{L2}), within each block, observations are arranged from least to greatest. Let R_j be the sum of the ranks for j^{th} treatment and R_1 indicates the sum of the ranks in the control sample. The modified Page's test is given below:

$$T_{L2} = R_1 + 2\sum_{j=2}^{k} R_j = R_1 + 2[R_2 + R_3 + \cdots R_k] \tag{6}$$

Under H_0, the expected value and variance of the modified Page's test (T_{L2}) are, respectively,

$$E(T_{L2}) = n_b E[L_1] = n_b\left(k^2 + \frac{k-1}{2}\right) \tag{7}$$

and

$$Var(T_{L2}) = n_b Var(L_1) = n_b\left(\frac{k^2-1}{12}\right)$$

where (L_1) is modified Page's test for one block with k treatment and n_b is the number of blocks. The standardized version of the modified Page's test is:

$$Z_{L2} = \frac{T_{L2} - E(T_{L2})}{\sqrt{Var(T_{L2})}} \tag{8}$$

Under H_0, the standardized version of the modified Page's test (Z_{L2}) has an asymptotic standard normal distribution. The null hypothesis is rejected only when $Z_{L2} \geq Z_\alpha$.

1.1.3. Modified Ansari-Bradley Test for CRD

Alsubie and Magel [13] introduced a modified version of the Ansari Bradley test (AB) for CRD for the simple tree alternative. The null and alternative hypotheses for the modified Ansari-Bradley test [14] for simple tree alternative are given below:

$$H_0: \sigma_1 = \sigma_2 = \cdots = \sigma_k \tag{9}$$
$$H_a: \sigma_1 \leq [\sigma_2, \dots, \sigma_k] \text{ with at least one strict inequality}$$

where σ_i is the scale parameter of the i^{th} population with $i = 1,2, \dots, k$ and k is the total number of populations. Population one, $i = 1$, indicates the control population, while populations 2 through k are the treatment populations.

Let n_c refer to the number of observations from the control population and n_t indicates the number of observations from the combined treatment populations where $N = n_c + n_t$. To calculate the modified AB test, arrange all observations in order from smallest to largest. Then, assign a rank of 1 to the smallest and largest observations, and assign a rank of 2 to the second smallest and second-largest observation, and so on. Let R_i be the rank of the i^{th} observation in the control sample, the modified Ansari-Bradley test for the simple tree alternative in CRD is the sum of the ranks in the control sample and given by:

$$T_{S1} = \sum_{i=1}^{n_c} R_i \tag{10}$$

When the null hypothesis is true, the expected value and variance of (T_{S1}) are, respectively,
If $N = n_c + n_t$ is an even number:

$$E(T_{S1}) = \frac{n_c\,(N+2)}{4} \tag{11}$$

and

$$Var(T_{S1}) = \frac{n_c\, n_t (N+2)(N-2)}{48(N-1)}$$

If $N = n_c + n_t$ is an odd number:

$$E(T_{S1}) = \frac{n_c\,(N+1)^2}{4N} \tag{12}$$

and

$$Var(T_{S1}) = \frac{n_c n_t (N+1)(3+N^2)}{48N^2}$$

The standardized version of modified Ansari-Bradley for CRD is:

$$Z_{S1} = \frac{T_{S1} - E(T_{S1})}{\sqrt{Var(T_{S1})}} \tag{13}$$

Under H_0, the standardized version of modified Ansari-Bradley for CRD (Z_{S1}) has an asymptotic standard normal distribution. The null hypothesis is rejected when $Z_{S1} \geq Z_\alpha$.

II. PROPOSED TESTS

2.1 Modified Ansari-Bradley Test for RCBD

We propose a new version of the modified Ansari Bradley test for a simple tree alternative in an RCBD. In this case of RCBD, we assume there is only one observation for each treatment in each block. The null and alternative hypotheses for modified Ansari-Bradley test for RCBD for simple tree alternative are given below:

$$H_0: \sigma_1 = \sigma_2 = \cdots = \sigma_k \tag{14}$$
$$H_a: \sigma_1 \leq [\sigma_2, \dots, \sigma_k] \text{ with at least one strict inequality}$$

where σ_i is the scale parameter of the i^{th} population with $i = 1,2, \dots, k$ and k is the total number of populations. Population one, $i = 1$, indicates the control population, while populations 2 through k are the treatment populations.

