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The Krylov-TPWL Method of Accelerating Reservoir Numerical Simulation

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Abstract: *Because of the large number of system unknowns, reservoir simulation of realistic reservoir can be computationally demanding. Model order reduction (MOR) technique represents a promising approach for accelerating the simulations. In this work, we focus on the application of a MOR technique called Krylov trajectory piecewise-linear (Krylov-TPWL). First, the nonlinear system is represented as a weighted combined piecewise linear system using TPWL method, and then reducing order of each linear model using Krylov subspace. We apply Krylov-TPWL method for a two-phase (oil-water) reservoir model which is solved by full implicit. The example demonstrates that which can greatly reduce the dimension of reservoir model, so as to reduce the calculation time and improve the operation speed.*
Keywords: *reservoir simulation; model order reduction; Krylov subspace; Trajectory piecewise-linear*

I. Introduction

Numerical reservoir simulation enables engineers better understand the reservoir physical properties and fluid flow law and to predict hydrocarbon recovery. It is an indispensable tool. As the reservoir simulation models arising from real fields consist of hundreds of thousands or millions of grid blocks, the numerical solutions of traditional reservoir simulators can be quite time consuming. In addition, if reservoir simulation is used in closed-loop reservoir management [1-4], computational costs are even higher. Where production optimization and the history matching apply repeatedly reservoir simulator, it is extremely time consuming if traditional simulators are used. Therefore, in the case of ensuring the sufficient accuracy of numerical solution, how to greatly accelerate the reservoir simulation speed is the urgent problem to be solved.

Model order reduction (MOR) techniques have shown promise in alleviating computational demands with minimal loss of accuracy [5]. Its task is to reduce the dimension of the state space vector and keep the input and output characteristics of the system at the same time. The proper orthogonal decomposition method (POD) is the most widely used in nonlinear system model reduction method. For now, POD is also widely applied to reservoir simulation [6-8].

Although the POD method can be applied to the nonlinear reservoir simulation system, the acceleration is limited, because in the simulation process, each iteration step requires the construction and projection of the full order Jacobian matrix. At present, trajectory piecewise-linear (TPWL) [9] reduced order method is be widely used in nonlinear system. The nonlinear system is represented as a weighted combined piecewise linear system. In this paper, the TPWL model reduction method is applied to reservoir simulator, and then reducing order of each linear model using Krylov subspace. This method is called Krylov trajectory piecewise-linear (Krylov-TPWL). It can greatly reduce the dimension of reservoir model, so as to reduce the calculation time and improve the operation speed.

II. Equation Of Reservoir Model

In this paper, the mathematical model of reservoir model is transformed into the state space equation by means of space discrete in order to explain the reduction process of TPWL method. Two dimensional oil-water two phase reservoir model is used. It is assumed that oil and water do not exchange material, the process is isothermal, the fluid is compressible, and the mass conservation equation and Darcy's law can be used to obtain [10]:

$$-\nabla \bullet \left[\frac{k_{ri}\rho_i}{\mu_i} \mathbf{K} (\nabla p_i - \rho_i g \nabla d) \right] + \frac{\partial(\phi S_i \rho_i)}{\partial t} - \rho_i q_i''' = 0 \qquad (1)$$

Where $\mathbf{K}$ is permeability tensor; μ is fluid viscosity; k_r is relative permeability; p is pressure; g is gravity acceleration; d is depth; fluid density; ϕ is porosity; S is fluid saturation; t is time; q''' is a source term expressed as flow rate per unit volume; superscript $i \in \{o, w\}$ is respectively oil phase and water phase. In the equation (1), there are four unknown quantities, p_w and S_o are eliminated by using the auxiliary equation

(2) and (3), so that only the state variables p_o, S_w are included in the equation,

$$S_o + S_w = 1 \tag{2}$$

$$p_o - p_w = p_c(S_w) \tag{3}$$

Where $p_c(S_w)$ is oil-water two-phase capillary pressure.

