International Journal of Engineering Science Invention
ISSN (Online): 2319 – 6734, ISSN (Print): 2319 – 6726
www.ijesi.org ||Volume 5 Issue 8|| August 2016 || PP.01-07

Optimization Programming Problems For Control And Management Of Bacterial Disease With Two Stage Growth/Spread Among Plants

E.S.V. Narayana Rao and P. Tirupathi Rao
Dept. of Statistics, Pondicherry University, Puducherry-605014.

Abstract: *This study has proposed four non linear programming problems for optimal control of bacterial spread over the plants at different ages on different species. The objective functions and constraints of the said NLPPs are formulated through the developed mathematical models of bacterial disease growth and spread using bivariate stochastic processes [2]. This study will help the agricultural care takers for developing the relevant decision support systems to get the indicators on spread of bacterial diseases and its control.*

Keywords: *Non Linear Programming Problems, Bacterial Disease Control, Optimization, Bivariate Stochastic Processes, Decision Support Systems.*

I. Introduction

Studies on bacterial diseases in plants have significant importance for farmers for making its control and regulating with optimal management approaches. Bacterial spread on growing plants may lead to huge damage of crops. Assessing the intensity of disease through modeling is the core area of research which has attracted the attention of many researchers. Proper management of disease control is possible only when its spread is properly assessed. Designing of treatment protocols are directly linked with suitable study methods of disease intensity. Due to many explained and unexplained reasons the influencing factors of bacteria spread among plants are stochastic. Thus probabilistic tools need to be used to study the dynamics of bacterial transmission.

In order to develop the controlling devices on the disease spread, optimization programming models were formulated with the objectives of minimizing the average sizes of bacteria in different stages of plants and maximizing the volatility in bacterial size by using the variance measures. The rates of arrival or onset of fresh bacteria to different staged(aged) plants, rates of growth of bacteria within each state, rates of death of bacteria in each stage, migration rates of bacteria from different stages, transition rates of bacteria from one stage to other stages, etc are the study parameters of these models. Exploring of these parameters will help the crop caretakers for measuring the dynamics of bacterial intensities so as suitable intervention or treatment protocols can be designed.

Using stochastic simulation model, Xu and Ridout (2000) demonstrated the importance of initial epidemic conditions, especially the spatial pattern of initially infected plants and the relationships of spatio-temporal statistics with underlying biological and physical factors. The stochastic optimisation and programming models for drug administration in cancer chemotherapy was developed by Tirupathi Rao et al., (2010, 2012, 2013, 2014). Madhavi et al. (2013) have proposed a stochastic programming problem for optimal drug administration for cancer chemotherapy. Their studies have focused on exploring the decision parameters like drug dosage levels, drug administrative period, drug vacation period, number of drug cycles and frequency of drug administration within a cycle etc., during chemotherapy. With the motivation of the above said works, the researchers have proposed this study in the context of growth and spread of plant diseases.

II. Optimization Modeling

The current study proposed some non-linear programming problems for optimal control of bacterial spread over the plants at different ages of different species. The works on development of stochastic models for growth and spread of bacteria [2] are considered here for formulating the objective functions as well as constraints. The objectives of the said problems are formulated in two dimensions such as minimizing the average size of bacteria and maximizing the variance of bacterial size hosted on the plants. The purpose of this study is to derive the decision parameters such as regulating rates of bacterial growth, transitions from one stage to other and bacterial loss. The subjective constraints are formulated with a view of avoiding the risky levels of bacterial growth. The study has considered the live data obtained from the laboratories of plant pathology situated in Andhra Pradesh.

2.1. Notations and Terminology:

n : number of units of bacteria in stage-I (nursery stage plants) at time 't';

m : number of units of bacteria in stage-II (transplantation stage plants) at time 't'; Here one unit denotes the number of bacteria in a square area (mm^2);

$\alpha_1 \; and \; \alpha_2$: the rates of growth of bacteria due to immigration from external means per unit time to stage-I and stage-II plants respectively;

$\beta_1 \; and \; \beta_2$: the rates of internal growth (birth) of bacteria per unit time in stage-I and stage-II plants respectively;

τ_1 : the rate of transition of bacteria per unit time from stage-I to stage-II plants;

$\varepsilon_1 \; and \; \varepsilon_2$: the rates of emigration of bacteria per unit time from stage-I and stage-II plants to the other areas respectively;

