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On A Curvature Inheritance in A R−⍰ Recurrent Finsler Space

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Abstract: *Singh S.P.* ***[1]*** *discussed curvature inheritance in a Finsler space and established necessary condition for existence of some transformation in a Finsler Space. Further, Mishra C.K. , Yadav D.D.S.* ***[2]*** *discussed the fundamental properties of Projective Curvature Inheritance in an NP-* F_n *space. The objective of this paper is to discuss the existence of curvature inheritance in a R-⍰ recurrent Finsler Space. Certain useful results have been obtained in this paper.*

Keywords: *Affine Motion, Curvature Inheritance, Finsler Space, skew symmetric Finsler space , symmetric Finsler space.*
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I. Prelemneries

We consider an n- dimensional Finsle space**[3]** having 2n line elements $(x^i, \dot{x}^i)$ (i= 1,2,3,……,n) equipped with non – symmetric connection Γ^i_{jk}. The non – symmetric connection Γ^i_{jk} is based on non – symmetric Fundamental metric tensor

$$g_{ij}\left(x^i,\dot{x}^i\right) \neq g_{ji}\left(x^i,\dot{x}^i\right)$$

We assume that Γ^i_{jk} are homogenous of degree zero in its directional arguments $\dot{x}^i$. Γ^i_{jk} can be written as below**[4]** :-

(1.1) $$\Gamma^i_{jk}\left(x^i,\dot{x}^i\right) = M^i_{jk}(x^i,\dot{x}^i) + \frac{1}{2} N^i_{jk}(x^i,\dot{x}^i)$$

where M^i_{jk} and $\frac{1}{2} N^i_{jk}$ denotes symmetric and skew symmetric parts of Γ^i_{jk} respectively.

The covariant derivative of a tensor field $T_j^{\ i}(x^i, \dot{x}^i)$ with respect to x^k are defined in two ways:-

(1.2) $$T^i_{j\,\overset{+}{|}\,k} = \partial_k T^i_j - (\dot{\partial}_m T^i_j)\,\Gamma^m_{pk}\,\dot{x}^p + T^m_j\,\Gamma^i_{mk} - T^i_m\,\Gamma^m_{jk}$$

and

(1.3) $$T^i_{j\,\overset{-}{|}\,k} = \partial_k T^i_j - (\dot{\partial}_m T^i_j)\,\overline{\Gamma}^m_{pk}\,\dot{x}^p + T^m_j\,\overline{\Gamma}^i_{mk} - T^i_m\,\overline{\Gamma}^m_{jk},$$

where another connection $\overline{\Gamma}^i_{jk}(x^i, \dot{x}^i)$ **[6]** defined as below:-

(1.4) $$\overline{\Gamma}^i_{jk}\left(x^i,\dot{x}^i\right) = M^i_{jk}\left(x^i,\dot{x}^i\right) - \frac{1}{2} N^i_{jk}\left(x^i,\dot{x}^i\right),$$

From (1.1) and (1.4),it is clearly seen that

$$\overline{\Gamma}^i_{jk} = \Gamma^i_{kj}$$

The commutation formula involving R-⍰ covariant derivative (1.2) is given by

(1.5) $$T^i_{j\,\overset{+}{|}\,hk} - T^i_{j\,\overset{+}{|}\,kh} = -\left(\dot{\partial}_m T^i_j\right)R^m_{shk}\,\dot{x}^s + T^m_j R^m_{jhk} + (T^i_j)_{\overset{+}{|}\,m} N^m_{kh},$$

where curvature tensor R^h_{ijk} given as

(1.6) $$R^h_{ijk} = \partial_k\Gamma^h_{ij} - \partial_j\Gamma^h_{ik} + \left(\dot{\partial}_m\Gamma^h_{ik}\right)\Gamma^m_{sj}\dot{x}^s - \left(\dot{\partial}_m\Gamma^h_{ij}\right)\Gamma^m_{sk}\dot{x}^s + \Gamma^p_{ij}\Gamma^h_{pk} - \Gamma^p_{ik}\Gamma^h_{pj},$$

Let us consider an infinitesimal point transformation

(1.7) $$\overline{x}^i = x^i + v^i(x)dt,$$

where $v^i(x)$ is any vector field and dt is an infinitesimal point constant.

