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An Interval Type-2 Fuzzy Approach for Process Plan Selection

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***ABSTRACT*:** *Process planning is a function in a manufacturing organization that systematically determines the detailed methods such as the manufacturing processes and process parameters to be used to convert a part from its initial design to the finished product. In a real manufacturing workplace the number of feasible sequences for a part increases exponentially as the complexity of the product increases. The manufacturing of several parts in a single facility sharing constrained resources and the existence of several alternative feasible process plan for each part leads to careful selection of best process plan. This paper proposes the method of process plan selection with the objective of minimizing the total processing time, total cost and the total number of setup changes by using an interval type-2 fuzzy technique for order preference by similarity to Ideal solutions. For this each process plan is evaluated and its likelihood closeness coefficient to shop floor performance is calculated using interval type 2 fuzzy set theory.*

KEYWORDS-*Fuzzy, IT2TrF Numbers, Likelihood approach, Linguistic Variables, MCDA.*

I. INTRODUCTION

Process Plan is set of instructions that are used to transform a component from initial raw material to final finished product so that customer requirements are met.In doing so it translates design specifications into manufacturing process details. Various attributes such as dimensions, geometry, and tolerances are continuously transformed step by step to get final finished product in reasonable cost and limited time. Having various options in alternative machines, alternative processes, and alternative setups to produce the same part, several process plans can be generated for a single product. The process plan selection has become a critical problem and such kind of problems is solved either by trial & error method or heuristic approaches [1]. In general manufacturing cost, number of setups, processing steps, processing time and flow rate of parts are main criteria's for designing or selecting a good process plan [2,3]. Considering the intrinsic differences in unevenness of raw material, machining parameters, and other manufacturing activities and information, the process planning can be vague and contradictory in nature.In process planning problem objectives are conflicting and information is imprecise and ambiguous. Considering this ambiguity and vagueness in process plan selection problem fuzzy based approaches can be considered as natural solution procedure for process plan selection. Fuzzy based approaches apply fuzzy logic to enumerate the role of each process plan to the shop floor performance in terms of fuzzy membership function [4].

Selection of most optimum process plan on the basis of cost, machining time, machine setups etc. makes the problem in category of multi criteria decision analysis problem. The technique for order preference by similarity to ideal solutions (TOPSIS), introduced by Hwang and Yoon, is an extensively used method for handling multiple criteria decision analysis (MCDA) problems [5]. In TOPSIS each criteria is given a performance rating against each alternative and selection of best alternative depends upon the relative importance of criteria's. In most practical cases it is often difficult for decision makers to assign precise performance values to an alternative with respect to a criteria or an accurate value of relative importance among criteria under consideration. The advantage of using a fuzzy approach in the TOPSIS methodology is to assign the linguistic ratings using fuzzy numbers instead of precise numbers [6]. The traditional fuzzy sets are represented by its membership functions that are chosen for a specified criteria. But it is often difficult to quantify membership function value as a number in interval [0,1]. Therefore, it is more suitable to represent this degree of certainty by an interval. In this regard type 2 fuzzy sets are the extension of ordinary fuzzy set concept, in which the membership function falls into an interval consisting of lower and upper limit of degree of membership [7].Interval type-2 trapezoidal fuzzy (IT2TrF) numbers provide more reasonable and computationally feasible method for handling complicated interval type-2 fuzzy data [7].Therefore, to formulateimprecisions and uncertainties, this paper attempts to advance a TOPSIS based on IT2TrF numbers for quantifying the ambiguous nature ofprocess plan selection problem. Positive ideal and negative ideal solutions are represented using IT2TrF number and likelihood based comparison approach is adopted to rate different alternatives.Next section introduces the basic definitions and notations of the IT2TrF numbers and linguistic variables with likelihood approach. Section 3 shows the proposed algorithm for likelihood closeness coefficients calculation and rank allocation. Section 4 contains process planning problem and then, the proposed method is illustrated with an example. Finally, some conclusions are pointed out in the end of this paper.

II. INTERVAL TYPE-2 FUZZY SETS AND IT2TrF NUMBERS

Selected relevant definitions and properties are briefed here to explain the concepts of interval type-2 fuzzy sets and IT2TrF numbers used throughout this paper [8, 9, 10].

Definition 1.Let Int([0, 1]) be the set of all closed subintervals of [0, 1]. A mapping $A: X \rightarrow \text{Int}([0, 1])$ is known as an interval type-2 fuzzy set in X; where X is an ordinary finite nonempty set.

Definition 2.For type-2 fuzzy set A; lower fuzzy set is $A^- : X \rightarrow [0,1]$ and upper fuzzy set is $A^+ = X \rightarrow [0,1]$ .The value $A(x) = [A^-(x), A^+(x)] \subseteq [0,1]$ represent the degree of membership of $x \in X$ to A.

