

Review: Linear Techniques for Analysis of Heart Rate Variability

Mazhar B. Tayel¹, Eslam I AlSaba²

Electrical Engineering Department, Faculty of Engineering, Alexandria University, Alexandria, Egypt

ABSTRACT : Heart rate variability (HRV) is a measure of the balance between sympathetic mediators of heart rate that is the effect of epinephrine and norepinephrine released from sympathetic nerve fibres acting on the sino-atrial and atrio-ventricular nodes which increase the rate of cardiac contraction and facilitate conduction at the atrio-ventricular node and parasympathetic mediators of heart rate that is the influence of acetylcholine released by the parasympathetic nerve fibres acting on the sino-atrial and atrio-ventricular nodes leading to a decrease in the heart rate and a slowing of conduction at the atrio-ventricular node. Sympathetic mediators appear to exert their influence over longer time periods and are reflected in the low frequency power (LFP) of the HRV spectrum (between 0.04Hz and 0.15 Hz). Vagal mediators exert their influence more quickly on the heart and principally affect the high frequency power (HFP) of the HRV spectrum (between 0.15Hz and 0.4 Hz). Thus at any point in time the LFP:HFP ratio is a proxy for the sympatho- vagal balance. Thus HRV is a valuable tool to investigate the sympathetic and parasympathetic function of the autonomic nervous system. Study of HRV enhance our understanding of physiological phenomenon, the actions of medications and disease mechanisms but large scale prospective studies are needed to determine the sensitivity, specificity and predictive values of heart rate variability regarding death or morbidity in cardiac and non-cardiac patients. This paper presents the linear techniques to analysis the HRV.

KEYWORDS -Heart Rate Variability, Physiology of Heart Rate Variability, Linear techniques, Time domain analysis, Frequency domain analysis, Time-frequency analysis.

I. INTRODUCTION

Heart rate variability (HRV) is the temporal variation between sequences of consecutive heart beats. On a standard electrocardiogram (ECG), the maximum upwards deflection of a normal QRS complex is at the peak of the R-wave, and the duration between two adjacent R-wave peaks is termed as the R-R interval. The ECG signal requires editing before HRV analysis can be performed, a process requiring the removal of all non sinus-node originating beats. The resulting period between adjacent QRS complexes resulting from sinus node depolarizations is termed the N-N (normal-normal) interval. HRV is the measurement of the variability of the N-N intervals [1].

Linear HRV parameters were obtained in agreement with the standards of measurement, proposed by [7]. Mean and standard deviation (SD) of the tachogram, the standard deviation of the 5 minute average of RR intervals (SDANN), the square root of the mean of the sum of the squares of differences between consecutive RR intervals (rMSSD) and the percentage of intervals that vary more than 50 ms from the previous interval (pNN50) were calculated in the time domain.

After resampling of the tachogram at 2 Hz, power spectral density was computed by using fast Fourier transformation. In the frequency domain, low frequency power (0.04 – 0.15 Hz), high frequency power (0.16 – 0.40 Hz) and total power (0.01 – 1.00 Hz), as well as the ratio of low frequency over high frequency, were calculated. In addition, the power can be expressed in absolute values or in normalized units (NU).

The present study introduce briefly the physiology of heart rate variability (HRV) and linear techniques that be used to analysis HRV.

II. PHYSIOLOGY OF HEART RATE VARIABILITY

Heart rate variability, that is, the amount of heart rate fluctuations around the mean heart rate [2] is produced because of the continuous changes in the sympathetic parasympathetic balance that in turn causes the sinus rhythm to exhibit fluctuations around the mean heart rate. Frequent small adjustments in heart rate are made by cardiovascular control mechanisms. This results in periodic fluctuations in heart rate. The main periodic fluctuations found are respiratory sinus arrhythmia and baroreflex related and thermoregulation related heart rate variability [3]. Due to inspiratory inhibition of the vagal tone, the heart rate shows fluctuations with a frequency equal to the respiratory rate [4]. The inspiratory inhibition is evoked primarily by central irradiation of impulses from the medullary respiratory to the cardiovascular center. In addition peripheral reflexes due to hemodynamic changes and thoracic stretch receptors contribute to respiratory sinus arrhythmia. This is parasympathetically mediated [5]. Therefore HRV is a measure of the balance between sympathetic mediators