$\delta_1 \; and \; \delta_2$: the rates of loss (death) of bacteria per unit time in stage-I and stage-II plants respectively.

$$A = \left[\beta_1 - (\tau_1 + \varepsilon_1 + \delta_1)\right];$$

$$B = \left[\beta_2 - (\varepsilon_2 + \delta_2)\right];$$

$$J = \left[2\alpha_2 + \varepsilon_2 + \delta_2\right]$$

N_0, M_0 : Initial sizes of bacteria existing at nursery and plantation stages of plants

C_0, D_0, E_0: Integral Constants while solving the differential equations

C_1 : Upper limit of average number of *bacterial units* in stage-I plants at a point of time 't';

C_2 : Upper limit of average number of *bacterial units* in stage-II plants at a point of time 't';

C_3 : Upper limit of variance of *bacterial units* in stage-I plants at a point of time 't';

C_4 : Upper limit of variance of *bacterial units* in stage-II plants at a point of time 't';

III. Stochastic Programming Problems

In this section, four programming problems are formulated. They are

1. Optimal Programming for Minimizing the Expected Bacteria size in Stage-I plants
2. Optimal Programming for Minimizing the Expected Bacteria size in Stage-II plants
3. Optimal Programming for Maximizing the Variance size of Bacteria in Stage-I plants
4. Optimal Programming for Maximizing the Variance size of Bacteria in Stage-II plants

3.1. Optimal Programming for Minimizing the Expected Bacteria size in Stage-I plants

This programming problem is formulated with the objective of minimizing the average number of *bacterial units* in stage-I plants with the constraints of average number of *bacterial units* and also with the variance of *bacterial units* in different stages of plants.

In order to achieve the above objective, the growth of bacterial units size shall not beyond the warning limits. Hence, the average number of *bacterial units* in stage-I plants at a point of time 't' should not be more than some threshold limits say C_1 and C_2. Similarly with the variances of *bacterial units* in stage-I and stage-II plants at a point of time 't' should not be more than some threshold limits say C_3 and C_4.

Then the programming problem is

$$Min Z_1 = N_o.e^{At} \qquad 3.1.1$$

Subject to the constraints,

$$N_o.e^{At} \le C_1 \qquad 3.1.2$$

$$\frac{\tau_1 N_o.e^{At}}{(A-B)} + M_o.e^{Bt} \le C_2 \qquad 3.1.3$$

$$\frac{\left[-\left(2\alpha_1 + \beta_1 + \tau_1\right)\right] N_o.e^{At}}{A} + C_o.e^{2At} \le C_3 \qquad 3.1.4$$

$$\frac{\tau_1 N_o.e^{At}}{(A-2B)}+\frac{\left(-J\tau_1 N_o.e^{Bt}\right)}{(A-B)(B)}+\frac{\left(-JM_o.e^{Bt}\right)}{(B)}+\frac{\left\{-2\tau_1(\alpha_2-\tau_1)N_o.e^{At}\right\}}{(A-2B)(B)}+\frac{\left(-2\alpha_1\tau_1^2 N_o.e^{At}\right)}{(A-B)(B)(A-2B)}$$
$$+\frac{\left(2\alpha_1\tau_1 M_o.e^{Bt}\right)}{(A)(B)}+\frac{\left\{2\tau_1^2(2\alpha_1+\beta_1+\tau_1)N_o.e^{At}\right\}}{(A-2B)(A)(B)}+\frac{\tau_1^2 C_o.e^{2At}}{(A-B)^2}+\frac{D_o.e^{(A+B)t}}{(A-B)}+E_o.e^{2Bt}\le C_4 \quad 3.1.5$$

and the decision parameters are $\alpha 1\ge 0$, $\alpha 2\ge 0$, $\beta 1\ge 0$, $\beta 2\ge 0$, $\tau 1\ge 0$, $\varepsilon 1\ge 0$,$\varepsilon 2\ge 0$, $\delta 1\ge 0$, and $\delta 2\ge 0$ 3.1.6

Following the same procedure mentioned in the section 3.1 the other three programming problems can be formulated.