Definition 3.For IT2TrF the lower and upper membership functions $A^-(x)$ and $A^+(x)$ respectively are defined as follows:

$$A^-(x) = \begin{cases} \dfrac{h_A^-(x - a_1^-)}{a_2^- - a_1^-} & if\ a_1^- < x < a_2^-, \\ h_A^- & if\ a_2^- < x < a_3^-, \\ \dfrac{h_A^-(a_4^- - x)}{a_4^- - a_3^-} & if\ a_3^- < x < a_4^-, \\ 0 & otherwise; \end{cases} \tag{1}$$

$$A^+(x) = \begin{cases} \dfrac{h_A^+(x - a_1^+)}{a_2^- - a_1^-} & if\ a_1^+ < x < a_2^+, \\ h_A^+ & if\ a_2^+ < x < a_3^+, \\ \dfrac{h_A^+(a_4^+ - x)}{a_4^+ - a_3^+} & if\ a_3^+ < x < a_4^+, \\ 0 & otherwise. \end{cases} \tag{2}$$

Where $a_1^-, a_2^-, a_3^-, a_4^-, a_1^+, a_2^+, a_3^+$ and a_4^+ are real and $a_1^+ \le a_2^+ \le a_3^+ \le a_4^+, a_1^+ \le a_1^-$ and $a_4^- \le a_4^+$. h_A^- and h_A^+ represents the heights of A^- and A^+ respectively such as $0 \le h_A^- \le h_A^+ \le 1$ Then A is an IT2TrF number in X and is expressed as follows:

$$A = [A^-, A^+] = [(a_1^-, a_2^-, a_3^-, a_4^-; h_A^-), (a_1^+, a_2^+, a_3^+, a_4^+; h_A^+)] \tag{3}$$

Definition 4.Let $A = [A^-, A^+]$ and $B = [B^-, B^+]$ be any two IT2TrF in X. Let ς be a positive integer. Assume that at least one of $h_A^- \ne h_B^+, a_4^- \ne a_1^-, b_4^+ \ne b_1^+$ and $a_\varsigma^- \ne b_\varsigma^+$ holds and at least one of $h_A^+ \ne h_B^-, a_4^+ \ne a_1^+, b_4^- \ne b_1^-$ and $a_\varsigma^+ \ne b_\varsigma^-$ Where $\varsigma = 1,2,3,4$.

The lower (LI^-) and upper (LI^+) likelihoodof an IT2TrF binary relation $A \ge B$ is defined as follows

$$LI^-(A \ge B) = \max\left\{1 - \max\left[\frac{\sum_{\varsigma=1}^{4}\max(b_\varsigma^+ - a_\varsigma^-, 0) + (b_4^+ - a_1^-) + 2\max(h_B^+ - h_A^-, 0)}{\sum_{\varsigma=1}^{4}\left|b_\varsigma^+ - a_\varsigma^-\right| + (a_4^- - a_1^-) + (b_4^+ - b_1^+) + 2\left|h_B^+ - h_A^-\right|}, 0\right], 0\right\} \tag{4}$$

$$LI^+(A \ge B) = \max\left\{1 - \max\left[\frac{\sum_{\varsigma=1}^{4}\max(b_\varsigma^- - a_\varsigma^+, 0) + (b_4^- - a_1^+) + 2\max(h_B^- - h_A^+, 0)}{\sum_{\varsigma=1}^{4}\left|b_\varsigma^- - a_\varsigma^+\right| + (a_4^+ - a_1^+) + (b_4^- - b_1^-) + 2\left|h_B^- - h_A^+\right|}, 0\right], 0\right\} \tag{5}$$

The likelihood $L(A \ge B)$ of an IT2TrF binary relation $A \ge B$ is given by the following:

$$LI(A \ge B) = \frac{LI^-(A \ge B) + LI^+(A \ge B)}{2} \tag{6}$$

This paper determines the lower likelihood of $L(A \geq B)$ via the relation $A^{-} \geq B^{+}$ because the minimal possibility of the event $A \geq B$ generally occurs in the comparison of A^{-} and B^{+}. Additionally, this paper determines the upper likelihood of $L(A \geq B)$ via the relation $A^{+} \geq B^{-}$ because the maximal possibility of the event $A \geq B$. Generally occurs in the comparison of A^{+} & B^{-}. The proposed likelihood $L^{+}(A \geq B)$, $L^{-}(A \geq B)$ and $L(A \geq B)$ in Definition 4 possess the following properties.

Property 1.The upper and lower likelihoods $L^{+}(A \geq B)$, $L^{-}(A \geq B)$ respectively, of an IT2TrF binary relation $A \geq B$ satisfy the following properties

(1) $0 \leq L^{-}(A \geq B) \leq 1;$

(2) $0 \leq L^{+}(A \geq B) \leq 1;$

(3) $L^{-}(A \geq B) + L^{+}(B \geq A) = 1;$

(4) $L^{-}(A \geq B) = 0 \ and \ L^{+}(B \geq A) = 1 \ if \ a_4^{+} \leq b_1^{+} \ and \ h_A^{-} \leq h_B^{+};$

(5) $L^{+}(A \geq B) = 0 \ and \ L^{-}(B \geq A) = 1 \ if \ b_1^{-} - a_4^{+} \geq 2\max(h_A^{+} - h_B^{-}, 0).$

Property 2.The likelihood of $L(A \geq B)$ an IT2TrF binary relation $A \geq B$ satisfies the following properties:

(1) $0 \leq L(A \geq B) \leq 1;$

(2) $L(A \geq B) + L(B \geq A) = 1;$

(3) $L(A \geq B) = L(B \geq A) = 0.5 \ if \ L(A \geq B) = L(B \geq A);$

(4) $L(A \geq A) = 0.5$.

III. PROPOSED ALGORITHM

This section develops the interval type 2 fuzzy TOPSIS method based on IT2TrF numbers.

Step 1: Formulate an MCDA problem. Specify the alternative set $Z = \{z_1, z_2, .. z_m\}$ and the criterion set $C = \{c_1, c_2, .. c_n\}$ which is divided into C_{I} (benefit criteria) and C_{II} (cost criteria).