of the heart rate (HR) i.e. the effect of epinephrine and norepinephrine released from sympathetic nerve fibres, acting on the sino-atrial and atrioventricular nodes, which increase the rate of cardiac contraction and facilitate conduction at the atrioventricular node and parasympathetic mediators of HR i.e. the influence of acetylcholine released by the parasympathetic nerve fibres, acting on the sino-atrial and atrioventricular nodes, leading to a decrease in the HR and a slowing of conduction at the atrioventricular node. Sympathetic mediators appears to exert their influence over longer time periods and are reflected in the low frequency power (LFP) of the HRV spectrum [6]. Vagal mediators exert their influence more quickly on the heart and principally affect the high frequency power (HFP) of the HRV spectrum. Thus at any point in time, the LFP:HFP ratio is a proxy for the sympatho-vagal balance.

III. LINEAR TECHNIQUES

It is customary to study HRV in the time-domain or in the frequency domain. For instance, the variance of an HRV-signal within a time-interval is a time-domain parameter expressing the power of the HRV in that interval. The variance is computed by integrating the squared amplitude of the HRV-signal over that interval. The squared amplitude of a signal is called the instantaneous power. The power spectral density of a signal computed over a time-interval is defined by the squared amplitude of the finite Fourier transform of the signal averaged over that time-interval. This reflects the power of each frequency component averaged over that time-interval. The power of an HRV-signal within a frequency band is a frequency-domain parameter and is obtained by integrating the power spectral density of the signal over that frequency band. However, the instantaneous power and the power spectral density, alone and in combination, are not sufficient to fully describe the properties of an HRV-signal. The instantaneous power describes which time components are present, but not the frequency range of a time component. In other words, the power spectral density does not reveal the frequency components in time and the instantaneous power does not show the time components in frequency. Statistics measured in a 5 min interval of HRV probably differ from those measured in the next 5 min and from those measured in the entire 10 min interval. When the differences are statistically significant, the signal is called non-stationary. If this is the case, there may be significant changes of the frequency components of the signal in time. Their presence in time cannot be derived from the power spectral density computed over the 10 min interval. And their powers cannot be interpreted unambiguously, because frequency components of different duration and amplitude may produce similar peaks in the power spectral density function. For instance, if a frequency component is present during 20 s within the 10 min interval with an amplitude of, say, 10 units, the power spectral density is similar to a situation where the frequency component is present during 5 s with an amplitude of 20 units. Thus the need arises for a description that represents the power of the signal simultaneously in the time- and frequency-domains. Such time-frequency representations are often called 'distributions' for historical reasons. The phase spectrum of the raw signal contains the information that is necessary to localize the frequency components in time. Especially, the instantaneous frequency is a means to localize the frequency components in time. Therefore, the phase information of the raw signal is included in the computation of a time-frequency representation. Time-frequency signal analysis does not assume stationarity, whereas power spectral analysis does. If there are transients in HRV, which is often the case, a time-frequency representation can be employed to describe them. This means a greatly improved interpretation of HRV-signals of experimental and clinical populations. Especially if a time-frequency method can be used that provides a qualitative and quantitative description of the dynamic changes in frequency and amplitude with a high resolution in time.

One example will be used throughout the following sections to explain more visually, if possible, what the technique does and how it can be calculated on the tachogram. The chosen example is given in **(Fig.1)**, being an RR interval time series extracted from an ECG signal monitored during a stress test. The tachogram has a length of 2712 seconds (45 minutes) containing 3984 heart beats. As indicated in the figure, some irregular or faulty RR intervals were corrected this way, changing the shortest RR interval from 256 ms to 443ms. In other words, the impossible instantaneous heart rate of 234 bpm in such condition was corrected by the preprocessing algorithm to a maximal instantaneous heart rate of 135 bpm which was probably correct. The linear time and frequency domain techniques for HRV were standardized in a report of the Task Force of the European Society of Cardiology and the North American Society of Pacing and Electrophysiology [7].