To compute the modified Ansari-Bradley for a randomized complete block design (RCBD), arrange observations from smallest to largest separately within each block. Next, assign a rank of 1 to both the smallest and largest observations, assign a rank of 2 to the second smallest and second-largest observations, and continue in the manner separately within each block. Let C_j be the rank of observation receiving the control in j^{th} block. The modified Ansari-Bradley test for RCBD is the sum of the ranks in the control sample and given by:

$$T_{S2} = \sum_{j=1}^{n_b} C_j \tag{15}$$

where n_b is the number of blocks.

Under H_0, the expected value and variance of T_{S2} are, respectively,
If k is an even number:

$$E(T_{S2}) = n_b \left(\frac{(k+2)}{4} \right) \tag{16}$$

$$\mathrm{Var}(T_{S2}) = n_b\left(\frac{(k-1)(k+2)(k-2)}{48(k-1)}\right)$$

If k is an odd number:

$$E(T_{S2}) = n_b\left(\frac{(k+1)^2}{4k}\right) \tag{17}$$

and

$$\mathrm{Var}(T_{S2}) = n_b\left(\frac{(k-1)(k+1)(3+k^2)}{48k^2}\right)$$

The standardized version of modified Ansari-Bradley for RCBD is:

$$Z_{S2} = \frac{T_{S2} - E(T_{S2})}{\sqrt{\mathrm{Var}(T_{S2})}} \tag{18}$$

where k is the number of treatments, and n_b is the number of blocks. Under the null hypothesis, the asymptotic distribution of Z_{S2} is standard normal distribution. The null hypothesis is rejected when $Z_{S2} \geq Z_\alpha$.

2.2 Proposed Mixed Design Tests

We propose linear combinations of test statistics previously mentioned. In some proposed tests, we sum the standardized versions of the previously mentioned test statistics first, then re-standardized, while other linear combinations are developed by adding the previously mentioned test statistics first and then standardizing. Two situations are considered for each of these sets of tests: one situation is to weight each of the tests in the linear combination equally, and the other is to weight them differently based on sample sizes

2.2.1 Proposed Test One

The first proposed test for the mixed design Z_1 will be developed using the standardized Fligner-Wolf test for CRD Z_{L1} given in (5), the standardized modified Pages test for RCBD Z_{L2} given in (8), the standardized modified Ansari-Bradley test for CRD Z_{S1} given in (13), and the standardized modified Ansari-Bradley test for RCBD Z_{S2} given in (18). We added these standardized tests together and then re-standardized them. As previously mentioned, under the null hypothesis, all these standardized tests have an asymptotic standard normal distribution. Thus, The asymptotic distribution of the summation of these tests $Z_{L1} + Z_{L2} + Z_{S1} + Z_{S2}$ under the null hypothesis is normal with mean 0 and variance 4. The proposed test one can be written as follows:

$$Z_1 = \frac{(Z_{L1} + Z_{L2} + Z_{S1} + Z_{S2}) - E(Z_{L1} + Z_{L2} + Z_{S1} + Z_{S2})}{\sqrt{\mathrm{Var}(Z_{L1} + Z_{L2} + Z_{S1} + Z_{S2})}}$$

$$Z_1 = \frac{(Z_{L1} + Z_{L2} + Z_{S1} + Z_{S2})}{\sqrt{(1+1+1+1)}} = \frac{(Z_{L1} + Z_{L2} + Z_{S1} + Z_{S2})}{\sqrt{(4)}} \tag{19}$$

2.2.2 Proposed Test Two

The second proposed test for mixed design Z_2 will be developed using the Fligner-Wolf test for CRD T_{L1} given in (3), the modified Pages test for RCBD T_{L2} given in (6), the modified Ansari-Bradley test for CRD T_{S1} given in (10), and the modified Ansari-Bradley test for RCBD T_{S2} given in (15). We added these unstandardized tests together and then standardized them. The proposed test two can be written as follows:

$$Z_2 = \frac{(T_{L1} + T_{L2} + T_{S1} + T_{S2}) - E(T_{L1} + T_{L2} + T_{S1} + T_{S2})}{\sqrt{\mathrm{Var}(T_{L1} + T_{L2} + T_{S1} + T_{S2})}}$$

$$Z_2 = \frac{(T_{L1} + T_{L2} + T_{S1} + T_{S2}) - \left(E(T_{L1}) + E(T_{L2}) + E(T_{S1}) + E(T_{S2})\right)}{\sqrt{\mathrm{Var}(T_{L1}) + \mathrm{Var}(T_{L2}) + \mathrm{Var}(T_{S1}) + \mathrm{Var}(T_{S2})}} \tag{20}$$