Step 2: Select the appropriate linguistic variable giving a rating of selected alternative (z_i) with respect to the given criteria (c_j) with appropriate IT2TrF number A_{ij} define by (3).

Step 3: Define the relative importance or criteria weight W_j in terms of IT2TrF number expressed as follows:

$$W_j = [W_j^{-}, W_j^{+}] = [(w_{1j}^{-}, w_{2j}^{-}, w_{3j}^{-}, w_{4j}^{-}; h_{W_j}^{-}), (w_{1j}^{+}, w_{2j}^{+}, w_{3j}^{+}, w_{4j}^{+}; h_{W_j}^{+})] \tag{7}$$

Step 4: Determine the weighted evaluative rating using (8)

$$\begin{aligned} \overline{A}_{ij} &= W_j \otimes A_{ij} \\ &= \left[\overline{A}_{ij}^{-}, \overline{A}_{ij}^{+}\right] = \left[\left(\overline{a}_{1ij}^{-}, \overline{a}_{2ij}^{-}, \overline{a}_{3ij}^{-}, \overline{a}_{4ij}^{-}; h_{\overline{A}_{ij}}^{-}\right), \left(\overline{a}_{1ij}^{+}, \overline{a}_{2ij}^{+}, \overline{a}_{3ij}^{+}, \overline{a}_{4ij}^{+}; h_{\overline{A}_{ij}}^{+}\right)\right] \\ &= \begin{bmatrix} \left(w_{1j}^{-} \times a_{1j}^{-}, w_{2j}^{-} \times a_{2j}^{-}, w_{3j}^{-} \times a_{3j}^{-}, w_{4j}^{-} \times a_{4j}^{-}; \min\left\{h_{W_j}^{-}, h_{A_j}^{-}\right\}\right), \\ \left(w_{1j}^{+} \times a_{1j}^{+}, w_{2j}^{+} \times a_{2j}^{+}, w_{3j}^{+} \times a_{3j}^{+}, w_{4j}^{+} \times a_{4j}^{+}; \min\left\{h_{W_j}^{+}, h_{A_j}^{+}\right\}\right) \end{bmatrix} \end{aligned} \tag{8}$$

Step 5: Apply (9) and (10) to derive the weighted evaluative rating $\overline{A}_{\rho j}$ for the approximate positive-ideal solution z_ρ with respect to criterion $c_j \in C$. in (11)

$$\overline{A}_{\rho j}^{-} = \left\{ \begin{array}{ll} \left(\overset{m}{\underset{i=1}{\vee}} \overline{a}_{1ij}^{-}, \overset{m}{\underset{i=1}{\vee}} \overline{a}_{2ij}^{-}, \overset{m}{\underset{i=1}{\vee}} \overline{a}_{3ij}^{-}, \overset{m}{\underset{i=1}{\vee}} \overline{a}_{4ij}^{-}; \overset{m}{\underset{i=1}{\wedge}} h_{\overline{A}_{ij}}^{-} \right) & if \ c_j \in C_{\mathrm{I}}, \\ \left(\left(\overset{m}{\underset{i=1}{\wedge}} \overline{a}_{1ij}^{-}, \overset{m}{\underset{i=1}{\wedge}} \overline{a}_{2ij}^{-}, \overset{m}{\underset{i=1}{\wedge}} \overline{a}_{3ij}^{-}, \overset{m}{\underset{i=1}{\wedge}} \overline{a}_{4ij}^{-}; \overset{m}{\underset{i=1}{\wedge}} h_{\overline{A}_{ij}}^{-} \right) \right) & if \ c_j \in C_{\mathrm{II}} \end{array} \right\} \tag{9}$$

$$\overline{A}_{\rho j}^{+}=\left\{\begin{array}{ll}\left(\bigvee_{i=1}^{m}\overline{a}_{1ij}^{+},\bigvee_{i=1}^{m}\overline{a}_{2ij}^{+},\bigvee_{i=1}^{m}\overline{a}_{3ij}^{+},\bigvee_{i=1}^{m}\overline{a}_{4ij}^{+};\bigwedge_{i=1}^{m}h_{\overline{A}_{ij}}^{+}\right) & if\ c_j\in C_{\mathrm{I}},\\ \left(\left(\bigwedge_{i=1}^{m}\overline{a}_{1ij}^{+},\bigwedge_{i=1}^{m}\overline{a}_{2ij}^{+},\bigwedge_{i=1}^{m}\overline{a}_{3ij}^{+},\bigwedge_{i=1}^{m}\overline{a}_{4ij}^{+};\bigwedge_{i=1}^{m}h_{\overline{A}_{ij}}^{+}\right)\right) & if\ c_j\in C_{\mathrm{II}}\end{array}\right\} \tag{10}$$

$$\overline{A}_{\rho j}=[\overline{A}_{\rho j}^{-},\overline{A}_{\rho j}^{+}]=[(\overline{a}_{1\rho j}^{-},\overline{a}_{2\rho j}^{-},\overline{a}_{3\rho j}^{-},\overline{a}_{4\rho j}^{-};h_{\overline{A}_{\rho j}}^{-}),(\overline{a}_{1\rho j}^{+},\overline{a}_{2\rho j}^{+},\overline{a}_{3\rho j}^{+},\overline{a}_{4\rho j}^{+};h_{\overline{A}_{\rho j}}^{+})] \tag{11}$$

Step 6: Apply (12) and (13)to derive the weighted evaluative rating $\overline{A}_{\eta j}$ for the approximate negative-ideal solution z_{η} with respect to criteria $c_j \in C$ in (14).