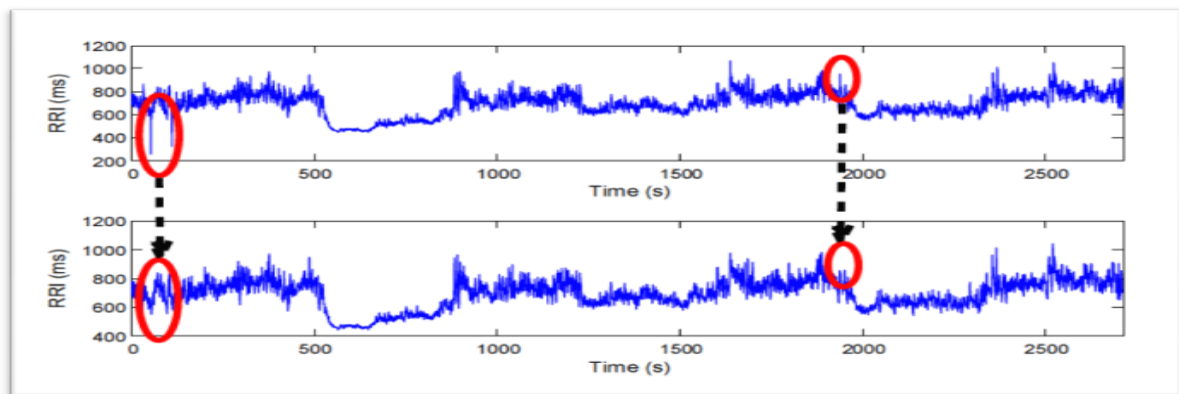


Figure 1 Thetachogram used as example [7].

IV. TIME DOMAIN ANALYSIS

Simple time domain variables that can be calculated include the mean NN interval, the mean heart rate and the difference between the longest and shortest NN interval.

The methods may be divided into two classes, (a) those derived from direct measurements of the NN intervals, and (b) those derived from the differences between NN intervals.

The simplest variable to calculate is the standard deviation of the NN interval (SDNN), the square root of variance. Since variance is mathematically equal to total power of spectral analysis, SDNN reflects all the cyclic components responsible for variability in the period of recording. In many studies, SDNN is calculated over a 24h period and thus encompasses both short-term high frequency variations, as well as the lowest frequency components seen in a 24h period. As the period of monitoring decreases, SDNN estimates shorter and shorter cycle lengths. It should also be noted that the total variance of HRV increases with the length of analyzed recording [8]. Thus, on arbitrarily selected ECGs, SDNN is not a well-defined statistical quantity because of its dependence on the length of recording period. Consequently, in practice, it is inappropriate to compare SDNN measures obtained from recordings of different durations. However, durations of the recordings used to determine SDNN values (and similarly other HRV measures) should be standardized. 5-minute recordings for short-term and nominal 24h for long-term recordings seem to be appropriate options.

Other commonly used statistical variables calculated from segments of the total monitoring period include SDANN, the standard deviation of the average NN interval calculated over short periods, usually 5 minutes, which is an estimate of the changes in heart rate due to cycles longer than 5 minutes, and the SDNN index, the mean of the 5-minute standard deviation of the NN interval calculated over 24h, which measures the variability due to cycles shorter than 5 minutes.

The most commonly used measures derived from interval differences include RMSSD, the square root of the mean squared differences of successive NN intervals, pNN50, the number of interval differences of successive NN intervals greater than 50ms divided by the total number of NN intervals and multiplied by 100 (expressed in %) and SDD, the standard deviation of differences between adjacent NN intervals. All these measurements of short-term variation estimate high frequency variations in heart rate and thus are highly correlated.

For the given example, the statistical time domain parameters are: mean RR = 680.82 ms, SDNN = 106.93 ms, SDANN = 69.92 ms, SDNN index = 76.36 ms, SDD = 29.91 ms; RMSSD = 41.74 ms and pNN50 = 17.15 [8].

V. FREQUENCY DOMAIN ANALYSIS

Various spectral methods for the analysis of the tachogram have been applied. Power spectral density (PSD) analysis provides the basic information of how power, and therefore the variance, distributes as a function of frequency. Independent of the method employed, only an estimate of the true PSD of the signals can be obtained by proper mathematical algorithms.

VI. PARAMETRIC VERSUS NONPARAMETRIC

Methods for the calculation of PSD may be generally classified as nonparametric and parametric. In most instances, both methods provide comparable results. The advantages of the nonparametric methods are: (a) the simplicity of the algorithm employed (Fast Fourier Transform – FFT – in most of the cases) and (b) the high processing speed, whilst the advantages of parametric methods are: (a) smoother spectral components which can be distinguished independently of preselected frequency bands, (b) easy post-processing of the spectrum with

an automatic calculation of low and high frequency power components and easy identification of the central frequency of each component, and (c) an accurate estimation of PSD even on a small number of samples on which the signal is supposed to maintain stationarity. The basic disadvantage of parametric methods is the need to verify the suitability of the chosen model and its complexity (the order of the model).