In view of transformation (1.7), The Lie-Derivative of tensor field $T^i_j(x^i, \dot{x}^i)$ and connection coefficient Γ^i_{jk} are given by **[4]**

(1.8) $$D_L T^i_j = T^i_{j\,|\,h} v^h - T^h_j v^i_{|h} + T^i_h v^h_{|j} + \dot{\partial}_h T^i_j v^h_{|s}\, \dot{x}^s ,$$

(1.9) $$D_L \Gamma^i_{jk} = v^i_{|j|k} + R^i_{jkh} v^h + \dot{\partial}_r \Gamma^i_{jk} v^r_{|s} \dot{x}^s .$$

The curvature tensor R^i_{hjk} satisfies the following two identities, called Bianchi's identities**[5]**

(1.10) $$R^i_{ljk\,|\,h} + R^i_{hlk\,|\,j} + R^i_{hjl\,|\,k} = -\left(R^m_{jk}\dot{\partial}_m \Gamma^i_{hl} + R^m_{lk}\dot{\partial}_m \Gamma^i_{hj} + R^m_{jl}\dot{\partial}_m \Gamma^i_{hk}\right),$$

(1.11) $$R^i_{hjk} + R^i_{jkh} + R^i_{khj} = N^i_{hj\,|\,k} + N^i_{jk\,|\,h} + N^i_{kh\,|\,j} ,$$

We have the following commutation formulae for Lie- Derivative of Mixed tensor T^i_{jk} and connection coefficient Γ^i_{jh} as

(1.12) $$(D_L T^i_{jk})_{|\,l} - D_L\left(T^i_{jk\,|\,l}\right) = T^i_{sk}\left(D_L \Gamma^s_{jl}\right) + T^i_{js}\left(D_L \Gamma^s_{kl}\right) - T^s_{jk}\left(D_L T^i_{sl}\right) + \dot{\partial}_B T^i_{jk}\left(D_L \Gamma^s_{rl}\right)\dot{x}^r,$$
$$(D_L \Gamma^i_{jh})_{|\,k} - (D_L \Gamma^i_{kh})_{|\,j} = D_L R^i_{hjk} + \dot{\partial}_r \Gamma^i_{hj}\left(D_L \Gamma^r_{ks}\right)\dot{x}^s - \dot{\partial}_r \Gamma^i_{hk}\left(D_L \Gamma^r_{js}\right)\dot{x}^s.$$

Definition(1.1) – An n-dimensional Finsler space F_n is said to be R-ℏ recurrent Finsler space if its curvature tensor R^i_{hjk} satisfies the relation

$$\mathbf{R}^i_{hjk\,|\,s} = \beta_s \mathbf{R}^i_{hjk} ,$$

where $\beta_s(x)$ denotes a non zero covariant recurrent vector field. we shall denote it F_n^* throughout the paper.**[4]**

Definition(1.2)- R^* − curvature inheritance is defined as an infinitesimal transformation with respect to which the Lie- Derivative of curvature tensor satisfies the following relation:

$\pounds_v \mathbf{R}^{*i}_{jkh} = \alpha(\mathbf{x})\mathbf{R}^i_{jkh}$, **[1]**

where $\alpha(x)$ is non zero scalar function.

II. Curvature inheritance in a R-ℏ recurrent finsler space

Definition(2.1):- Curvature inheritance in a R-ℏ recurrent finsler space is defined as infinitesimal transformation given by (1.7) with respect to which Lie- derivative of curvature tensor R^i_{hjk} satisfies the relation

(2.1) $$\pounds_v R^i_{hjk} = \alpha(x) R^i_{hjk},$$

where $\alpha(x)$ is a non-zero scalar function.

In view of result (1.8), The Lie-derivative of R^i_{hjk} is given by

(2.2) $$D_L R^i_{hjk} = R^i_{hjk\,|\,l} v^l - R^l_{hjk} v^i_{|l} + R^i_{ljk} v^l_{|h} + R^i_{hlk} v^l_{|j} + R^i_{hjl} v^l_{|k} + \dot{\partial}_l R^i_{hjk} v^l_{|s} \dot{x}^s,$$

If a R-ℏ recurrent Finsler space admits a curvature inheritance in (2.1), then (2.2) becomes

(2.3) $$\alpha(x) R^i_{hjk} = \beta_l R^i_{hjk} - R^l_{hjk} v^i_{|l} + R^i_{ljk} v^l_{|h} + R^i_{hlk} v^l_{|j} + R^i_{hjl} v^l_{|k} + \dot{\partial}_l R^i_{hjk} v^l_{|s} \dot{x}^s,$$

where $$\mathbf{v}^i_{|j} = \lambda \delta^i_j$$

Consider the infinitesimal point transformation given by(1.7) takes the con-circular transformation as