$$\overline{A}_{\eta j}^{-}=\left\{\begin{array}{ll}\left(\bigwedge_{i=1}^{m}\overline{a}_{1ij}^{-},\bigwedge_{i=1}^{m}\overline{a}_{2ij}^{-},\bigwedge_{i=1}^{m}\overline{a}_{3ij}^{-},\bigwedge_{i=1}^{m}\overline{a}_{4ij}^{-};\bigwedge_{i=1}^{m}h_{\overline{A}_{ij}}^{-}\right) & if\ c_j\in C_{\mathrm{I}},\\ \left(\left(\bigvee_{i=1}^{m}\overline{a}_{1ij}^{-},\bigvee_{i=1}^{m}\overline{a}_{2ij}^{-},\bigvee_{i=1}^{m}\overline{a}_{3ij}^{-},\bigvee_{i=1}^{m}\overline{a}_{4ij}^{-};\bigwedge_{i=1}^{m}h_{\overline{A}_{ij}}^{-}\right)\right) & if\ c_j\in C_{\mathrm{II}}\end{array}\right\} \tag{12}$$

$$\overline{A}_{\eta j}^{+}=\left\{\begin{array}{ll}\left(\bigwedge_{i=1}^{m}\overline{a}_{1ij}^{+},\bigwedge_{i=1}^{m}\overline{a}_{2ij}^{+},\bigwedge_{i=1}^{m}\overline{a}_{3ij}^{+},\bigwedge_{i=1}^{m}\overline{a}_{4ij}^{+};\bigwedge_{i=1}^{m}h_{\overline{A}_{ij}}^{+}\right) & if\ c_j\in C_{\mathrm{I}},\\ \left(\left(\bigvee_{i=1}^{m}\overline{a}_{1ij}^{+},\bigvee_{i=1}^{m}\overline{a}_{2ij}^{+},\bigvee_{i=1}^{m}\overline{a}_{3ij}^{+},\bigvee_{i=1}^{m}\overline{a}_{4ij}^{+};\bigwedge_{i=1}^{m}h_{\overline{A}_{ij}}^{+}\right)\right) & if\ c_j\in C_{\mathrm{II}}\end{array}\right\} \tag{13}$$

$$\overline{A}_{\eta j}=[\overline{A}_{\eta j}^{-},\overline{A}_{\eta j}^{+}]=[(\overline{a}_{1\eta j}^{-},\overline{a}_{2\eta j}^{-},\overline{a}_{3\eta j}^{-},\overline{a}_{4\eta j}^{-};h_{\overline{A}_{\eta j}}^{-}),(\overline{a}_{1\eta j}^{+},\overline{a}_{2\eta j}^{+},\overline{a}_{3\eta j}^{+},\overline{a}_{4\eta j}^{+};h_{\overline{A}_{\eta j}}^{+})] \tag{14}$$

Step 7: Determine the likelihood-based comparison indices $LI^{-}(\overline{A}_{ij}\Re\overline{A}_{\rho j}), LI^{+}(\overline{A}_{ij}\Re\overline{A}_{\rho j})$ and $LI(\overline{A}_{ij}\Re\overline{A}_{\rho j})$ of $\overline{A}_{ij}$ relative to $\overline{A}_{\rho j}$ by using (15), (16) and (17) respectively for each $z_i \in Z$ with respect to $c_j \in C$.

$$LI^{-}(\bar{A}_{ij}\Re\bar{A}_{\rho j})=\begin{cases}\max\left\{1-\max\left[\dfrac{\sum_{\varsigma=1}^{4}\max(\bar{a}^{+}_{\varsigma ij}-\bar{a}^{-}_{\varsigma\rho j},0)+(\bar{a}^{+}_{4ij}-\bar{a}^{-}_{1\rho j})+2\max(h^{+}_{\bar{A}_{ij}}-h^{-}_{\bar{A}_{\rho j}},0)}{\sum_{\varsigma=1}^{4}\left|\bar{a}^{+}_{\varsigma ij}-\bar{a}^{-}_{\varsigma\rho j}\right|+(\bar{a}^{-}_{4\rho j}-\bar{a}^{-}_{1\rho j})+(\bar{a}^{+}_{4ij}-\bar{a}^{+}_{1ij})+2\left|h^{+}_{\bar{A}_{ij}}-h^{-}_{\bar{A}_{\rho j}}\right|},0\right],0\right\} & if\ \ c_j\in C_{\mathrm{I}},\\ \max\left\{1-\max\left[\dfrac{\sum_{\varsigma=1}^{4}\max(\bar{a}^{+}_{\varsigma\rho j}-\bar{a}^{-}_{\varsigma ij},0)+(\bar{a}^{+}_{4\rho j}-\bar{a}^{-}_{1ij})+2\max(h^{+}_{\bar{A}_{\rho j}}-h^{-}_{\bar{A}_{ij}},0)}{\sum_{\varsigma=1}^{4}\left|\bar{a}^{+}_{\varsigma\rho j}-\bar{a}^{-}_{\varsigma ij}\right|+(\bar{a}^{-}_{4ij}-\bar{a}^{-}_{1ij})+(\bar{a}^{+}_{4\rho j}-\bar{a}^{+}_{1\rho j})+2\left|h^{+}_{\bar{A}_{\rho j}}-h^{-}_{\bar{A}_{ij}}\right|},0\right],0\right\} & if\ c_j\in C_{\mathrm{II}}\end{cases}\tag{15}$$