In most cases, first a resampling of the RR interval time series is necessary to have equidistant time points. However, via the Lomb periodogram the spectrum can also be calculated directly from the unevenly spaced time series [9].

VII. SHORT TERM RECORDINGS

One has to distinguish between short term and long term recordings. During short term recordings, three main spectral bands are distinguished in a spectrum: very low frequency (VLF) below 0.04 Hz, low frequency (LF) from 0.04 to 0.15 Hz, and high frequency (HF) fluctuations from 0.15 to 0.4 Hz (**Fig.2**). The distribution of the power and the central frequency of LF and HF are not fixed but may vary in relation to changes in autonomic modulations of the heart period. In each frequency band, the power is calculated as the area under the PSD curve between the corresponding lower and upper bound. The total power (TP) is defined as the power in the frequency band going from 0 Hz to 1 Hz. Calculation of VLF, LF and HF powers are usually made in absolute values of power (ms^2), but LF and HF may also be measured in normalized units (n.u.):

$$LF(n.u.) = \frac{LF}{TP-VLF}, \quad HF(n.u.) = \frac{HF}{TP-VLF} \quad (1)$$

The representation of LF and HF in n.u. emphasizes the controlled and balanced behavior of the two branches of the autonomic nervous system. Moreover, normalization tends to minimize the effect on the values of LF and HF components of the changes in total power. Another measure is LF/HF, calculated as the ratio of the power in LF and HF band [10].

Vagal activity is the major contributor to the HF component. Disagreement exists concerning the LF component. While some studies suggest that LF, when expressed in normalized units, is a quantitative marker for sympathetic modulations, most studies view LF as reflecting both sympathetic and vagal activity. Consequently, the LF/HF ratio is considered to mirror mainly sympathovagal balance. The physiological explanation of the VLF component is much less defined and the existence of a specific physiological process attributable to these heart period changes might even be questioned. The non-harmonic component which does not have coherent properties and which is affected by algorithms of baseline or trend removal is commonly accepted as a major constituent of VLF. Thus VLF assessed from ≈ 5 minute recordings is a dubious measure and should be avoided when interpreting the PSD of short-term ECGs [10].

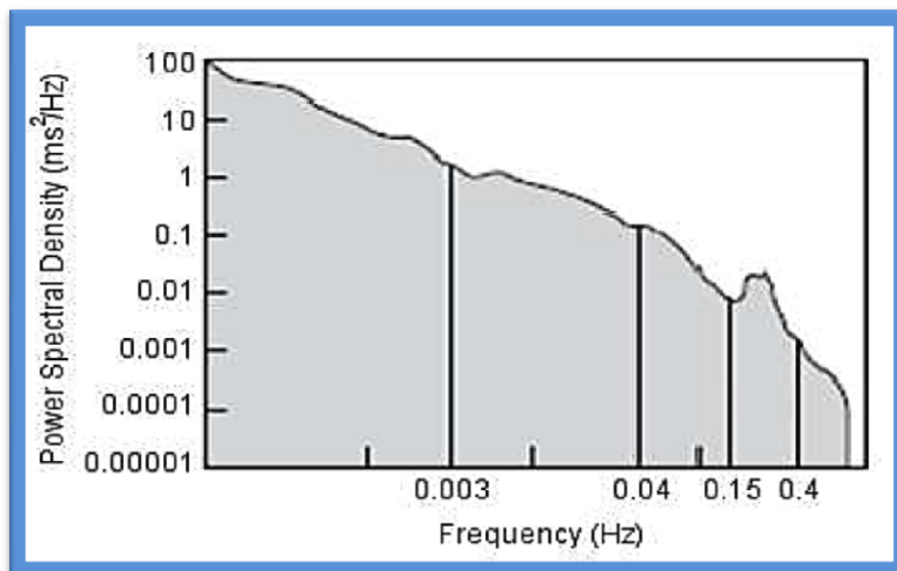


Figure 2 Example of an estimate of power spectral density obtained from the entire 24h interval of a long-term Holter recording. The different frequency bands are clearly indicated: ultra-low frequency component (ULF), very low frequency component (VLF), low frequency component (LF) and high frequency component (HF) [7].