where $E(T_{L1})$ and $\mathrm{Var}(T_{L1})$ are the expected value and variance of the Fligner Wolfe test T_{L1} given in (4), $E(T_{L2})$ and $\mathrm{Var}(T_{L2})$ are the expected value and variance of the modified Page's test T_{L2} given in (7), $E(T_{S1})$ and $\mathrm{Var}(T_{S1})$ are the expected value and variance of the modified Ansari-Bradley test T_{S1} for CRD given in ((11) and (12)), and $E(T_{S2})$ and $\mathrm{Var}(T_{S2})$ are the expected value and variance of the modified Ansari-Bradley test T_{S2} for RCBD given in ((16) and (17)).

2.2.3 Proposed Test Three

The third proposed test for mixed design Z_3 will be developed using the similar way that used to develop the first proposed test Z_1 except we will add weight. We consider the sample size for each treatment n_a under the CRD portion along with the number of blocks n_b under the RCBD portion as weight. The weight $\frac{n_a}{n}$ is used in standardized tests for CRD (Z_{L1} and Z_{S1}), and the weight $\frac{n_b}{n}$ is used in standardized tests for RCBD

(Z_{L2} and Z_{S2}). The weights are assigned to standardized test statistics and then re-standardized. Proposed test three can be written as follows:

$$Z_3 = \frac{\left(\frac{n_a}{n} Z_{L1} + \frac{n_b}{n} Z_{L2} + \frac{n_a}{n} Z_{S1} + \frac{n_b}{n} Z_{S2}\right) - E\left(\frac{n_a}{n} Z_{L1} + \frac{n_b}{n} Z_{L2} + \frac{n_a}{n} Z_{S1} + \frac{n_b}{n} Z_{S2}\right)}{\sqrt{Var\left(\frac{n_a}{n} Z_{L1} + \frac{n_a}{n} Z_{L2} + \frac{n_a}{n} Z_{S1} + \frac{n_a}{n} Z_{S2}\right)}}$$

$$Z_3 = \frac{\left(\frac{n_a}{n} Z_{L1} + \frac{n_b}{n} Z_{L2} + \frac{n_a}{n} Z_{S1} + \frac{n_b}{n} Z_{S2}\right)}{\sqrt{\left(\frac{n_a^2}{n^2} + \frac{n_b^2}{n^2} + \frac{n_a^2}{n^2} + \frac{n_b^2}{n^2}\right)}} \quad (21)$$

where n is the sum of the sample size for each treatment n_a under the CRD portion and the number of blocks n_b under the RCBD portion $n = n_a + n_b$.

When the sample size for each treatment n_a under the CRD portion and the number of blocks n_b under the RCBD portion are equal, then the equal weight is assigned to all standardized tests. Thus, the proposed test three is equal to the proposed test one. However, If the sample size for each treatment n_a under the CRD portion greater than the number of blocks n_b under the RCBD portion, the standardized Fligner-Wolfe test and the standardized modified Ansari-Bradley test for CRD are assigned greater weight than the standardized modified Pages test and the standardized modified Ansari-Bradley test for RCBD. Also, If the sample size for each treatment n_a under the CRD portion less than the number of blocks n_b under the RCBD portion, then the standardized Fligner-Wolfe test and the standardized modified Ansari-Bradley test for CRD are assigned less weight than the standardized modified Pages test and the standardized modified Ansari-Bradley test for RCBD.

2.2.4 Proposed Test Four

The fourth proposed test for mixed design Z_4 will be developed using the similar way that used to develop the second proposed test Z_2 except we will add weight. We consider the sample size for each treatment n_a under the CRD portion along with the number of blocks n_b under the RCBD portion as weight. The weight $\frac{n_a}{n}$ is used in CRD tests (T_{L1} and T_{S1}), and the weight $\frac{n_b}{n}$ is used in RCBD tests (T_{L2} and T_{S2}). The weights are assigned to test statistics and then standardized. Proposed test four can be written as follows:

$$Z_4 = \frac{\left(\frac{n_a}{n} T_{L1} + \frac{n_b}{n} T_{L2} + \frac{n_a}{n} T_{S1} + \frac{n_b}{n} T_{S2}\right) - E\left(\frac{n_a}{n} T_{L1} + \frac{n_b}{n} T_{L2} + \frac{n_a}{n} T_{S1} + \frac{n_b}{n} T_{S2}\right)}{\sqrt{Var\left(\frac{n_a}{n} T_{L1} + \frac{n_b}{n} T_{L2} + \frac{n_a}{n} T_{S1} + \frac{n_b}{n} T_{S2}\right)}}$$

$$Z_4 = \frac{\left(\frac{n_a}{n} T_{L1} + \frac{n_b}{n} T_{L2} + \frac{n_a}{n} T_{S1} + \frac{n_b}{n} T_{S2}\right) - \left(\frac{n_a}{n} E(T_{L1}) + \frac{n_b}{n} E(T_{L2}) + \frac{n_a}{n} E(T_{S1}) + \frac{n_b}{n} E(T_{S2})\right)}{\sqrt{\frac{n_a^2}{n^2} Var(T_{L1}) + \frac{n_b^2}{n^2} Var(T_{L2}) + \frac{n_a^2}{n^2} Var(T_{S1}) + \frac{n_b^2}{n^2} Var(T_{S2})}} \quad (22)$$

where $n = n_a + n_b$.

If $n_a = n_b$, then the equal weight is assigned to all tests. Thus, the proposed test four is equal to the proposed test two. If $n_a > n_b$, the Fligner-Wolfe test and the modified Ansari-Bradley test for CRD are assigned greater weight than the modified Pages test and the modified Ansari-Bradley test for RCBD. If $n_a < n_b$, the Fligner-Wolfe test and the modified Ansari-Bradley test for CRD are assigned less weight than the modified Pages test and the modified Ansari-Bradley test for RCBD.

2.2.5 Proposed Test Five

The fifth proposed test for mixed design Z_5 is similar to proposed test three Z_3 with different weight values. The weight $\frac{n_b}{n}$ is used in standardized tests for CRD (Z_{L1} and Z_{S1}), and the weight $\frac{n_a}{n}$ is used in standardized tests for RCBD (Z_{L2} and Z_{S2}). Proposed test five is given below:

$$Z_5 = \frac{\left(\frac{n_b}{n} Z_{L1} + \frac{n_a}{n} Z_{L2} + \frac{n_b}{n} Z_{S1} + \frac{n_a}{n} Z_{S2}\right) - E\left(\frac{n_b}{n} Z_{L1} + \frac{n_a}{n} Z_{L2} + \frac{n_b}{n} Z_{S1} + \frac{n_a}{n} Z_{S2}\right)}{\sqrt{Var\left(\frac{n_b}{n} Z_{L1} + \frac{n_a}{n} Z_{L2} + \frac{n_b}{n} Z_{S1} + \frac{n_a}{n} Z_{S2}\right)}}$$

$$Z_5 = \frac{\left(\frac{n_b}{n} Z_{L1} + \frac{n_a}{n} Z_{L2} + \frac{n_b}{n} Z_{S1} + \frac{n_a}{n} Z_{S2}\right)}{\sqrt{\left(\frac{n_b^2}{n^2} + \frac{n_a^2}{n^2} + \frac{n_b^2}{n^2} + \frac{n_a^2}{n^2}\right)}} \tag{23}$$

where $n = n_a + n_b$.

If $n_a = n_b$, then the equal weight is assigned to all standardized tests. Thus, the proposed test five is equal to the proposed test one. If $n_a > n_b$, then the standardized modified Pages test and the standardized modified Ansari-Bradley test for RCBD are assigned greater weight than the standardized Fligner-Wolfe test and the standardized modified Ansari-Bradley test for CRD. If $n_a < n_b$, then the standardized modified Pages test and the standardized modified Ansari-Bradley test for RCBD are assigned less weight than the standardized Fligner-Wolfe test and the standardized modified Ansari-Bradley test for CRD.