$$LI^{+}(\bar{A}_{ij}\Re\bar{A}_{\rho j})=\begin{cases}\max\left\{1-\max\left[\dfrac{\sum_{\varsigma=1}^{4}\max(\bar{a}^{-}_{\varsigma ij}-\bar{a}^{+}_{\varsigma\rho j},0)+(\bar{a}^{-}_{4ij}-\bar{a}^{+}_{1\rho j})+2\max(h^{-}_{\bar{A}_{ij}}-h^{+}_{\bar{A}_{\rho j}},0)}{\sum_{\varsigma=1}^{4}\left|\bar{a}^{-}_{\varsigma ij}-\bar{a}^{+}_{\varsigma\rho j}\right|+(\bar{a}^{+}_{4\rho j}-\bar{a}^{+}_{1\rho j})+(\bar{a}^{-}_{4ij}-\bar{a}^{-}_{1ij})+2\left|h^{-}_{\bar{A}_{ij}}-h^{+}_{\bar{A}_{\rho j}}\right|},0\right],0\right\} & if\ \ c_j\in C_{\mathrm{I}},\\ \max\left\{1-\max\left[\dfrac{\sum_{\varsigma=1}^{4}\max(\bar{a}^{-}_{\varsigma\rho j}-\bar{a}^{+}_{\varsigma ij},0)+(\bar{a}^{-}_{4\rho j}-\bar{a}^{+}_{1ij})+2\max(h^{-}_{\bar{A}_{\rho j}}-h^{+}_{\bar{A}_{ij}},0)}{\sum_{\varsigma=1}^{4}\left|\bar{a}^{-}_{\varsigma\rho j}-\bar{a}^{+}_{\varsigma ij}\right|+(\bar{a}^{+}_{4ij}-\bar{a}^{+}_{1ij})+(\bar{a}^{-}_{4\rho j}-\bar{a}^{-}_{1\rho j})+2\left|h^{-}_{\bar{A}_{\rho j}}-h^{+}_{\bar{A}_{ij}}\right|},0\right],0\right\} & if\ c_j\in C_{\mathrm{II}}\end{cases}\tag{16}$$

$$LI(\bar{A}_{ij}\Re\bar{A}_{\rho j})=\frac{LI^{-}(\bar{A}_{ij}\Re\bar{A}_{\rho j})+LI^{+}(\bar{A}_{ij}\Re\bar{A}_{\rho j})}{2}\tag{17}$$

Step 8: Determine the likelihood-based comparison indices $LI^{-}(\bar{A}_{ij}\Re\bar{A}_{\eta j})$, $LI^{+}(\bar{A}_{ij}\Re\bar{A}_{\eta j})$ and $LI(\bar{A}_{ij}\Re\bar{A}_{\eta j})$ of $\bar{A}_{ij}$ relative to $\bar{A}_{\eta j}$ by using (18), (19) and (20) respectively for each $z_i\in Z$ with respect to $c_j\in C$.

$$LI^{-}(\bar{A}_{ij}\Re\bar{A}_{\eta j})=\begin{cases}\max\left\{1-\max\left[\dfrac{\sum_{\varsigma=1}^{4}\max(\bar{a}^{+}_{\varsigma\eta j}-\bar{a}^{-}_{\varsigma ij},0)+(\bar{a}^{+}_{4\eta j}-\bar{a}^{-}_{1ij})+2\max(h^{+}_{\bar{A}_{\eta j}}-h^{-}_{\bar{A}_{ij}},0)}{\sum_{\varsigma=1}^{4}\left|\bar{a}^{+}_{\varsigma\eta j}-\bar{a}^{-}_{\varsigma ij}\right|+(\bar{a}^{-}_{4ij}-\bar{a}^{-}_{1ij})+(\bar{a}^{+}_{4\eta j}-\bar{a}^{+}_{1\eta j})+2\left|h^{+}_{\bar{A}_{\eta j}}-h^{-}_{\bar{A}_{ij}}\right|},0\right],0\right\} & if\ \ c_j\in C_{\mathrm{I}},\\ \max\left\{1-\max\left[\dfrac{\sum_{\varsigma=1}^{4}\max(\bar{a}^{+}_{\varsigma ij}-\bar{a}^{-}_{\varsigma\eta j},0)+(\bar{a}^{+}_{4ij}-\bar{a}^{-}_{1\eta j})+2\max(h^{+}_{\bar{A}_{ij}}-h^{-}_{\bar{A}_{\eta j}},0)}{\sum_{\varsigma=1}^{4}\left|\bar{a}^{+}_{\varsigma ij}-\bar{a}^{-}_{\varsigma\eta j}\right|+(\bar{a}^{-}_{4\eta j}-\bar{a}^{-}_{1\eta j})+(\bar{a}^{+}_{4ij}-\bar{a}^{+}_{1ij})+2\left|h^{+}_{\bar{A}_{ij}}-h^{-}_{\bar{A}_{\eta j}}\right|},0\right],0\right\} & if\ c_j\in C_{\mathrm{II}}\end{cases}\tag{18}$$