VIII. LONG TERM RECORDINGS

For long-term recordings, spectral analysis may also be used to analyze thesequence of NN intervals in the entire 24h period. The result then includes an ultralowfrequency component (ULF) below 0.003 Hz, in addition to VLF (0.003 to0.04 Hz), LF and HF components (**Fig.2**). Now, the problem of 'stationarity'comes in. A deterministic signal is said to be stationary if it can be written as adiscrete sum of sinusoids :

$$x(t) = \sum_{k \in N} A_k \cos(2\pi f_k t + \varphi_k) \quad \text{for a real signal} \quad (2)$$

$$x(t) = \sum_{k \in N} A_k e^{j(2\pi f_k t + \varphi_k)} \quad \text{for a complex signal} \quad (3)$$

which A_k is the amplitude, f_k is the frequency and φ_k is the phase of the k^{th} sinusoid. If mechanisms responsible for heart period modulations of a certain frequency remain unchanged during the whole period of recording, the corresponding frequency component of HRV may be used as a measure of these modulations. If the modulations are not stable, interpretation of the results of frequency analysis is less well defined. In particular, physiological mechanisms of heart period modulations responsible for LF and HF power components cannot be considered stationary during the 24h period [11]. Therefore, spectral analysis performed in the entire 24h period as well as spectral results obtained from shorter segments averaged over the entire 24h period (the LF and HF results of these two computations are equal) provide averages of the modulations attributable to the LF and HF components (**Fig.2**). Such averages obscure detailed information about autonomic modulation of RR intervals available in shorter recordings [11]. It should be noticed that the components of HRV provide measurements of the degree of autonomic modulations rather than of the level of autonomic tone [12] and averages of modulations do not represent an averaged level of tone.

IX. CORRELATION AND DIFFERENCES BETWEEN TIME AND FREQUENCY DOMAIN MEASURES

When analyzing stationary short-term recordings, more experience and theoretical knowledge exists on the physiological interpretation of the frequency domain measures compared to the time domain measures derived from the same recordings. However, many time and frequency domain variables measured over a long term period are strongly correlated with each other as summarized in (**Table 1**). These strong correlations exist because of both mathematical and physiological relationships. In addition, the physiological interpretation of the spectral components calculated over 24 h is difficult, for the reasons mentioned before. Therefore, unless special investigations are performed which use the 24h HRV signal to extract information other than the usual frequency components, the results of frequency domain analysis are equivalent to those of time domain analysis, which is easier to perform.

Table 1 Approximate correspondence of time domain and frequency domain methods applied to long term ECG recordings.

Time domain variable	Frequency domain correlate
SDNN	Total power
HRV triangular index	Total power
TINN	Total power
SDANN	ULF
SDNN index	Mean of 5 minute total power
RMSSD	HF
SDSD	HF
pNN50	HF

X. ANALYSIS BY GEOMETRICAL METHOD

Geometrical methods present RR intervals in geometric patterns and various approaches have been used to derive measures of HRV from them. The triangular index is a measure, where the length of RR intervals serves as the x-axis of the plot and the number of each RR interval length serves as the y-axis. The length of the base of the triangle is used and approximated by the main peak of the RR interval frequency distribution diagram. The triangular interpolation of NN interval histogram (TINN) is the baseline width of the distribution measured as a base of a triangle, approximating the NN interval distribution (the minimum of HRV). Triangular interpolation approximates the RR interval distribution by a linear function and the baseline width of this approximation triangle is used as a measure of the HRV index [13, 14]. This triangular index had a high correlation with the standard deviation of all RR intervals. But it is highly insensitive to artifacts and ectopic beats, because they are left outside the triangle. This reduces the need for preprocessing of the recorded data [14]. The major advantage of geometric methods lies in their relative insensitivity to the analytical quality of the series of NN intervals. The typical values of different statistical and geometric parameters of HRV signal (**Fig. 3**) is shown in **Table 2**.

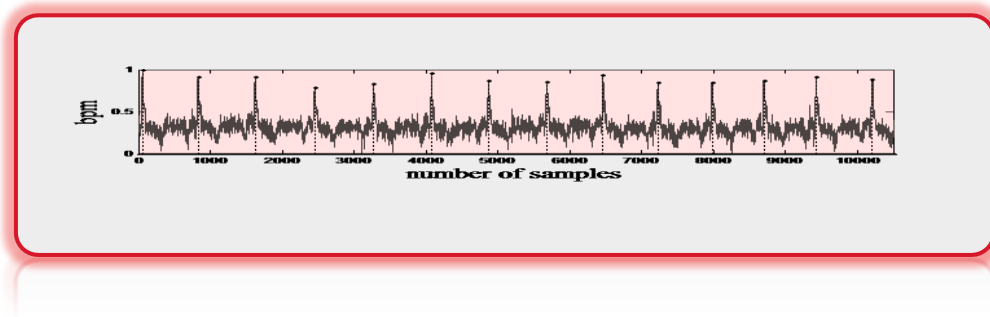


Figure 3 Heart rate variation of a normal subject [15].