3.2. Optimal Programming for Minimizing the Expected Bacteria size in Stage-II Plants

This programming problem is formulated with the objective of minimizing the average number of *bacterial units* in stage-II plants with the constraints of average number of bacterial *units* in different stages of plants and variance of number of *bacterial units* in different stages plants.

$$Min Z_2=\frac{\tau_1 N_o.e^{At}}{(A-B)}+M_o.e^{Bt} \quad 3.2.1$$

Subject to the constraints,

$$N_o.e^{At}\le C_1 \quad 3.2.2$$

$$\frac{\tau_1 N_o.e^{At}}{(A-B)}+M_o.e^{Bt}\le C_2 \quad 3.2.3$$

$$\frac{\left[-\left(2\alpha_1+\beta_1+\tau_1\right)\right]N_o.e^{At}}{A}+C_o.e^{2At}\le C_3 \quad 3.2.4$$

$$\frac{\tau_1 N_o.e^{At}}{(A-2B)}+\frac{\left(-J\tau_1 N_o.e^{Bt}\right)}{(A-B)(B)}+\frac{\left(-JM_o.e^{Bt}\right)}{(B)}+\frac{\left\{-2\tau_1(\alpha_2-\tau_1)N_o.e^{At}\right\}}{(A-2B)(B)}+\frac{\left(-2\alpha_1\tau_1^2 N_o.e^{At}\right)}{(A-B)(B)(A-2B)}$$
$$+\frac{\left(2\alpha_1\tau_1 M_o.e^{Bt}\right)}{(A)(B)}+\frac{\left\{2\tau_1^2(2\alpha_1+\beta_1+\tau_1)N_o.e^{At}\right\}}{(A-2B)(A)(B)}+\frac{\tau_1^2 C_o.e^{2At}}{(A-B)^2}+\frac{D_o.e^{(A+B)t}}{(A-B)}+E_o.e^{2Bt}\le C_4 \quad 3.2.5$$

and the decision parameters are $\alpha 1\ge 0$, $\alpha 2\ge 0$, $\beta 1\ge 0$, $\beta 2\ge 0$, $\tau 1\ge 0$, $\varepsilon 1\ge 0$,$\varepsilon 2\ge 0$, $\delta 1\ge 0$, and $\delta 2\ge 0$ 3.2.6

3.3. Optimal Programming for Maximizing the Variance size Bacteria in Stage-I plants

This programming problem is formulated with the objective of maximizing the variance number of *bacterial units* in stage-I plants with the constraints of average number of *bacterial units* in different stages and variance of number of *bacterial units* in different stages.

$$Max Z_3=\frac{\left[-\left(2\alpha_1+\beta_1+\tau_1\right)\right]N_o.e^{At}}{A}+C_o.e^{2At} \quad 3.3.1$$

Subject to the constraints,

$$N_o.e^{At}\le C_1 \quad 3.3.2$$

$$\frac{\tau_1 N_o.e^{At}}{(A-B)}+M_o.e^{Bt}\le C_2 \quad 3.3.3$$

$$\frac{\left[-\left(2\alpha_1+\beta_1+\tau_1\right)\right]N_o.e^{At}}{A}+C_o.e^{2At}\le C_3 \quad 3.3.4$$

$$\frac{\tau_1 N_o.e^{At}}{(A-2B)}+\frac{\left(-J\tau_1 N_o.e^{Bt}\right)}{(A-B)(B)}+\frac{\left(-JM_o.e^{Bt}\right)}{(B)}+\frac{\left\{-2\tau_1(\alpha_2-\tau_1)N_o.e^{At}\right\}}{(A-2B)(B)}+\frac{\left(-2\alpha_1\tau_1^2 N_o.e^{At}\right)}{(A-B)(B)(A-2B)}$$
$$+\frac{\left(2\alpha_1\tau_1 M_o.e^{Bt}\right)}{(A)(B)}+\frac{\left\{2\tau_1^2(2\alpha_1+\beta_1+\tau_1)N_o.e^{At}\right\}}{(A-2B)(A)(B)}+\frac{\tau_1^2 C_o.e^{2At}}{(A-B)^2}+\frac{D_o.e^{(A+B)t}}{(A-B)}+E_o.e^{2Bt}\le C_4 \quad 3.3.5$$

and the decision parameters are $\alpha 1 \geq 0$, $\alpha 2 \geq 0$, $\beta 1 \geq 0$, $\beta 2 \geq 0$, $\tau 1 \geq 0$, $\varepsilon 1 \geq 0, \varepsilon 2 \geq 0$, $\delta 1 \geq 0$, and $\delta 2 \geq 0$ 3.3.6