$$LI^{+}(\bar{A}_{ij}\Re\bar{A}_{\eta j})=\begin{cases}\max\left\{1-\max\left[\dfrac{\sum_{\varsigma=1}^{4}\max(\bar{a}^{-}_{\varsigma\eta j}-\bar{a}^{+}_{\varsigma ij},0)+(\bar{a}^{-}_{4\eta j}-\bar{a}^{+}_{1ij})+2\max(h^{-}_{\bar{A}_{\eta j}}-h^{+}_{\bar{A}_{ij}},0)}{\sum_{\varsigma=1}^{4}\left|\bar{a}^{-}_{\varsigma\eta j}-\bar{a}^{+}_{\varsigma ij}\right|+(\bar{a}^{+}_{4ij}-\bar{a}^{+}_{1ij})+(\bar{a}^{-}_{4\eta j}-\bar{a}^{-}_{1\eta j})+2\left|h^{-}_{\bar{A}_{\eta j}}-h^{+}_{\bar{A}_{ij}}\right|},0\right],0\right\} & if\ \ c_j\in C_{\mathrm{I}},\\ \max\left\{1-\max\left[\dfrac{\sum_{\varsigma=1}^{4}\max(\bar{a}^{-}_{\varsigma ij}-\bar{a}^{+}_{\varsigma\eta j},0)+(\bar{a}^{-}_{4ij}-\bar{a}^{+}_{1\eta j})+2\max(h^{-}_{\bar{A}_{ij}}-h^{+}_{\bar{A}_{\eta j}},0)}{\sum_{\varsigma=1}^{4}\left|\bar{a}^{-}_{\varsigma ij}-\bar{a}^{+}_{\varsigma\eta j}\right|+(\bar{a}^{+}_{4\eta j}-\bar{a}^{+}_{1\eta j})+(\bar{a}^{-}_{4ij}-\bar{a}^{-}_{1ij})+2\left|h^{-}_{\bar{A}_{ij}}-h^{+}_{\bar{A}_{\eta j}}\right|},0\right],0\right\} & if\ c_j\in C_{\mathrm{II}}\end{cases}\tag{19}$$

$$LI(\bar{A}_{ij}\Re\bar{A}_{\eta j})=\frac{LI^{-}(\bar{A}_{ij}\Re\bar{A}_{\eta j})+LI^{+}(\bar{A}_{ij}\Re\bar{A}_{\eta j})}{2}\tag{20}$$

Step 9: Derive the likelihood-based closeness coefficient LC_i using (21)

$$LC_i=\frac{\sum_{j=1}^{n}LI(\bar{A}_{ij}\Re\bar{A}_{\eta j})}{\sum_{j=1}^{n}(LI(\bar{A}_{ij}\Re\bar{A}_{\rho j})+\sum_{j=1}^{n}LI(\bar{A}_{ij}\Re\bar{A}_{\eta j})}\tag{21}$$

For each alternative $z_i\in Z$ with respect to $c_j\in C$.

Step 10: Rank the *m* alternatives in accordance with the LC_i values. The alternative with the largest LC_i value is the best choice.

IV. PROBLEM FORMULATION

Three sample parts from namely Job 1 from [11, 12], Job 2 from [13], and Job 3 from[14] are considered for process plan selection problem. There are seven feasible process plan for Job 1, five feasible process plan for Job 2 and four feasible process plan for Job 3considering criteria such as total cost, setup changes, machine changes and tool changes. Time consumed in machining a job at various machines and tool combination is calculated from the data given in [15].Total cost involved is the sum of the machine usage cost,tool usage cost,setup cost,machine change cost, tool change cost.Total time to complete one process plan involves working time of machine, material handling time and setup time.The set of all alternative process plans is denoted by $Z = \{z_1,z_2,z_3,z_4,z_5,z_6,z_7,z_8,z_9,z_{10},z_{11},z_{12},z_{13},z_{14},z_{15},z_{16}\}$and five criteria's cost(c_1), setup change(c_2),tool Change(c_3),machine change(c_4) and Time (c_5) and are given in Table I. All five criteria denote minimize type.

Table I: Process Plans & their criteria for all the jobs

Job no.	Process Plans	Cost	Setup Change	Tool change	Machine Change	Time
1	z_1	2537	10	9	2	473
	z_2	2535	10	9	2	483
	z_3	2527	10	10	2	587
	z_4	2567	10	9	2	625
	z_5	2720	8	16	1	454
	z_6	3215	7	15	1	543
	z_7	3205	6	15	0	516
2	z_8	1739	3	8	1	644
	z_9	2664	11	13	0	543
	z_{10}	3799	0	10	8	550
	z_{11}	5014	5	0	12	677
	z_{12}	1784	3	9	1	649
3	z_{13}	745	1	5	0	299
	z_{14}	1198	1	5	0	292
	z_{15}	833	2	5	0	319
	z_{16}	1308	2	3	2	378

V. RESULTS

In this paper five point linguistic rating scales is takento establish the evaluative ratings of the alternatives with respect to criteria's. These linguistic variables expressed in IT2TrF numbers and are given in Table II.

Table II: Linguistic terms and their corresponding interval Trapezoidal type-2 fuzzy sets

Linguistic terms	Symbol	Interval type-2 fuzzy sets
Very low (VL)	VL	((0,0.1,0.2,0.3;1),(0,0.13,0.17,0.27;0.8))
Low (L)	L	((0.1,0.2,0.3,0.4;1),(0.13,0.23,0.27,0.37;0.8))
Medium (M)	M	((0.3,0.4,0.5,0.6;1),(0.33,0.43,0.47,0.57;0.8))
High (H)	H	((0.5,0.6,0.7,0.8;1),(0.53,0.63,0.67,0.77;0.8))
Very high (VH)	VH	((0.7,0.8,0.9,1;1),(0.73,0.83,0.87,0.97;0.8))

The relative weightage ratings of criteria in terms of IT2TrF number are presented in Table III.