Table 2 Result of statistical and geometric parameters of heartrate

Time domain statistics		
Variable	Units	Value
Statistical measures		
SDNN	ms	30.00
SENN	ms	4.120
SDSD	ms	36.60
RMSSD	ms	33.30
NN50	Count	0
Geometric measures		
RR triangular index		0.011
TINN	ms	20.00

XI. TIME-FREQUENCY ANALYSIS

Because of the problem of stationarity as discussed in the previous paragraph, frequency domain HRV parameters are not reliable in case of quick changes in heart rate or its autonomic modulation. The spectrum essentially tells which frequencies are contained in the signal, as well as their corresponding amplitudes and phases, but does not tell at which times these frequencies occur. Luckily, there exist techniques which combine time and frequency information simultaneously, the so called time-frequency representations (TFR). An overview of TFRs is given by Auger et al [16]. The basic short-time Fourier transform (STFT) and the more advanced continuous wavelet transform (CWT) is discussed. A schematic illustration of the differences between time series analysis, Fourier transform, STFT and CWT is shown in (Fig.4) and will be explained further in the following subsections.

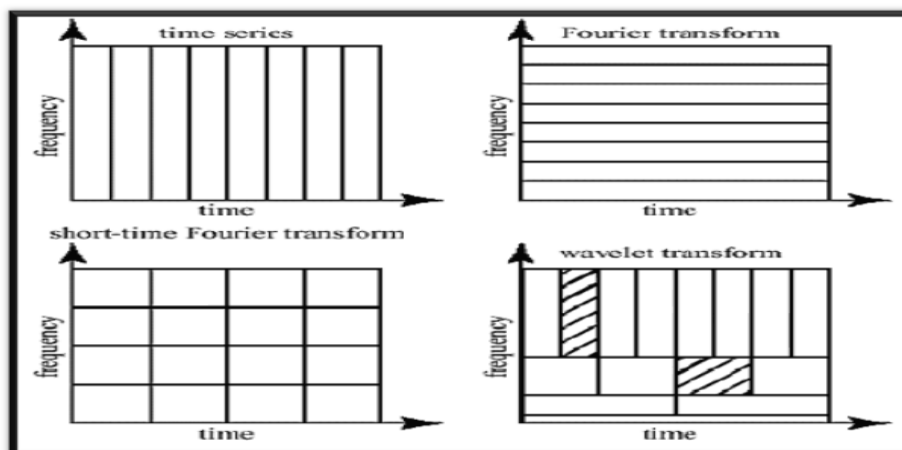


Fig. 4 A schematic illustration of the differences between time series analysis, Fourier transform, short-time Fourier transform and continuous wavelet transform regarding the time and frequency resolution

XII. SHORT-TIME FOURIER TRANSFORM

In order to introduce time-dependency in the Fourier transform, a simple and intuitive solution consists in pre-windowing the signal $x(u)$ around a particular time t , calculating its Fourier transform, and doing that for each time instant t . The resulting transform, called the short-time Fourier transform (STFT), is:

$$F_x(t, f; h) = \int_{-\infty}^{+\infty} x(u) \cdot h^*(u - t) \cdot e^{-2j\pi fu} \cdot du \quad (4)$$

where $h(t)$ is a short-time analysis window localized around $t = 0$ and $f = 0$. The STFT is also invertible, but this type of time-frequency representation has a tradeoff between time and frequency resolutions. On one hand, a good time resolution requires a short window $h(t)$ while on the other hand, a good frequency resolution requires a narrowband filter and therefore a long window $h(t)$. This limitation is a consequence of the Heisenberg-Gabor inequality.

If it be considered the squared modulus of the STFT, it be obtained a spectral energy density of the locally windowed signal $x(u) \cdot h^*(u - t)$:

$$S_x(t, f) = \left| \int_{-\infty}^{+\infty} x(u) \cdot h^*(u - t) \cdot e^{-2j\pi fu} \cdot du \right|^2 \quad (5)$$

This defines the spectrogram, which is a real-valued and non-negative distribution. Since the window h of the STFT is assumed of unit energy, the spectrogram satisfies the global energy distribution property :

$$E_x = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} S_x(t, f) \cdot dt \cdot df \quad (6)$$

Therefore, the spectrogram can be interpreted as a measure of the energy of the signal contained in the time-frequency domain centered at the point (t, f) .