2.2.6 Proposed Test Six

The sixth proposed test for mixed design Z_6 is similar to proposed test four Z_4 with different weight values. The weight $\frac{n_b}{n}$ is used in tests for CRD (T_{L1}, and T_{S1}), and the weight $\frac{n_a}{n}$ is used in tests for RCBD (T_{L2}, and T_{S2}). Proposed test six is given below:

$$Z_6 = \frac{\left(\frac{n_b}{n} T_{L1} + \frac{n_a}{n} T_{L2} + \frac{n_b}{n} T_{S1} + \frac{n_a}{n} T_{S2}\right) - E\left(\frac{n_b}{n} T_{L1} + \frac{n_a}{n} T_{L2} + \frac{n_a}{n} T_{S1} + \frac{n_b}{n} T_{S2}\right)}{\sqrt{Var\left(\frac{n_b}{n} T_{L1} + \frac{n_a}{n} T_{L2} + \frac{n_a}{n} T_{S1} + \frac{n_b}{n} T_{S2}\right)}}$$

$$Z_6 = \frac{\left(\frac{n_b}{n} T_{L1} + \frac{n_a}{n} T_{L2} + \frac{n_b}{n} T_{S1} + \frac{n_a}{n} T_{S2}\right) - \left(\frac{n_a}{n} E(T_{L1}) + \frac{n_a}{n} E(T_{L2}) + \frac{n_a}{n} E(T_{S1}) + \frac{n_a}{n} E(T_{S2})\right)}{\sqrt{\frac{n_b^2}{n^2} Var(T_{L1}) + \frac{n_a^2}{n^2} Var(T_{L2}) + \frac{n_b^2}{n^2} Var(T_{S1}) + \frac{n_a^2}{n^2} Var(T_{S2})}} \tag{24}$$

If $n_a = n_b$, then equal weight is assigned to all tests in each of the proposed linear combinations. Thus, proposed tests two, four, and six are all equivalent. If $n_a > n_b$, then the modified Pages test and the modified Ansari-Bradley test for RCBD are assigned greater weight than the Fligner-Wolfe test and the modified Ansari-Bradley test for CRD. If $n_a < n_b$, then the modified Pages test and the modified Ansari-Bradley test for RCBD are assigned less weight than the Fligner-Wolfe test and the modified Ansari-Bradley test for CRD.

2.2.7. Rejection Region for All Tests

Under H_0, all of the proposed tests have an asymptotic standard normal distribution. The null hypothesis is rejected when they are $\geq Z_\alpha$ at the α level of significance where Z_α is the $(1-\alpha)100\%$ of the standard normal distribution.

III. Simulation Study

The simulation study was performed using SAS version 9.4. The random variables were generated from different distributions using the RAND function in the data step. The seed value for the RAND function was set by using the STREAMINIT subroutine. In this study, the seed was set to be zero, meaning each code run would produce a different data set.

In this study, the significant level α for each proposed test statistic was estimated and compared to the stated alpha value, which was 0.05. The significant level of each proposed test was calculated by counting the number of times that the null hypothesis was rejected under H_0 then divided by the number of replications, which was 10,000 samples. If the estimated alpha value was approximately 0.05, the power of tests was compared to each other. The power of each proposed test was estimated by counting the number of times that the null hypothesis was rejected under H_a then divided by the number of replications, which was 10,000 samples.

The study considered the following underlying distributions: normal, and t distribution with three degrees of freedom. The call function (X= Rand ("Normal", 0,1)) was used to generate the random sample from a standard normal distribution with a mean equal to zero and a standard deviation equal to one. The call function (X= Rand ("T",3)) was used to generate the random sample from t-distribution with three degrees of freedom.

This study considered three cases related to the number of blocks in the RCBD portion and the sample size for each treatment in the CRD portion. Powers and significance levels for the proposed tests were estimated in each case. We started with the number of blocks in the RCBD portion , n_b, is equal to the sample size for each treatment in the CRD portion, n_a. In this case, we considered n_b and n_a both equal 10. Then, the number of blocks in the RCBD portion is greater than the sample size for each treatment in the CRD portion, $n_b =$

$10, n_a = 5$. Finally, we considered where the number of blocks in the RCBD portion is less than the sample size for each treatment in the CRD portion, $n_b = 5, n_a = 10$.

The data used in this study was generated from a mixed design consisting of a CRD and RCBD portion. The RCBD is commonly used to reduce error variation; hence the error variance associated with RCBD data is smaller than that associated with CRD data. We want to decide whether or not it makes a difference in which test statistic to use when the error variance in both the RCBD data and the CRD data is about the same and when the CRD data have a larger error variance. Two cases were considered related to the variance among the CRD portion and the RCBD portion. First, the variance of the CRD portion is equal to the variance of the RCBD portion. Second, the variance of the CRD portion is two times the variance of the RCBD portion.