Table III: Weightage Ratings for various Criteria

Criteria	Weightage Ratings
Cost (c_1)	((0.1,0.2,0.3,0.4;1),(0.13,0.23,0.27,0.37;0.8))
Setup change (c_2)	((0.3,0.4,0.5,0.6;1),(0.33,0.43,0.47,0.57;0.8))
Tool change (c_3)	((0.3,0.4,0.5,0.6;1),(0.33,0.43,0.47,0.57;0.8))
Machine change (c_4)	((0.3,0.4,0.5,0.6;1),(0.33,0.43,0.47,0.57;0.8))
Time (c_5)	((0.1,0.2,0.3,0.4;1),(0.13,0.23,0.27,0.37;0.8))

Linguistic performance rating of each alternative with respect to criteria is given in Table IV.

Table IV: Linguistic performance rating of alternatives and their relative ranking

		Criteria						
Job	Process Plan	c_1	c_2	c_3	c_4	c_5	LC	Rank
Job 1	z_1	VL	H	VL	M	VL	0.5293135	1
	z_2	VL	H	VL	M	L	0.5188261	2
	z_3	VL	H	L	M	M	0.4872964	5
	z_4	L	H	VL	M	N	0.4756051	6
	z_5	M	M	H	L	VL	0.4998666	4
	z_6	VH	L	H	L	M	0.4720732	7
	z_7	VH	L	H	VL	L	0.5054797	3
Job 2	z_8	VL	L	M	VL	M	0.5756089	1
	z_9	M	VH	VH	VL	L	0.4932366	4
	z_{10}	H	VL	H	M	M	0.5093507	3
	z_{11}	VH	L	VL	VH	VH	0.4712634	5
	z_{12}	VL	L	H	VL	H	0.5481439	2
Job 3	z_{13}	VL	M	H	VL	M	0.5505214	1
	z_{14}	H	M	H	VL	L	0.5299625	2
	z_{15}	L	H	H	VL	M	0.5101461	3
	z_{16}	VH	H	L	H	VH	0.4293073	4

The IT2TrF numbers are utilized to perform the calculations for each alternative process plan according to steps given in the algorithm.Table V shows the calculated values of positive and negative likelihood based comparison indices of each criteria within a process plan.Likelihood based closeness coefficients of each process plan can be obtained by using equation (21). Rank ordering for most optimum process plan is done on the basis of Likelihood-based closeness coefficients obtained. For rank ordering, value of likelihood closeness coefficient (LC) near to one is considered as best in each job. That is for job 1 process plan (alternative) $z_1$is having LC value maximum hence selected, similarly for job 2 process plan z_8 is selected and for job 3 process plan z_{13} is selected. LC values and their relative rankings are presented in Table IV.

Table V: Positive and negative Likelihood Values for various criteria in a process plan

Process Plan	Criteria	$LI^{-}(\bar{A}_{ij}\Re\bar{A}_{\rho j})$	$LI^{+}(\bar{A}_{ij}\Re\bar{A}_{\rho j})$	$LI(\bar{A}_{ij}\Re\bar{A}_{\rho j})$	$LI^{-}(\bar{A}_{ij}\Re\bar{A}_{\eta j})$	$LI^{+}(\bar{A}_{ij}\Re\bar{A}_{\eta j})$	$LI(\bar{A}_{ij}\Re\bar{A}_{\eta j})$
z_1	c_1	0.165361	0.834639	0.5	0.709552	0.980762	0.845157
	c_2	0.704436	0.96283	0.833633	0.306128	0.693872	0.5
	c_3	0.213317	0.786683	0.5	0.764969	0.99785	0.881409
	c_4	0.641912	0.954167	0.79804	0.280382	0.719618	0.5
	c_5	0.165361	0.834639	0.5	0.628908	0.961443	0.795175
z_2	c_1	0.165361	0.834639	0.5	0.709552	0.980762	0.845157
	c_2	0.704436	0.96283	0.833633	0.306128	0.693872	0.5
	c_3	0.213317	0.786683	0.5	0.764969	0.99785	0.881409
	c_4	0.641912	0.954167	0.79804	0.280382	0.719618	0.5
	c_5	0.300862	0.88388	0.592371	0.571316	0.928206	0.749761
z_3	c_1	0.165361	0.834639	0.5	0.709552	0.980762	0.845157
	c_2	0.704436	0.96283	0.833633	0.306128	0.693872	0.5
	c_3	0.408219	0.872819	0.640519	0.704436	0.96283	0.833633
	c_4	0.641912	0.954167	0.79804	0.280382	0.719618	0.5
	c_5	0.506924	0.932412	0.719668	0.43208	0.847978	0.640029
z_4	c_1	0.300862	0.88388	0.592371	0.66626	0.954502	0.810381
	c_2	0.704436	0.96283	0.833633	0.306128	0.693872	0.5
	c_3	0.213317	0.786683	0.5	0.764969	0.99785	0.881409
	c_4	0.641912	0.954167	0.79804	0.280382	0.719618	0.5
	c_5	0.628908	0.961443	0.795175	0.280382	0.719618	0.5
z_5	c_1	0.506924	0.932412	0.719668	0.565172	0.893128	0.72915
	c_2	0.544355	0.900772	0.722563	0.540594	0.868139	0.704366
	c_3	0.764969	0.99785	0.881409	0.306128	0.715986	0.511057
	c_4	0.408219	0.872819	0.640519	0.544355	0.900772	0.722563
	c_5	0.165361	0.834639	0.5	0.628908	0.961443	0.795175