XIII. CONTINUOUS WAVELET TRANSFORM

The idea of the continuous wavelet transform (CWT) is to project a signal x on a family of zero-mean functions (the wavelets) deduced from an elementary function (the mother wavelet) by translations and dilations:

$$T_x(t, a; \psi) = \int_{-\infty}^{+\infty} x(s) \psi_{t,a}^*(s) \cdot ds \quad (7)$$

where

$$\psi_{t,a}(s) = |a|^{-\frac{1}{2}} \cdot \psi\left(\frac{s-t}{a}\right) \quad (8)$$

Several types of wavelet functions exist such as the Morlet wavelet, Haar wavelet, Shannon wavelet, Daubechies wavelet, Gaussian wavelet, Meyer wavelet and Mexican hat wavelet. Some of them are illustrated in **(Fig.5)**.

The variable a corresponds now to a scale factor, in the sense that taking $|a| > 1$ dilates the wavelet ψ and taking $|a| < 1$ compresses ψ . By definition, the wavelet transform is more a time-scale than a time-frequency representation. However, for wavelets which are well localized around a non-zero frequency f_0 at scale $a = 1$, a time-frequency interpretation is possible thanks to the formal identification $f = \frac{f_0}{a}$. The basic difference between the wavelet transform and the short-time Fourier transform is as follows: when the scale factor a is changed, the duration and the bandwidth of the wavelet are both changed but its shape remains the same. And in contrast to the STFT, which uses a single analysis window, the CWT uses short windows at high frequencies and long windows at low frequencies. This partially overcomes the resolution limitation of the STFT as the bandwidth B is proportional to f . The CWT can also be seen as a filter bank analysis composed of band-pass filters with constant relative bandwidth. As the STFT was reversible, the signal x can also be reconstructed from its continuous wavelet transform according to the formula:

$$x(t) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} T_x(s, a, \varphi) \cdot \psi_{s,a}(t) \cdot ds \cdot \frac{da}{a^2} \quad (9)$$

A similar distribution to the spectrogram can be defined in the wavelet case. Since the continuous wavelet transform behaves like an orthonormal basis decomposition, it can be shown that it preserves energy:

$$E_x = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} |T_x(t, a, \psi)|^2 \cdot dt \cdot \frac{da}{a^2} \quad (10)$$

where E_x is the energy of x . This leads to define the scalogram of x as the squared modulus of the continuous wavelet transform. It is an energy distribution of the signal in the time-scale plane, associated with the measured $dt \cdot \frac{da}{a^2}$. As for the wavelet transform, time and frequency resolutions of the scalogram are related via the Heisenberg-Gabor principle: time and frequency resolutions depend on the considered frequency.