We consider the relatively simple cases and ignore gravity and capillary force. Format to discrete in space by using five point block centered finite difference, we may have the nonlinear first-order differential equation (4), see the specific derivation of literature [11]:

$$\underbrace{\begin{bmatrix} \mathbf{V}_{wp} & \mathbf{V}_{ws} \\ \mathbf{V}_{op} & \mathbf{V}_{os} \end{bmatrix}}_{\mathbf{V}} \begin{bmatrix} \dot{\mathbf{p}} \\ \dot{\mathbf{s}} \end{bmatrix} + \underbrace{\begin{bmatrix} \mathbf{T}_w & \mathbf{0} \\ \mathbf{T}_o & \mathbf{0} \end{bmatrix}}_{\mathbf{T}} \begin{bmatrix} \mathbf{p} \\ \mathbf{s} \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{F}_w(\mathbf{s}) \\ \mathbf{F}_o(\mathbf{s}) \end{bmatrix}}_{\mathbf{F}} \mathbf{q}_{well,t} \tag{4}$$

Where: vector $\mathbf{p}$ and $\mathbf{s}$ is grid center oil pressure p_o and water saturation S_w respectively; $\dot{\mathbf{p}}$ and $\dot{\mathbf{s}}$ is the time t derivative of vector $\mathbf{p}$ and $\mathbf{s}$ respectively; $\mathbf{V}$ is the cumulative matrix; $\mathbf{T}$ is transmission matrix; $\mathbf{F}$ is divided flow matrix; Vector $\mathbf{q}_{well,t}$ is the total flow of oil-water well.

Define the state vector $\mathbf{x}$, input vector $\mathbf{u}$ and output vector $\mathbf{y}$

$$\mathbf{x} \triangleq \begin{bmatrix} \mathbf{p} \\ \mathbf{s} \end{bmatrix} \qquad \mathbf{u} \triangleq \begin{bmatrix} \breve{\mathbf{q}}_{well,t} \\ \breve{\mathbf{p}}_{well} \end{bmatrix} \qquad \mathbf{y} \triangleq \begin{bmatrix} \bar{\mathbf{p}}_{well} \\ \bar{\mathbf{q}}_{well,w} \\ \bar{\mathbf{q}}_{well,o} \end{bmatrix} \tag{5,6,7}$$

Where vector $\breve{\mathbf{q}}_{well,t}$ and $\breve{\mathbf{p}}_{well}$ represent the well of the constant flow and the bottom hole pressure respectively; The vector $\bar{\mathbf{p}}_{well}$ indicates the output bottom hole flow pressure of the constant flow well; Vector $\bar{\mathbf{q}}_{well,o}$ and $\bar{\mathbf{q}}_{well,w}$ indicate the output oil and water flow of the constant bottom hole pressure respectively. The equation (4) can be written as the form of state space equation [11]:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x},\mathbf{u}) = \mathbf{A}(\mathbf{x})\mathbf{x} + \mathbf{B}(\mathbf{x})\mathbf{u} \tag{8}$$

$$\mathbf{y} = \mathbf{h}(\mathbf{x},\mathbf{u}) = \mathbf{C}(\mathbf{x})\mathbf{x} + \mathbf{D}(\mathbf{x})\mathbf{u} \tag{9}$$

In the control system, $\mathbf{A}$ is called the system matrix, $\mathbf{B}$ is called the input matrix, $\mathbf{C}$ is called the output matrix, $\mathbf{D}$ is called the direct transfer matrix. Because the elements of the matrix $\mathbf{V}$, $\mathbf{T}$, $\mathbf{F}$, $\mathbf{J}$ are function of the state variables, the system is a nonlinear system.

III. Krylov-Tpwl Reduced Order Method

By using the TPWL method, a set of linearized points is obtained by using a kind of linear expansion point selection algorithm: $\hat{\mathbf{x}}_0, \hat{\mathbf{x}}_1, \cdots, \hat{\mathbf{x}}_{s-1}$ .Near the linearization points, a set of linear models are obtained by the linear expansion of the nonlinear term $\mathbf{f}(\mathbf{x}) = \mathbf{A}(\mathbf{x})\mathbf{x}$:

$$\dot{\mathbf{x}} = \mathbf{G}_i\mathbf{x} + (\mathbf{f}(\hat{\mathbf{x}}_i) - \mathbf{G}_i\hat{\mathbf{x}}_i) + \mathbf{B}_i\mathbf{u}\ , \quad i = 0, 1, \cdots, (s-1) \tag{10}$$

Where: $\mathbf{G}_i$ is Jacobian matrix of $\mathbf{f}(\mathbf{x})$ at $\hat{\mathbf{x}}_i$, $\mathbf{B}_i = \mathbf{B}(\hat{\mathbf{x}}_i)$.