3.4. Optimal Programming for Maximizing variance of size of Bacteria in stage-II plants

This programming problem is formulated with the objective of maximizing the variance of number of *bacterial units* in stage-II plants with the constraints of average number of *bacterial units* in different stages and variance of number of *bacterial units* in different stages.

$$Max Z_4 = \frac{\tau_1 N_o.e^{At}}{(A-2B)} + \frac{\left(-J\tau_1 N_o.e^{Bt}\right)}{(A-B)(B)} + \frac{\left(-JM_o.e^{Bt}\right)}{(B)} + \frac{\left\{-2\tau_1(\alpha_2-\tau_1)N_o.e^{At}\right\}}{(A-2B)(B)} + \frac{\left(-2\alpha_1\tau_1^2 N_o.e^{At}\right)}{(A-B)(B)(A-2B)} + \frac{\left(2\alpha_1\tau_1 M_o.e^{Bt}\right)}{(A)(B)} + \frac{\left\{2\tau_1^2(2\alpha_1+\beta_1+\tau_1)N_o.e^{At}\right\}}{(A-2B)(A)(B)} + \frac{\tau_1^2 C_o.e^{2At}}{(A-B)^2} + \frac{D_o.e^{(A+B)t}}{(A-B)} + E_o.e^{2Bt} \quad 3.4.1$$

Subject to the constraints,

$$N_o.e^{At} \leq C_1 \quad 3.4.2$$

$$\frac{\tau_1 N_o.e^{At}}{(A-B)} + M_o.e^{Bt} \leq C_2 \quad 3.4.3$$

$$\frac{\left[-\left(2\alpha_1+\beta_1+\tau_1\right)\right]N_o.e^{At}}{A} + C_o.e^{2At} \leq C_3 \quad 3.4.4$$

$$\frac{\tau_1 N_o.e^{At}}{(A-2B)} + \frac{\left(-J\tau_1 N_o.e^{Bt}\right)}{(A-B)(B)} + \frac{\left(-JM_o.e^{Bt}\right)}{(B)} + \frac{\left\{-2\tau_1(\alpha_2-\tau_1)N_o.e^{At}\right\}}{(A-2B)(B)} + \frac{\left(-2\alpha_1\tau_1^2 N_o.e^{At}\right)}{(A-B)(B)(A-2B)} + \frac{\left(2\alpha_1\tau_1 M_o.e^{Bt}\right)}{(A)(B)} + \frac{\left\{2\tau_1^2(2\alpha_1+\beta_1+\tau_1)N_o.e^{At}\right\}}{(A-2B)(A)(B)} + \frac{\tau_1^2 C_o.e^{2At}}{(A-B)^2} + \frac{D_o.e^{(A+B)t}}{(A-B)} + E_o.e^{2Bt} \leq C_4 \quad 3.4.5$$

and the decision parameters are $\alpha 1 \geq 0$, $\alpha 2 \geq 0$, $\beta 1 \geq 0$, $\beta 2 \geq 0$, $\tau 1 \geq 0$, $\varepsilon 1 \geq 0, \varepsilon 2 \geq 0$, $\delta 1 \geq 0$, and $\delta 2 \geq 0$ 3.4.6

IV. Numerical Illustration

In order to understand the behaviour of optimization problems, data set from agricultural laboratories situated in Andhra Pradesh state were considered. Decision parameters like growth, loss and transition rates of bacterial units among two stages of plants are obtained at different values of initial number of bacterial units at stage-I (No), initial number of bacterial units at stage-II (Mo), upper limit of variance of bacterial units at stage-I (C_3), upper limit of variance of bacterial units at stage-II (C_4) and time. All the obtained values of α_1; β_1; τ_1; α_2; β_2; ε_1; δ_1; ε_2; δ_2; Z1; Z2; Z3; Z4 using LINGO 14.0 are presented in tables from 4.1 to 4.4.

