Effect of Longitudinal Velocity of the Particle of the Dusty Fluid with Volume Fraction in the Incompressible Fluid

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Abstract: The effect of finite volume fraction of suspended particulate matter on axially symmetrical jet mixing of incompressible dusty fluid has been considered. Here we are assuming the velocity and temperature in the jet to differ only slightly from that of surrounding stream, a perturbation method has been employed to linearize the equation those have been solved by using Laplace Transformation technique. Numerical computations have been made to find the solutions of the longitudinal perturbed fluid velocity and longitudinal perturbed particle velocity. **Keyword and phrase:** particulate suspension, boundary layer characteristics, volume fraction, incompressible flow.

Nomenclature :

Space coordinates
Velocity components of fluid phase
Velocity components of particle phase
Dimensionless velocity components of fluid phase
Dimensionless velocity components of particle phase
Temperature of fluid phase
Temperature of particle phase
Skin friction coefficients at the lower and upper plates respectively
Specific heats of fluid and SPM respectively
Thermal conductivity
Fluid phase Reynolds number
Particle phase Reynolds number
Eckret number

I. Introduction

Many researchers have been studied, incompressible laminar jet mixing of a dusty fluid issuing from a circular jet with negligible volume fraction of SPM. However, this assumption is not justified when the fluid density is high or particle mass fraction is large. In the present paper, we find a solution of Longitudinal perturbed velocity of fluid and particle phase of boundary layer equations in axi symmetrical jet mixing of an incompressible fluid along with the effect of volume fraction of SPM. Assuming the velocity and temperature in the jet to differ only slightly from that of the surrounding stream, a perturbation method has been employed to liberalize the governing differential equations. The resulting liberalize equations have been solved by using Laplace transformation technique. Numerical computations have been made to find the solution of velocity profiles particle phase.

II. Mathematical Formulation

The equation governing the study two-phase boundary layer flow in axi-symmetric case can be written in cylindrical polar coordinates as

(1)

Equation of Continuity in particle phase

$$\frac{\partial}{\partial z} \left(r \rho_{p} u_{p} \right) + \frac{\partial}{\partial r} \left(r \rho_{p} v_{p} \right) = 0$$

Equation of Motion in particle phase

$$\rho_{p}\left(u_{p}\frac{\partial u_{p}}{\partial z}+v_{p}\frac{\partial u_{p}}{\partial r}\right)=-\rho_{p}\left(\frac{u_{p}-u}{\tau_{m}}\right)$$
(2)

$$\rho_{p}\left(u_{p}\frac{\partial v_{p}}{\partial z}+v_{p}\frac{\partial v_{p}}{\partial r}\right)=-\rho_{p}\left(\frac{v_{p}-v}{\tau_{m}}\right)$$
Equation of heat in particle phase (3)

Equation of heat in particle phase

$$\rho_{p} C_{p} \left(u_{p} \frac{\partial T_{p}}{\partial z} + v_{p} \frac{\partial T_{p}}{\partial r} \right) = -\rho_{p} C_{s} \frac{\left(T_{p} - T \right)}{\tau_{T}}$$
(4)

To study the boundary layer flow, we introduce the dimensionless variables are

$$\overline{z} = \frac{z}{\lambda}, \ \overline{r} = \frac{r}{\left(\tau_{\rm m}\upsilon\right)^{\frac{1}{2}}}, \ \overline{u} = \frac{u}{U}, \ \overline{v} = v\left(\frac{\tau_{\rm m}}{\upsilon}\right)^{\frac{1}{2}}, \ \overline{u}_{\rm p} = \frac{u_{\rm p}}{U}, \ \overline{v}_{\rm p} = v_{\rm p}\left(\frac{\tau_{\rm m}}{\upsilon}\right)^{\frac{1}{2}}, \ \alpha = \frac{\rho_{\rm p_0}}{\rho} = \text{const}$$
$$\overline{\rho}_{\rm p} = \frac{\rho_{\rm p_0}}{\rho_{\rm p_0}}, \ \overline{T} = \frac{T}{T_0}, \ \overline{T}_{\rm p} = \frac{T_{\rm p}}{T_0}, \ \lambda = \tau_{\rm m}U, \ \tau_{\rm m} = \frac{2}{3}\frac{C_{\rm p}}{C_{\rm s}}\frac{1}{p_{\rm r}}\tau_{\rm r}, \ p_{\rm r} = \frac{\mu C_{\rm p}}{K} \cdot$$

Now considering the flow from the orifice under full expansion we can assume that the pressure in the mixing region to be approximately constant. Hence, the pressure at the exit is equal to that of the surrounding stream. Therefore, both the velocity and the temperature in the jet is only slightly different from that of the surrounding stream. The coefficient of viscosity u and thermal conductivity K are assumed to be constant. Then it is possible to write

 $u = u_0 + u_1$, $v = v_1$, $u_p = u_{p_0} + u_{p_1}$, $v_p = v_{p_1}$, $T = T_0 + T_1$, $T_p = T_{p_0} + T_{p_1}$, $\rho_p = \rho_{p_1}$ Where the Subscripts 1 denote the perturbed values which are much smaller than the basic values with subscripts '0' of the surrounding stream, i.e. $u_0 >> u_1$, $u_{p_0} >> u_{p_1}$, $T_0 >> T_1$, $T_{p_0} >> T_{p_1}$.