Powers for the proposed tests were estimated under a variety of location and scale parameter arrangements. The following parameter arrangements were considered in the simulation study:

- For three populations:

1. The second and third populations have the same location and scale parameters that are different than the first population (control).
2. The first population (control) and second population have the same location and scale parameters that are different than the third population.
3. The three populations have different and unequally spaced location and scale parameters.

- For Four populations:

1. The last three populations have the same location and scale parameters that are different than the first population (control).
2. The first population (control) and second population have the same location and scale parameters that are different than the third and fourth populations.
3. The four populations have different and unequally spaced location and scale parameters.

- For five populations:

1. The last four populations have the same location and scale parameters that are different than the first population (control).
2. The first population (control) and second population have the same location and scale parameters that are different than the last three populations.
3. The five populations have different and unequally spaced location and scale parameters.

IV. Results

Selected tables and graphs are only a small portion of the findings but represent the overall findings of the study. Additional results may be obtained from the authors. Tables 1-3 show the simulation study results for three and five treatments (k=3,5) under the normal distribution and four treatments (k=4) under the t distribution. We estimated the alpha values for the proposed tests and tabled them in the first row of Tables (1, 2, and 3). The estimated alpha values for all proposed tests were around 0.05, meaning that all proposed tests maintained their alpha values, see the first row of Tables (1, 2, and 3). Moreover, we estimated and compared the powers of all proposed tests under different combinations of location parameters and scale parameters.

The first case we considered is when the number of blocks in the RCBD portion equal to the sample size for each treatment in the CRD portion. In this case, the weights will not affect, so the results of proposed tests one, three, and five are similar. Likewise, the proposed tests two, four, and six have the same results. When the populations have different location and scale parameters, the proposed test one, Z_1, has the highest powers (Table 1).

The second case we considered is when the number of blocks in the RCBD portion is greater than the sample size for each treatment in the CRD portion. When the populations have different location and scale parameters, the proposed test one, Z_1, has the highest powers (Table 2).

The last case we considered is when the number of blocks in the RCBD portion is less than the sample size for each treatment in the CRD portion. When the populations have different location and scale parameters, the proposed test three, Z_3, has the highest powers (Table 3).

In cases where the variance of the CRD portion is larger than the variance of the RCBD portion, the proposed tests that have the highest powers will be approximately the same as the cases where the variance of the CRD portion equal to the variance of the RCBD portion (Figure 1 and Figure 2). Figure 1 compares the powers of proposed tests when the number of blocks in the RCBD portion is greater than the sample size for each treatment in the CRD portion, and the populations have different location and scale parameters for the normal distribution. Figure 2 compares the powers of proposed tests when the number of blocks in the RCBD portion is less than the sample size for each treatment in the CRD portion, and the populations have different location and scale parameters for the normal distribution.

Table 1. Estimated power of tests for mixed design under the normal distribution with different means and variances; the variance in RCBD =CRD; K=3; $n_b = 10, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,1) (0,1)	0.0517	0.0509	0.0517	0.0509	0.0517	0.0509
(0,1) (1,2) (1,2)	**0.7449**	0.6998	**0.7449**	0.6998	**0.7449**	0.6998
(0,1) (0,1) (1,3)	**0.4272**	0.2826	**0.4272**	0.2826	**0.4272**	0.2826
(0,1) (0,2) (1,3)	**0.6306**	0.4358	**0.6306**	0.4358	**0.6306**	0.4358
(0,1) (0.5,2.5) (1,5)	**0.7748**	0.5756	**0.7748**	0.5756	**0.7748**	0.5756
(0,1) (0.25,2) (0.5,5)	**0.6430**	0.4224	**0.6430**	0.4224	**0.6430**	0.4224
(0,1) (1,3) (1.5,3.5)	**0.8525**	0.7466	**0.8525**	0.7466	**0.8525**	0.7466
(0,1) (1.5,6) (1.75,8)	**0.8955**	0.7381	**0.8955**	0.7381	**0.8955**	0.7381