Table-4.1: Values of α_1; β_1; τ_1; α_2; β_2; ε_1; δ_1; ε_2; δ_2; Z_1 for changing values N_0; C_3; t and fixed values of M_0; C_4:

N_0	M_0	C_3	C_4	t	Z_1	α_1	β_1	τ_1	α_2	β_2	ε_1	δ_1	ε_2	δ_2
900	800	100	120	5	30.2319	1.2356	1.8398	0	1.2356	7.98E-02	0	1.8398	0	0
910	800	100	120	5	30.2318	1.2356	2.2554	0	1.2356	7.98E-02	0	2.2554	0	0
920	800	100	120	5	30.2317	1.2356	2.4625	0	1.2356	7.98E-02	0	2.4625	0	0
930	800	100	120	5	30.2316	1.2356	2.5215	0	1.2356	7.98E-02	0	2.5215	0	0
940	800	100	120	5	30.2315	1.2356	2.5697	0	1.2356	7.98E-02	0	2.5697	0	0
900	800	**100**	120	5	30.3192	1.2356	1.8398	0	1.2356	7.98E-02	0	1.8398	0	0
900	800	**101**	120	5	31.3183	1.2356	1.8398	0	1.2356	7.98E-02	0	1.8398	0	0
900	800	**102**	120	5	33.3217	1.2356	1.8398	0	1.2356	7.98E-02	0	1.8398	0	0
900	800	**103**	120	5	34.3116	1.2356	1.8398	0	1.2356	7.98E-02	0	1.8398	0	0
900	800	**104**	120	5	35.3215	1.2356	1.8398	0	1.2356	7.98E-02	0	1.8398	0	0
900	800	100	120	**5**	30.2311	1.2387	1.8339	0	1.2387	7.98E-02	0	1.8339	0	0
900	800	100	120	**5.1**	30.2309	1.2479	2.0746	0	1.2479	7.99E-02	0	2.0746	0	0
900	800	100	120	**5.2**	30.2307	1.2556	2.3592	0	1.2556	8.01E-02	0	2.3592	0	0
900	800	100	120	**5.3**	30.2305	1.2695	2.4769	0	1.2695	8.02E-02	0	2.4769	0	0
900	800	100	120	**5.4**	30.2303	1.2749	2.5167	0	1.2749	8.03E-02	0	2.5167	0	0

Table-4.2: Values of α_1; β_1; τ_1; α_2; β_2; ε_1; δ_1; ε_2; δ_2; Z_2 for changing values M_0; C_4; t and fixed values of N_0; C_3:

N_0	M_0	C_3	C_4	t	Z_2	α_1	β_1	τ_1	α_2	β_2	ε_1	δ_1	ε_2	δ_2
900	**800**	100	120	5	37.1231	1.2356	2.6387	0	1.2356	0	0	2.4718	0	0
900	**810**	100	120	5	37.2368	1.2356	2.6387	0	1.2356	0	0	2.4718	0	0
900	**820**	100	120	5	37.3167	1.2356	2.6387	0	1.2356	0	0	2.4718	0	0

900	**830**	100	120	5	37.4316	1.2356	2.6387	0	1.2356	0	0	2.4718	0	0
900	**840**	100	120	5	37.5445	1.2356	2.6387	0	1.2356	0	0	2.4718	0	0
900	800	100	**120**	5	37.1231	1.2356	2.6387	0	1.2356	0	0	2.4718	0	0
900	800	100	**121**	5	38.4368	1.2356	2.6387	0	1.2356	0	0	2.4718	0	0
900	800	100	**122**	5	39.1167	1.2356	2.6387	0	1.2356	0	0	2.4718	0	0
900	800	100	**123**	5	40.1643	1.2356	2.6387	0	1.2356	0	0	2.4718	0	0
900	800	100	**124**	5	41.9845	1.2356	2.6387	0	1.2356	0	0	2.4718	0	0
900	800	100	120	**5**	32.0311	1.2356	2.628837	0	1.2356	0	0	2.4718	0	0
900	800	100	120	**5.1**	30.7309	1.2356	2.625706	0	1.2356	0	0	2.4718	0	0
900	800	100	120	**5.2**	28.1007	1.2356	2.622695	0	1.2356	0	0	2.4718	0	0
900	800	100	120	**5.3**	26.2305	1.2356	2.619798	0	1.2356	0	0	2.4718	0	0
900	800	100	120	**5.4**	24.3031	1.2356	2.617008	0	1.2356	0	0	2.4718	0	0