Using the dimensionless variable and the perturbation method the non linear equations (1) to (4) becomes

$$\rho_{p_{1}} = \rho_{p_{1}}(\mathbf{r})$$

$$u \quad \frac{\partial u_{p_{1}}}{\partial u_{p_{1}}} = (u_{0} - u_{0}) + (u_{1} - u_{0})$$
(6)

$$\mathbf{u}_{\mathbf{p}_{0}} \frac{\mathbf{v}_{1}}{\partial z} = \left(\mathbf{u}_{0} - \mathbf{u}_{\mathbf{p}_{0}}\right) + \left(\mathbf{u}_{1} - \mathbf{u}_{\mathbf{p}_{1}}\right) \tag{6}$$

$$\mathbf{u}_{\mathbf{p}_{0}} \frac{\partial \mathbf{v}_{\mathbf{p}_{1}}}{\partial z} = \left(\mathbf{v}_{1} - \mathbf{v}_{\mathbf{p}_{1}}\right)$$
(7)

$$u_{p_{0}} \frac{\partial T_{p_{1}}}{\partial z} = \frac{2}{3} \frac{1}{p_{r}} \left[\left(T_{0} - T_{p_{0}} \right) + \left(T_{1} - T_{p_{1}} \right) \right]$$
(8)

The boundary conditions for u_{p_1} and v_{p_2} are

$$u_{p_{1}}(0, r) = \begin{cases} u_{p_{10}}, r \leq 1 \\ 0, r > 1 \end{cases}$$
(9)
$$v_{p_{1}}(0, r) = 0$$
(10)

III. Method of Solution

The governing linearized equation (6) have been solved by using Laplace transform technique and using the relevant conditions from (9) and (10) we get

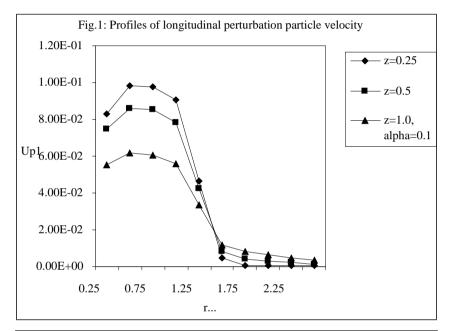
Laplace transform of
$$u_{p_0} \frac{\partial u_{p_1}}{\partial z} = (u_0 - u_{p_0}) + (u_1 - u_{p_1})$$
 is
i.e. $L\left\{u_{p_0} \frac{\partial u_{p_1}}{\partial z}\right\} = L\left\{(u_0 - u_{p_0}) + (u_1 - u_{p_1})\right\}$
 $u_{p_1}^* = \left\{\begin{array}{l}u_1^*(z, s) u_{p_0} + \frac{u_0 - u_{p_0}}{s} u_{p_0}^2 + \\ \left\{u_{p_{10}} - u_{10} u_{p_0}\right\} \frac{(1 - e^{-s})}{s} - \frac{u_0 - u_{p_0}}{s} u_{p_0}^2\right\} e^{-\frac{z}{u_{p_0}}}\right\}$ (11)

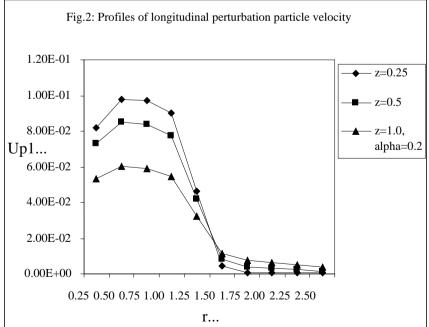
Laplace inverse Transformation of (11) gives

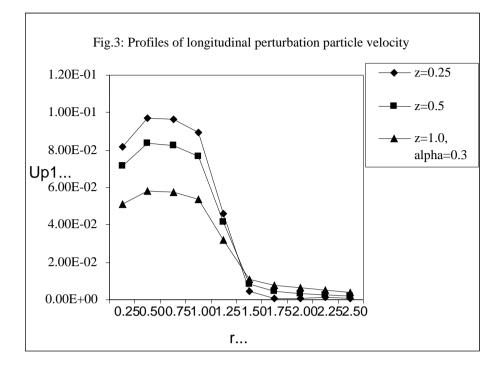
$$u_{p1} = L^{-1} \left[\left\{ u_{p10} \frac{(1 - e^{-s})}{s} - u_{10} \frac{(1 - e^{-s})}{s} u_{p0} - \frac{u_{0} - u_{p0}}{s} u_{p0}^{2} \right\} e^{-\frac{z}{u_{p0}}} u_{1}^{*}(z, s) \cdot u_{p0} + \frac{u_{0} - u_{p0}}{s} u_{p0}^{2} \right]$$
(12)

IV. Discussion of Result and Conclusion

The above result (12) only gives the solutions of the longitudinal perturbed particle velocity. However the magnitude of longitudinal velocity can be obtained by using different boundary conditions. Numerical computation have been made by taking $P_r = 0.72$, $u_{10} = up_{10} = T_{10} = Tp_{10} = \rho_{p10} = 0.1$, $\varphi = 0.01$. The velocity and temperature at the exit are taken nearly equal to unity. Figures 1, 2 and 3 show the profiles of longitudinal perturbation particle velocity u_{p1} for $\alpha = 0.1$, 0.2 and 0.3 and for Z=0.25, 0.5 and 1.0. It is observed that the magnitude of u_{p1} decreases with the increase of dust parameter. Hence we conclude that consideration of finite volume fraction shows that the magnitude of particle velocity reduces significantly.







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