Table 2. Estimated power of tests for mixed design under the t distribution with different means and variances; the variance in RCBD =CRD; K=4; $n_b = 10, n_a = 5, \sigma = \sqrt{3}$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) (μ_4,σ_4)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (0,σ) (0,σ) (0,σ)	0.0522	0.0535	0.0529	0.0512	0.0530	0.0480
(0,σ) (1,2σ) (1,2σ) (1,2σ)	**0.5518**	0.4811	0.5092	0.5478	0.5122	0.3947
(0,σ) (0,σ) (0,3σ) (1, 3σ)	**0.3387**	0.1939	0.3057	0.2100	0.3117	0.1633
(0,σ) (0,2σ) (1,3σ) (1,4σ)	**0.5470**	0.3391	0.4961	0.3721	0.5276	0.2843
(0,σ) (0.5,2.5σ) (1,5σ) (1.5,7.5σ)	**0.6937**	0.4351	0.6153	0.4794	0.6806	0.3627
(0,σ) (0.25,2σ) (0.5,5σ) (0.75,σ)	**0.4554**	0.2621	0.4223	0.3146	0.4168	0.2311
(0,σ) (1,3σ) (1.5,3.5σ) (2,2σ)	**0.7206**	0.6369	0.6707	0.7143	0.6855	0.5327
(0,σ) (1,4σ) (1.5,6σ) (1.75,8σ)	**0.7636**	0.5208	0.6814	0.5677	0.7588	0.4425

Table 3. Estimated power of tests for mixed design under the normal distribution with different means and variances; the variance in RCBD =CRD; K=5; $n_b = 5, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) $(\mu_4,\sigma_4)(\mu_5,\sigma_5)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,1) (0,1) (0,1) (0,1)	0.0545	0.0516	0.0525	0.0518	0.0532	0.0541
(0,1) (1,2) (1,2) (1,2) (1,2)	0.7411	0.7325	**0.7872**	0.7202	0.6062	0.7631
(0,1) (0,1) (0,3) (1,3) (1,3)	0.6462	0.4188	**0.7003**	0.4117	0.4983	0.4460
(0,1) (0,2) (1,3) (1,4) (1,5)	0.8393	0.6273	**0.8968**	0.6172	0.6714	0.6621
(0,1) (0.5,2.5) (1,5) (1.5,7.5) (2,10)	0.9259	0.758	**0.9629**	0.7501	0.7885	0.7849
(0,1) (0.25,2) (0.5,5) (0.75,1) (1,4.5)	0.7537	0.5714	**0.8053**	0.5607	0.5936	0.6056
(0,1) (1,3) (1.5,3.5) (2,2) (2.5,6)	0.8927	0.8730	**0.9196**	0.8623	0.7732	0.8981
(0,1) (1,4) (1.5,6) (1.75,8) (2.5,10)	0.9733	0.8444	**0.9796**	0.8276	0.919	0.8780