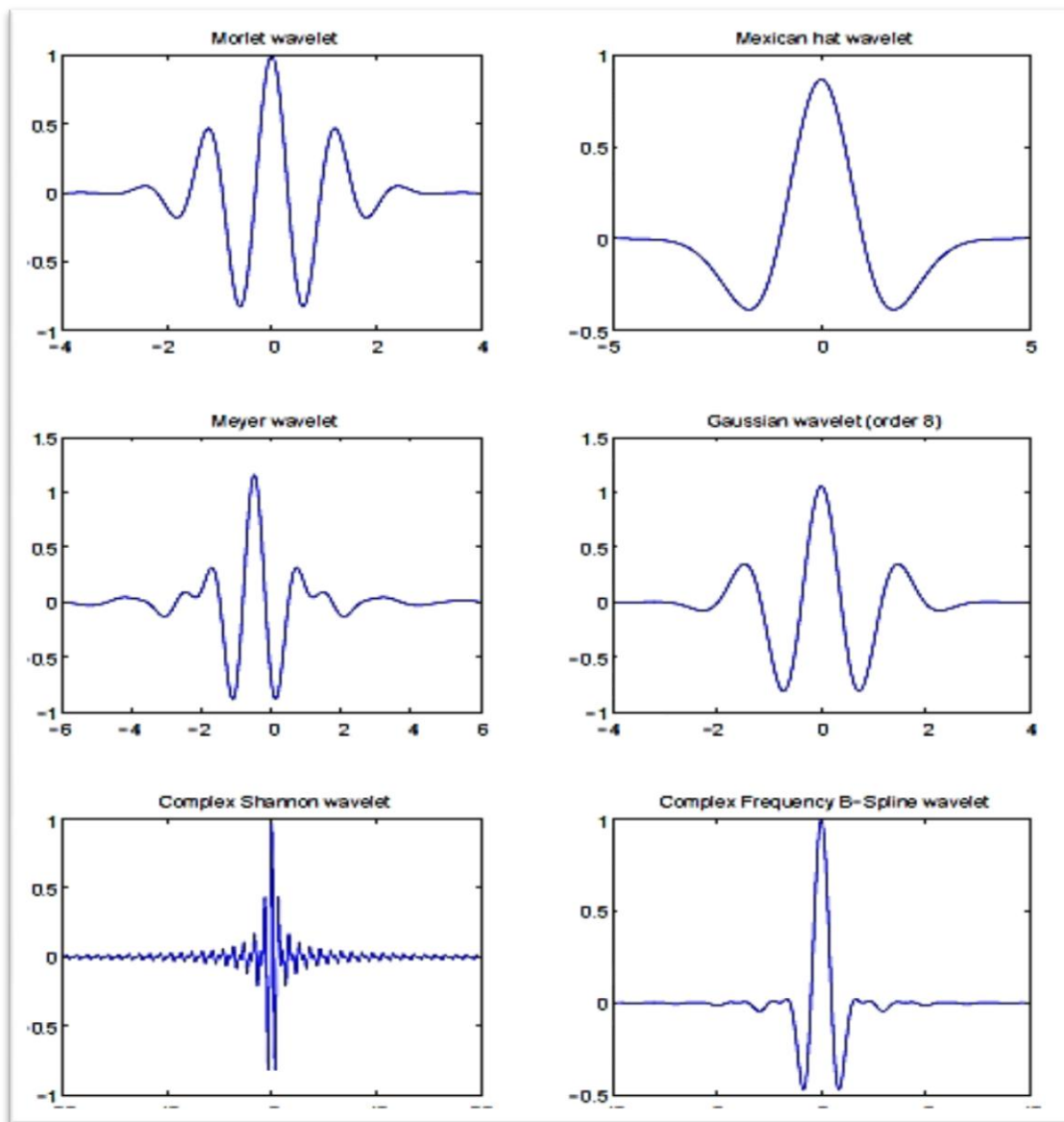


Fig. 5. Illustration of several types of mother wavelet functions: Morlet wavelet (top left), Mexican hat wavelet (top right), Meyer wavelet (middle left), Gaussian wavelet (middle right), complex Shannon wavelet (bottom left), complex frequency B-spline wavelet (bottom right).

XIV. INSTANTANEOUS FREQUENCY AND POWER

The instantaneous frequency of a signal calculated as the derivative of the phase of its analytical signal often produces results that, in some ways, may seem paradoxical [17], and which, in any case, make their physical interpretation difficult. This drawback can be avoided by defining instantaneous frequency as the mean frequency of the spectrum at each instant, where the spectrum is obtained as a section of the time frequency distribution at this instant

$$f_s(t) = \frac{\sum_{n=1}^N f_n \cdot TFR_{xx}(t, f_n)}{\sum_{n=1}^N TFR_{xx}(t, f_n)} \quad (11)$$

where N is the number of samples on the frequency axis. This way, instantaneous frequencies can be calculated for each RR signal and separately in each of the predefined frequency bands VLF, LF and HF. Analogously, in each frequency band the energy (power) can be calculated by integrating the spectrogram, expressed in absolute values or normalized units.

XV. CONCLUSION

Heart Rate Variability (HRV) play an important role in monitoring, predicting, and diagnosing cardiological and noncardiological diseases. Thus it needs to be analyzed to gest its merits. The present work present a review to support the study in area of liner techniques for analysis HRV. The linear techniques can be divided into three sections. These sections are the techniques used in time domain, frequency domain, and time-frequency domain. Each of these area has characteristics merits and demerits. Therefore, Time, Frequency, and Time-frequency have become a common tool in signal analysis. During the past decades their methods were developed and applied to various bio-medical signals. Each method has its own features and limitations and the literature of them analysis is extensive. The present work reviews the short-time Fourier transform. This is one of the oldest time-frequency representations and used in the study of heart rate variability (HRV). Also, the wavelet transform is a recent time-frequency representation and still in development.

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