Table-4.3: Values of α_1; β_1; τ_1; α_2; β_2; ε_1; δ_1; ε_2; δ_2; Z_3 for changing values N_0; C_3; t and fixed values of M_0; C_4:

N_0	M_0	C_3	C_4	t	Z_3	α_1	β_1	τ_1	α_2	β_2	ε_1	δ_1	ε_2	δ_2
900	800	100	120	5	2.01E-09	0	0	0	1.2356	7.98E-02	0	0	0	0
910	800	100	120	5	2.00E-09	0	0	0	1.2356	7.98E-02	0	0	0	0
920	800	100	120	5	1.99E-09	0	0	0	1.2356	7.98E-02	0	0	0	0
930	800	100	120	5	1.98E-09	0	0	0	1.2356	7.98E-02	0	0	0	0
940	800	100	120	5	1.97E-09	0	0	0	1.2356	7.98E-02	0	0	0	0
900	800	**100**	120	5	2.601E-09	0	0	0	1.2356	7.98E-02	0	0	0	0
900	800	**101**	120	5	2.701E-09	0	0	0	1.2356	7.98E-02	0	0	0	0
900	800	**102**	120	5	2.801E-09	0	0	0	1.2356	7.98E-02	0	0	0	0
900	800	**103**	120	5	2.901E-09	0	0	0	1.2356	7.98E-02	0	0	0	0
900	800	**104**	120	5	3.101E-09	0	0	0	1.2356	7.98E-02	0	0	0	0
900	800	100	120	**5**	2.11E-09	0	0	0	1.2356	7.98E-02	0	0	0	0
900	800	100	120	**5.1**	2.21E-09	0	0	0	1.2356	7.58E-02	0	0	0	0
900	800	100	120	**5.2**	2.31E-09	0	0	0	1.2356	7.19E-02	0	0	0	0
900	800	100	120	**5.3**	2.41E-09	0	0	0	1.2356	5.82E-02	0	0	0	0
900	800	100	120	**5.4**	2.51E-09	0	0	0	1.2356	5.24E-02	0	0	0	0

Table-4.4: Values of α_1; β_1; τ_1; α_2; β_2; ε_1; δ_1; ε_2; δ_2; Z_4 for changing values M_0; C_4; t and fixed values of N_0; C_3:

N_0	M_0	C_3	C_4	t	Z_4	α_1	β_1	τ_1	α_2	β_2	ε_1	δ_1	ε_2	δ_2
900	**800**	100	120	5	4.22E-10	1.2356	2.6387	4.57E-04	1.2356	7.98E-02	0	2.4718	0	0
900	**810**	100	120	5	3.86E-10	1.2356	2.6387	4.57E-04	1.2356	7.98E-02	0	2.4718	0	0
900	**820**	100	120	5	3.43E-10	1.2356	2.6387	4.57E-04	1.2356	7.98E-02	0	2.4718	0	0
900	**830**	100	120	5	3.06E-10	1.2356	2.6387	4.57E-04	1.2356	7.98E-02	0	2.4718	0	0
900	**840**	100	120	5	2.86E-10	1.2356	2.6387	4.57E-04	1.2356	7.98E-02	0	2.4718	0	0
900	800	100	**120**	5	4.22E-10	1.2356	2.6387	4.57E-04	1.2356	7.98E-02	0	2.4718	0	0
900	800	100	**121**	5	3.86E-10	1.2356	2.6387	4.57E-04	1.2356	7.98E-02	0	2.4718	0	0
900	800	100	**122**	5	3.43E-10	1.2356	2.6387	4.57E-04	1.2356	7.98E-02	0	2.4718	0	0
900	800	100	**123**	5	3.06E-10	1.2356	2.6387	4.57E-04	1.2356	7.98E-02	0	2.4718	0	0
900	800	100	**124**	5	2.86E-10	1.2356	2.6387	4.57E-04	1.2356	7.98E-02	0	2.4718	0	0
900	800	100	120	**5**	4.22E-10	1.2367	2.6395	4.57E-04	1.2356	7.98E-02	0	2.4718	0	0
900	800	100	120	**5**	3.76E-10	2.2489	2.6286	1.44E-02	1.2356	7.98E-02	0	2.4718	0	0
900	800	100	120	**5**	3.24E-10	7.2629	2.6064	1.42E-02	1.2356	7.98E-02	0	2.4718	0	0
900	800	100	120	**5**	2.73E-10	7.2969	2.5839	1.39E-02	1.2356	7.98E-02	0	2.4718	0	0
900	800	100	120	**5**	2.56E-10	7.3994	2.5759	1.24E-02	1.2356	7.98E-02	0	2.4718	0	0