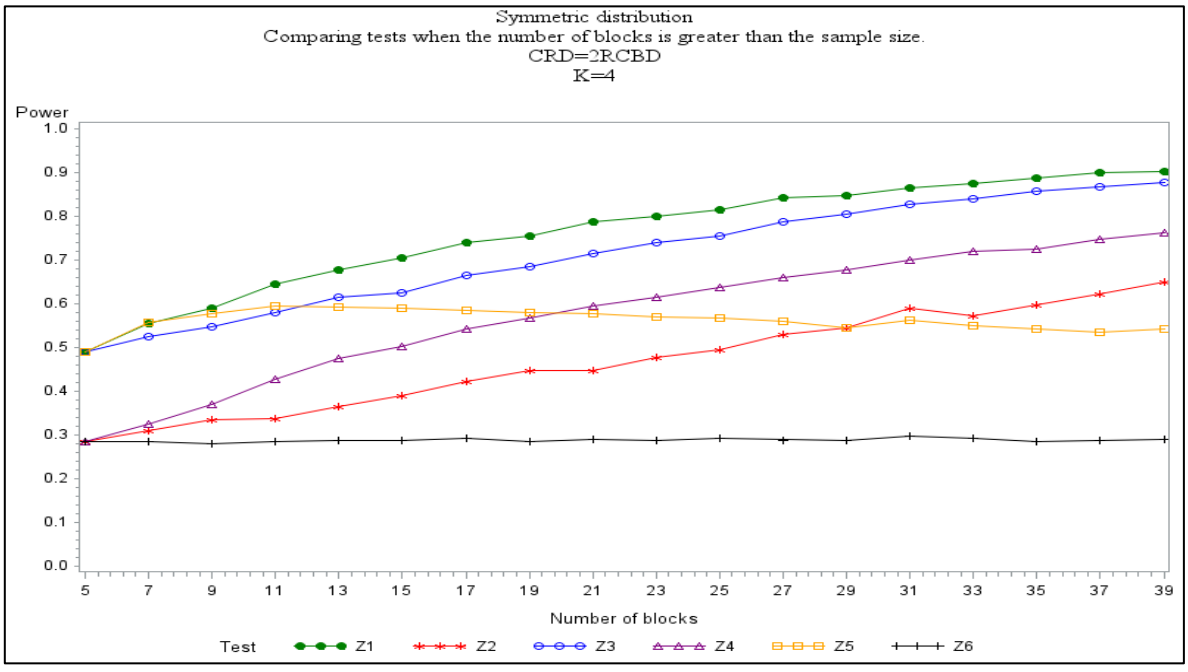

Figure 1. Estimated powers for proposed tests; CRD=2RCBD; K=4; n_a=5, and n_b=5,7,9,…,39.

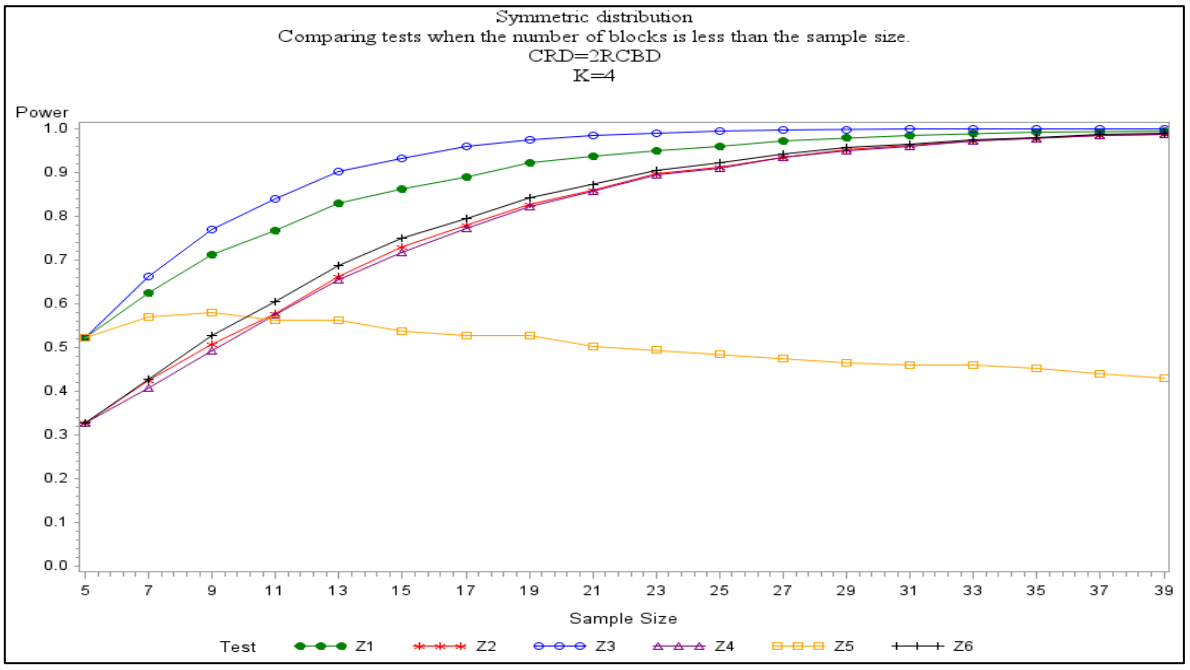

Figure 2. Estimated powers for proposed tests; CRD=2RCBD; K=4; n_a=5,7,9,…,39, and n_b=5

V. Conclusion

We proposed six nonparametric tests for a simple tree alternative for testing the location-scale problem when the data are a mixture of a randomized complete block design (RCBD) and a completely randomized design (CRD). The tests combined the Fligner-Wolf test, modified Pages test, and modified Ansari-Bradley test for CRD and RCBD. The simulation study showed that all proposed tests maintained their alpha values. Since the tests all maintained their alpha values, we compared them on the basis of estimated powers.

In the light of our findings, the overall recommendation for location and scale testing is to use the proposed test one , Z_1, and proposed test three , Z_3, if the observations are assumed to come from an approximately symmetric distribution. The proposed test one, Z_1, is recommended when the number of blocks of the RCBD portion is equal to or greater than the sample size for each treatment in the CRD portion. However, if the number of blocks for the RCBD portion is less than the sample size for each treatment in the CRD portion, the proposed test three, Z_3, is recommended.

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