(2.4) $$\bar{x}^i = x^i + v^i(x)dt, \quad v^i_{|j} = \lambda \delta^i_j,$$

where, λ is non zero constant.(2.4) in(2.3),

$$\alpha(x) R^i_{hjk} = \beta_s R^i_{hjk} - R^l_{hjk}(\lambda \delta^i_l) + R^i_{ljk}(\lambda \delta^l_h) + R^i_{hlk}(\lambda \delta^l_j) + R^i_{hjl}(\lambda \delta^l_k) + \dot{\partial}_l R^i_{hjk}(\lambda \delta^l_s)\dot{x}^s$$

(2.5) $$(\alpha(x) - \beta_s) R^i_{hjk} = -\lambda R^i_{hjk} + \lambda R^i_{hjk} + \lambda R^i_{hjk} + \lambda R^i_{hjk} + \lambda \dot{\partial}_s R^i_{hjk} \dot{x}^s,$$

Or

$$(\alpha(x) - \beta_s) R^i_{hjk} = 2\lambda R^i_{hjk} + \lambda \dot{\partial}_s R^i_{hjk} \dot{x}^s,$$

Due to Homogeneity property of curvature tensor R^i_{hjk}, we have,

(2.6) $$\dot{\partial}_s R^i_{hjk} \dot{x}^s = 0.$$

Due to (2.6), (2.5) reduces to,

(2.7) $$(\alpha(x) - \beta_s - 2\lambda) R^i_{hjk} = 0$$

For non-flat space,

$$\mathbf{R}^i_{hjk} \neq 0.$$

So we must have,

$$(\alpha(x) - \beta_s - 2\lambda) = 0$$

$$\alpha(x) = \beta_s + 2\lambda$$

Thus, We have following theorem,

Theorem(2.1) – If the R-⍰ recurrent Finsler space admits the curvature inheritance with con-circular form then scalar function $\alpha(x)$ is given by (2.7).

Theorem(2.2) – If the R-⍰ recurrent finsler space admits the curvature inheritance with con-circular form with scalar function $\alpha(x) = 0$ then, covariant recurrent vector becomes,

$$\beta_s = -2\lambda.$$

III. Recurrent form in a R-⍰ recurrent finsler space

Definition (3.1)- If the covariant derivative of the vector field $v^i(x)$ satisfies the relation:

(3.1) $$v^i_{|j} = \emptyset_j v^i,$$

where $\emptyset_j(x)$ denotes an arbitrary covariant vector.The vector field satisfying (3.1) is called a recurrent field.

Consider the infinitesimal point transformation given by(1.7) takes the following form

(3.2) $$\bar{x}^i = x^i + v^i(x)dt, \quad v^i_{|j} = \emptyset_j v^i,$$

Such a transformation is called recurrent transformation. In view of recurrent transformation(3.2) and (1.14), (2.2) reduces to:

$$\pounds_v R^i_{hjk} = \beta_l v^l R^i_{hjk} + R^i_{ljk}(\emptyset_h v^l) + R^i_{hlk}(\emptyset_j v^l) + R^i_{hjl}(\emptyset_k v^l) - R^l_{hjk}(\emptyset_l v^i) + \dot{\partial}_l R^i_{hjk}(\emptyset_s v^l)\dot{x}^s$$

$$= \beta_l v^l R^i_{hjk} + [R^i_{ljk}\emptyset_h + R^i_{hlk}\emptyset_j + R^i_{hjl}\emptyset_k]v^l - R^l_{hjk}\emptyset_l v^i + \dot{\partial}_l R^i_{hjk}(\emptyset_s v^l)\dot{x}^s$$

$$= \beta_l v^l R^i_{hjk} + \left(R^i_{ljk\,+|h} + R^i_{hlk\,+|j} + R^i_{hjl\,+|k} + \dot{\partial}_l R^i_{hjk\,+|s}\dot{x}^s\right)v^l - R^l_{hjk}\emptyset_l v^i$$

(3.3) $\Rightarrow \quad \pounds_v R^i_{hjk} \neq 0.$

Thus, We have

Theorem(3.1)- In R-⍰ Recurrent Finsler space, recurrent form (3.2) admits curvature inheritance.

If R-⍰ recurrent Finsler space admits curvature inheritance subject to recurrent form given by (3.2), then (3.3) reduces to

(3.4) $$(\alpha - \beta_l v^l)R^i_{hjk} = \left(R^i_{ljk\,+|h} + R^i_{hlk\,+|j} + R^i_{hjl\,+|k} + \dot{\partial}_l R^i_{hjk\,+|s}\dot{x}^s\right)v^l - R^l_{hjk}\emptyset_l v^i$$

Thus, We have

Theorem(3.2)- If R-⍰ recurrent finsler space admits curvature inheritance subject to recurrent transformation given by (3.2) , then results (3.4) holds good.

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