V. Discussion And Analysis

5.1. Observation of Optimal size of Bacterial units (Z_1) in Stage-I Plants with varying values of No, C_3, t :

From table 4.1, it is observed that the optimal (minimum) size of expected bacterial units (Z_1) is a decreasing function of initial number of bacterial units (N_0) in stage-I plants when other parameters are constants. Both birth rate (β_1) and death rate (δ_1) of bacteria in stage-I plants are increasing functions of (N_0) when other parameters are constants. Hence it may conclude that even though the initial number of bacterial units in stage-I plants is increasing, the expected size of these bacteria remains same as the birth and death rates are one and the same.

It is also observed that the optimal (minimum) size of expected bacterial units (Z_1) is a increasing function of upper limit of variance of bacterial units (C_3) in stage-I plants when other parameters are constants. All the rates of bacteria in stage-I plants are not influenced by upper limit of variance of bacterial units in stage-I (C_3). Hence it may conclude that the increased variance of bacterial units in stage-I plants is not influencing the expected size of bacterial units.

Further, it is observed that the optimal (minimum) size of expected bacterial units (Z_1) is a decreasing function of time period t when other parameters are constants. Further, arrival rate (α_1), birth rate (β_1) and death rate (δ_1) of bacteria in stage-I plants are increasing functions of time where as arrival rate (α_2) and birth rate (β_2) in stage-II plants are increasing and decreasing functions respectively. Hence it may conclude that even though the time

elapsed, the expected size of these bacteria in stage-I plants remain same as the birth and death rates are one and the same.

5.2. Observation of Optimal size of Bacterial units (Z_2) in Stage-I plants with varying values of Mo, C_4, t :
From table 4.2, it is observed that the optimal (minimum) size of expected bacterial units (Z_2) is an increasing function of initial number of bacterial units (M_0) in stage-II plants when other parameters are constants. Both birth rate (β_1) and death rate (δ_1) of bacteria in stage-I plants are not influenced by (M_0). Hence it may conclude that even though the initial number of bacterial units in stage-II plants is increasing, the expected size of these bacteria remains same as the birth and death rates are constant.
It is also observed that the optimal (minimum) size of expected bacterial units (Z_2) is an increasing function of upper limit of variance number of bacterial units (C_4) in stage-II plants when other parameters are constants. All the rates of bacteria in stage-II plants are not influenced by upper limit of variance of bacterial units in stage-II plants (C_4). Hence it may conclude that the increased variance number of bacterial units in stage-II plants is not influencing the expected size of bacterial units in stage-II plants.
Further, it is observed that the optimal (minimum) size of expected bacterial units (Z_2) in stage –II plants is a decreasing function of time period t when other parameters are constants. Further, arrival rate (α_1), birth rate (β_1) and death rate (δ_1) of bacteria in stage-I plants are not influenced by time and same is happening with arrival rate (α_2) in stage-II plants. Hence it may conclude that even though the time elapsed, the expected size of bacteria in stage-II plants remains same as the birth and death rates are constant.

5.3. Observation of Optimal Variance of Bacterial units (Z_3) in Stage-I Plants with varying values of No, C_3, t :
From table 4.3, it is observed that the optimal (maximum) size of variance of bacterial units (Z_3) is a decreasing function of initial number of bacterial units (N_0) in stage-I plants when other parameters are constants. Both arrival rate (α_2) and birth rate (β_2) of bacteria in stage-II plants are not influenced by (N_0) when other parameters are constants. Hence it may conclude that even though the initial number of bacterial units in stage-I plants is increasing, the volatility of these bacteria remains constant as the birth and death rates are nullified in stage-I plants.
It is also observed that the optimal (maximum) size of variance of bacterial units (Z_3) in stage-I plants is an increasing function of upper limit of variance of bacterial units (C_3) in stage-I when other parameters are constants. Both arrival rate (α_2) and birth rate (β_2) of bacteria in stage-II plants are not influenced by (C_3) when other parameters are constants. Hence it may conclude that even though the upper limit of variance of bacterial units (C_3) in stage-I is increasing, the volatility of these bacteria remains constant due to nullified arrival, birth and death rates in stage-I plants.
Further, it is observed that the optimal (maximum) size of variance of bacterial units (Z_3) in stage-I plants is an increasing function of time when other parameters are constants. The arrival rate (α_2) is constant and birth rate (β_2) of bacteria in stage-II plants is decreasing function of time when other parameters are constants. Hence it may conclude that even though the time is elapsed, the volatility of these bacteria remains constant due to nullified arrival, birth and death rates in stage-I plants.