z_6	c_1	0.709552	0.980762	0.845157	0.306128	0.693872	0.5
	c_2	0.246752	0.753248	0.5	0.704436	0.96283	0.833633
	c_3	0.764969	0.99785	0.881409	0.306128	0.715986	0.511057
	c_4	0.408219	0.872819	0.640519	0.544355	0.900772	0.722563
	c_5	0.506924	0.932412	0.719668	0.43208	0.847978	0.640029
z_7	c_1	0.709552	0.980762	0.845157	0.306128	0.693872	0.5
	c_2	0.246752	0.753248	0.5	0.704436	0.96283	0.833633
	c_3	0.764969	0.99785	0.881409	0.306128	0.715986	0.511057
	c_4	0.213317	0.786683	0.5	0.641912	0.954167	0.79804
	c_5	0.300862	0.88388	0.592371	0.571316	0.928206	0.749761
z_8	c_1	0.165361	0.834639	0.5	0.709552	0.980762	0.845157
	c_2	0.408219	0.872819	0.640519	0.798597	0.999563	0.89908
	c_3	0.641912	0.954167	0.79804	0.691306	0.934208	0.812757
	c_4	0.213317	0.786683	0.5	0.840918	1	0.920459
	c_5	0.423908	0.887521	0.655715	0.545599	0.893128	0.719363
z_9	c_1	0.506924	0.932412	0.719668	0.565172	0.893128	0.72915
	c_2	0.840918	1	0.920459	0.326471	0.673529	0.5
	c_3	0.840918	1	0.920459	0.326471	0.698092	0.512281
	c_4	0.213317	0.786683	0.5	0.840918	1	0.920459
	c_5	0.200959	0.799041	0.5	0.652807	0.954502	0.803654
z_{10}	c_1	0.628908	0.961443	0.795175	0.438667	0.815758	0.627213
	c_2	0.213317	0.786683	0.5	0.840918	1	0.920459
	c_3	0.764969	0.99785	0.881409	0.53742	0.840421	0.688921
	c_4	0.641912	0.954167	0.79804	0.691306	0.934208	0.812757
	c_5	0.423908	0.887521	0.655715	0.545599	0.893128	0.719363
z_{11}	c_1	0.709552	0.980762	0.845157	0.306128	0.693872	0.5
	c_2	0.408219	0.872819	0.640519	0.798597	0.999563	0.89908
	c_3	0.213317	0.786683	0.5	0.840918	1	0.920459
	c_4	0.840918	1	0.920459	0.326471	0.673529	0.5
	c_5	0.66626	0.954502	0.810381	0.292119	0.693872	0.492995
z_{12}	c_1	0.165361	0.834639	0.5	0.709552	0.980762	0.845157
	c_2	0.408219	0.872819	0.640519	0.798597	0.999563	0.89908
	c_3	0.764969	0.99785	0.881409	0.53742	0.840421	0.688921
	c_4	0.213317	0.786683	0.5	0.840918	1	0.920459
	c_5	0.571316	0.928206	0.749761	0.414743	0.815758	0.615251
z_{13}	c_1	0.165361	0.834639	0.5	0.709552	0.980762	0.845157
	c_2	0.280382	0.719618	0.5	0.540594	0.868139	0.704366
	c_3	0.704436	0.96283	0.833633	0.306128	0.715986	0.511057
	c_4	0.213317	0.786683	0.5	0.764969	0.99785	0.881409
	c_5	0.423908	0.887521	0.655715	0.545599	0.893128	0.719363
z_{14}	c_1	0.628908	0.961443	0.795175	0.438667	0.815758	0.627213
	c_2	0.280382	0.719618	0.5	0.540594	0.868139	0.704366
	c_3	0.704436	0.96283	0.833633	0.306128	0.715986	0.511057
	c_4	0.213317	0.786683	0.5	0.764969	0.99785	0.881409
	c_5	0.200959	0.799041	0.5	0.652807	0.954502	0.803654
z_{15}	c_1	0.300862	0.88388	0.592371	0.66626	0.954502	0.810381
	c_2	0.540594	0.868139	0.704366	0.306128	0.693872	0.5
	c_3	0.704436	0.96283	0.833633	0.306128	0.715986	0.511057
	c_4	0.213317	0.786683	0.5	0.764969	0.99785	0.881409
	c_5	0.423908	0.887521	0.655715	0.545599	0.893128	0.719363
z_{16}	c_1	0.709552	0.980762	0.845157	0.306128	0.693872	0.5
	c_2	0.540594	0.868139	0.704366	0.306128	0.693872	0.5
	c_3	0.246752	0.753248	0.5	0.680036	0.96283	0.821433
	c_4	0.764969	0.99785	0.881409	0.306128	0.693872	0.5
	c_5	0.66626	0.954502	0.810381	0.292119	0.693872	0.492995

VI. CONCLUSION

This paper provides the interval type-2 fuzzy TOPSIS algorithmic procedure for determining the priority ranking orders of alternative process plans for various jobs. The complexity in process plan selection problem can be solved with IT2TrF numbers, which reduces ambiguity & vagueness. The results establish that the proposed interval type-2 fuzzy TOPSIS method is effective and valid for addressing the process plan problems in fuzzy environment. Although the proposed method presented in this paper is illustrated by a process plan selection problem, however, it can also be applied to problems such as information project selection, material selection and many other areas of multiple decision problems.

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