By using weighted function, the approximate reduction system of the nonlinear system (8) is obtained by weighted summation of the formula (10)

$$\dot{\mathbf{x}} = \sum_{i=0}^{s-1} \omega_i(\mathbf{x})(\mathbf{G}_i\mathbf{x} + (\mathbf{f}(\hat{\mathbf{x}}_i) - \mathbf{G}_i\hat{\mathbf{x}}_i) + \mathbf{B}_i\mathbf{u}), \qquad i = 0, 1, \cdots, (s-1) \tag{11}$$

The implicit Euler discretization is used for each linear model

$$\frac{\mathbf{x}_{k+1} - \mathbf{x}_k}{\Delta t} = \mathbf{G}_i\mathbf{x}_{k+1} + (\mathbf{f}(\hat{\mathbf{x}}_i) - \mathbf{G}_i\hat{\mathbf{x}}_i) + \mathbf{B}_i\mathbf{u}_k$$

Further finishing:

$$(\mathbf{I} - \Delta t \mathbf{G}_i)\mathbf{x}_{k+1} = \mathbf{x}_k + \Delta t(\mathbf{f}(\hat{\mathbf{x}}_i) - \mathbf{G}_i \hat{\mathbf{x}}_i) + \Delta t \mathbf{B}_i \mathbf{u}_k$$

Using Krylov subspace method to generate reduced order matrix

① set $\mathbf{A}_i = \mathbf{I} - \Delta t \mathbf{G}_i$, $\mathbf{b}_i = \Delta t(\mathbf{f}(\hat{\mathbf{x}}_i) - \mathbf{G}_i \hat{\mathbf{x}}_i) + \Delta t \mathbf{B}_i \mathbf{u}_k$, $i = 0, 1, \cdots, (s-1)$

For each pair $\mathbf{A}_i$, $\mathbf{b}_i$, using Arnoldi method to generate Krylov base $\mathbf{V} = [\mathbf{V}_1 \ \mathbf{V}_2 \cdots \mathbf{V}_s]$;

② Orthogonalize the columns of $\mathbf{V}$ using the SVD algorithm and construct a new reduced order basis matrix $\boldsymbol{\Phi}_r = [\boldsymbol{\varphi}_1 \ \boldsymbol{\varphi}_2 \cdots \boldsymbol{\varphi}_r]$ 。

We can get the approximation of nonlinear system (8), (9) for order reduction system

$$\dot{\mathbf{z}} = \sum_{i=0}^{s-1} \omega_i(\mathbf{z})(\mathbf{G}_{ir}\mathbf{z} + \mathbf{V}^T(\mathbf{f}(\hat{\mathbf{x}}_i) - \mathbf{G}_i \hat{\mathbf{x}}_i) + \mathbf{B}_{ir}\mathbf{u}) \quad (12)$$

$$\mathbf{y} = \mathbf{C}_r \mathbf{z} + \mathbf{D}\mathbf{u} \quad (13)$$

In the literature [9], the proposed algorithm for generating the collection of linearized models may be summarized in the following steps:

1) Generate a linearized model about the initial state $\hat{\mathbf{x}}_0 = \mathbf{x}_0$, and set $i = 0$

2) Simulate the nonlinear system while $\min_{0 \le j \le i} \left\| \mathbf{x} - \mathbf{x}_j \right\| > \delta$ for some $\delta > 0$,

i.e. while the current state x is close enough to any of the previous linearization points;

3) Generate a new linearized model about $\hat{\mathbf{x}}_{i+1} = \mathbf{x}$, and set $i := i + 1$

4) If $i < s - 1$, return to step 2.

In the literature [9], the calculation of the weight function $\omega_i(\mathbf{z})$ of the current state $\mathbf{z}$ is as follows:

1) For $i = 0, 1, \cdots, (s-1)$ compute $d_i = \left\| \mathbf{z} - \hat{\mathbf{z}}_i \right\|_2$

2) Take $m = \min_{i=0,\cdots,(s-1)} d_i$

3) For $i = 0, 1, \cdots, (s-1)$ compute $\hat{\omega}_{ii} = e^{-\beta d_i / m}$,take $\beta = 25$

4) Normalize $\hat{\omega}_{ii}$ at the evaluation point:

a) compute $S(\mathbf{z}) = \sum_{j=0}^{s-1} \hat{\omega}_j(\mathbf{z})$;

b) For $i = 0, 1, \cdots, (s-1)$, set $\omega_i(\mathbf{z}) = \hat{\omega}_i(\mathbf{z}) / S(\mathbf{z})$.