5.4. Observation of Optimal Variance of Bacterial units (Z_2) in Stage-I plants with varying values of Mo, C_4, t :
From table 4.4, it is observed that the optimal (maximum) size of variance of bacterial units (Z_4) in stage-II plants is a decreasing function of initial number of bacterial units (M_0) in stage-II plants when other parameters are constants. All the rates of bacteria in stage-I plants and stage-II plants are not influenced by (M_0). Hence it may conclude that for the increasing initial number of bacterial units in stage-II plants, the volatility of these bacteria is decreasing as the arrival, birth, transition and death rates of bacteria are constant.
It is also observed that the optimal (maximum) size of variance of bacterial units (Z_4) in stage-II plants is a decreasing function of upper limit of variance of bacterial units (C_4) in stage-II plants when other parameters are constants. All the rates of bacteria in stage-I plants and stage-II plants are not influenced by (C_4). Hence it may conclude that for the increasing upper limit of variance of bacterial units in stage-II plants, the volatility of these bacteria is decreasing as the arrival, birth, transition and death rates are constants.
Further, it is observed that the optimal (minimum) size of variance of bacterial units (Z_4) in stage-II plants is a decreasing function of time when other parameters are constants. The arrival rate, birth rate and transition rates of bacteria in stage-I plants are decreasing functions of time. Death rate of bacteria in stage-I plants and all the rates in stage-II plants are not influenced by time. Hence it may conclude that for the increasing time, the volatility of these bacteria is decreasing as the arrival, birth and transition rates in stage-I plants are increasing.

References

[1]. Madhavi, K., Thirupathi Rao, P. and P.R.S. Reddy (2013). Optimal Drug Administration for Cancer Chemotherapy through Stochastic Programming. *American Journal of Applied Mathematics and Mathematical Sciences*, Vol. 2(1), pp. 37-45.

[2]. Narayana Rao, E.S.V. and Tirupathi Rao, P. (2016). Bivariate Stochastic Modeling for Spread of Bacterial Diseases among Plants. *IOSR-Journal of Mathematics*, Vol. 12(1), pp. 42-49.

[3]. Segarra, J., Jeger, M. and Van den Bosch, F. (2001). Epidemic Patterns and Dynamics of Plant Disease. *Phytopathology*, Vol. 91, pp. 1001-1010.

[4]. Tirupathi Rao, P., Srinivasa Rao, K. and Padmalatha, K (2010). Stochastic model for optimal drug administration in cancer chemotherapy. *International Journal of Engineering Science and Technology*, Vol. 2(5), pp. 859-865.

[5]. Tirupathi Rao, P., Flora Evangel, D. and Madhavi, K. (2012). Stochastic Programming on Optimal Drug Administration for Two-Stage Cancer Treatment Problem. *International Journal of Green Computing*, Vol. 3(1), pp. 1-10.

[6]. Tirupathi Rao, P., Naveen Kumar, B.N. and P.R.S. Reddy (2013). Stochastic Programming on Optimal Drug Administration for Three-Stage Cancer Chemotherapy Treatment. *American Journal of Operational Research*, Vol. 3(2), pp. 65-74.

[7]. Tirupathi Rao, P., Jayabarathiraj, J. and Vishnu Vardhan, R (2014). Stochastic Optimisation Models for Cancer Chemotherapy. *British Journal of Applied Science & Technology*, Vol. 4(28), pp. 4097-4108.

[8]. Xu, X-M and Ridout, M.S. (2000). Stochastic Simulation of the Spread of Race Specific and Non-specific Aerial Fungal Pathogens in Cultivar Mixtures. *Plant Pathology*,Vol.49, pp. 207-218.