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Greedy Edge Colouring for Lower Bound of an Achromatic Index of Simple Graphs

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ABSTRACT *: The Achromatic index ψ'(G) of a simple graph G is the maximum number of colours used in the proper edge colouring of G such that the colouring is complete. The number for the complete graph on n vertices is denoted by A(n). The number in general is not known for simple graphs. In this paper we give greedy way of colouring edges of simple graph G to find lower bound of an achromatic index of G.*

KEYWORDS - *Achromatic Index, Edge colouring, Complete edge Colouring*

I. INTRODUCTION

An algorithm is finite set of precise instructions for performing computations or for solving problems. The algorithm should work in such a way that it must always produce the correct answer and should be efficient. One of the simplest approaches is to select the best choice at each step instead considering all sequences of steps that may lead to the optimum solution. Such an algorithm is called greedy algorithm.But interestingly even greedy algorithm also fails many times to give optimization. For example [1] consider problem of exchanging of 30 cents into quarter (25 cents), dime (10 cents), and penny (1 cent). The solution is optimum in the sense that it uses the fewest coins. The greedy algorithm gives the exchange as1-quarter, 5-pennies which uses 6 coins but this is not the optimum solution. The optimum solution is 3 –dimes.

A proper k-edge colouring of a simple graph G is assigning k-colours to the edges of G so that any two adjacent edges receive different colours. If for each pair t_i, t_j of colours there exists adjacent edges with this colours then the colouring is said to be complete.The Achromatic index ψ'(G) of graph G is the maximum number of colours used in the proper edge colouring of G such that the colouring is complete. The number for the complete graph on n vertices is denoted by A(n)[2]. The number in general is not known for simple graphs.

Definition: colour-colour adjacency matrix (C-C matrix)[3]

Consider an proper edge colouring C of a simple graph G with k colours.

We define colour-colour adjacency matrix C_G with respect to the above colouring as the matrix of order $k \times k$ by $C_G=[c_{ij}]$

Where c_{ij} = the number of vertices at which colour i is adjacent to colour j in the colouring C of the graph G.

II. GREEDY EDGE COLOURING

Initially we colour the edges using the following procedure & form the C-C matrix using this colouring.

Step 1: Name the Vertices of given G(p, q) graph to be $V_1, V_2, V_3, \ldots, V_p$

Step 2: whenever the edge exits colour it in the following fashion.

Colour V_1V_2 as 1, V_1V_3 as 2, ………… ,V_1V_p as p-1,

Colour V_2V_3 as p, V_2V_4 as p+1, ………… ,V_2V_p as(p-1)+(p-2) = 2p-3,

.

.

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Colour $V_{p-1}V_p$ as $\frac{p(p-1)}{2}$

Step 3: form the colour- colour adjacency matrix with respect to the above colouring.

Now we will start recolouring in the graph, we must try to recolour in such a way that number of non adjacencies among the colours should decrease. i. e. whenever existing colour is to be recoloured number of zeros in the non vanishing colour columns are require to be counted and among all such possible recolouring the pair should be consider for recolouring of one by the other which will give us minimum number of zeros (hence the maximum adjacencies) in the colour- colour adjacency matrix. The algorithm for the method is explained below.

Step 4: For every non vanished and non adjacent pair of colours i, j count the number [i j] depending upon k such that {c[k,i]=0 and c[k, j]≠ 0 or c[k, i]≠0 and c[k, j]=0}for all non vanished k rows in the colour-colour adjacency matrix.

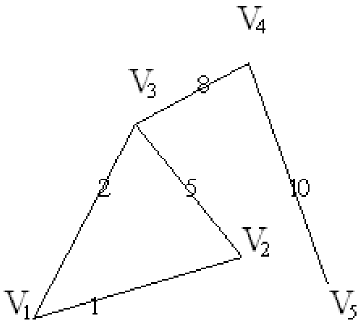
Step 5: recolour j_1 with i_1

where $[i_1\ j_1]$ = max { [i j]:over all existing mutual pairs of colours}

Step 6: reconstruct the C-C matrix and repeat steps 3, 4, 5 till complete colouring of G is obtained.

III. WORKING EXAMPLE

consider the following graph.



The numbers on the edges are colours.

The C-C matrix of the graph is written below where the natural order of rows and coloumns representing colours 1,2,5,8,10 respectively.

$$\begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Step 1 and Step 2 have been implemented.

Now [1 8]=1, [1 10]=2, [2 10]=2, [5 10]=2

Max{ [1 8], [1 10], [2 10], [5 10] }= [1 10]

Therefore recolour 10 as 1.

$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

Now we can note the colouring is complete with 4 colours.

Hence lower bound for achromatic index of graph is 4.

In this case it is simple to verify that $\psi'(G) = 4$[3]

IV. CONCLUSION

Though the method looks useful but a lower bound generated depends upon labeling of the vertices too. The complexity of the method is not the interest here but the worst case complexity of the method can be calculated as $o(p^4)$.

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