IV. Example Verification

A numerical example is that a two-dimensional oil-water two phase anisotropic reservoir is described. Its grid is divided into 21 * 21, and the distribution of permeability and porosity is shown in Figure 1, 2. The related parameters of reservoir model: thickness h=2m, length and width of grid $\Delta x = \Delta y$ =33.33m, the viscosity of the crude oil μ_o =5mPa·s, formation water viscosity μ_w =1mPa·s, comprehensive compression coefficient c_t =3.0×10^{-3}MPa^{-1}, the original formation pressure p_i =30MPa, borehole radius r_{well} =0.114m, the end point relative permeability of oil phase k_{ro}^0 =0.9, the end point relative permeability of water phase k_{rw}^0 =0.6, oil phase Corey index n_o =2.0, water phase Corey index n_w =2.0, residual oil saturation S_{or} =0.2, irreducible water saturation S_{wc} =0.2. We use anti five point method well pattern to produce. Center has a water injection well, and four corners have four production wells. We ignore gravity and capillary force.

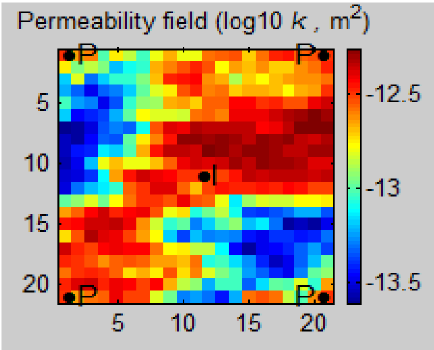

Fig.1 Permeability distribution of reservoir model

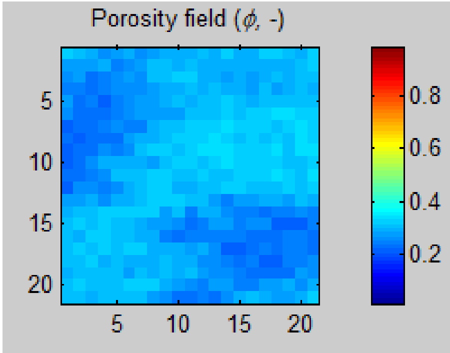

Fig.2 Porosity distribution of reservoir model

The numerical example is simulated by a fully implicit processing. It is divided into training and forecasting two processes:

(1) Training process

The bottom hole pressure of production well is 27.5MPa, the bottom hole flow of injection well is 0.0015m^3/s. We run the full order simulator for 1400 days and save the results of the 66 time steps. The number of selected linearization point is 12.

In the training process, the comparison between the full order reservoir simulator and the reduced order simulator using TPWL method is shown in figure 3, 4.

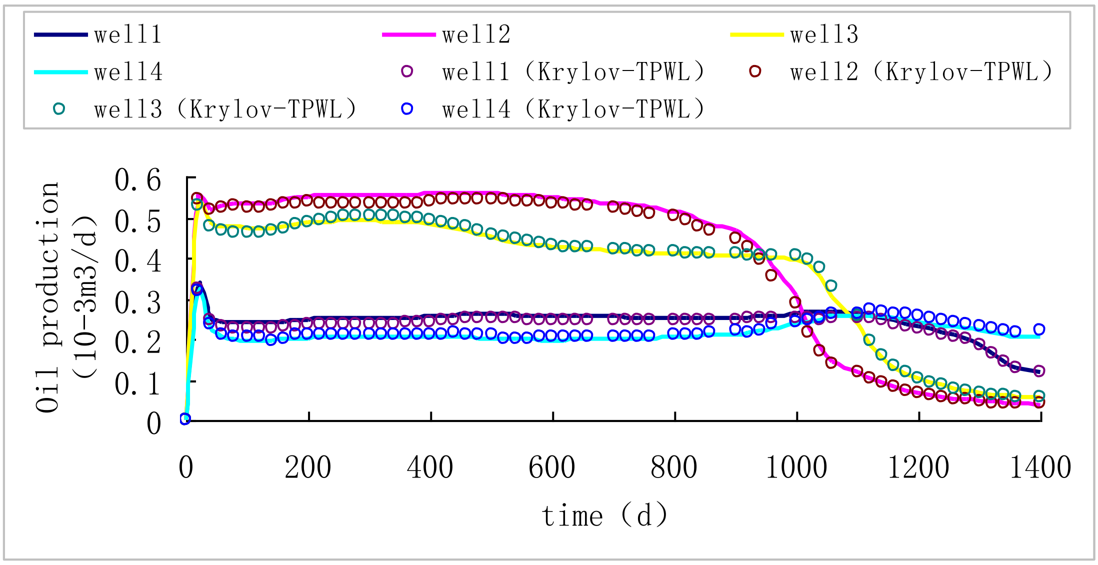

Fig.3 Oil production contrast of four production wells (training process)

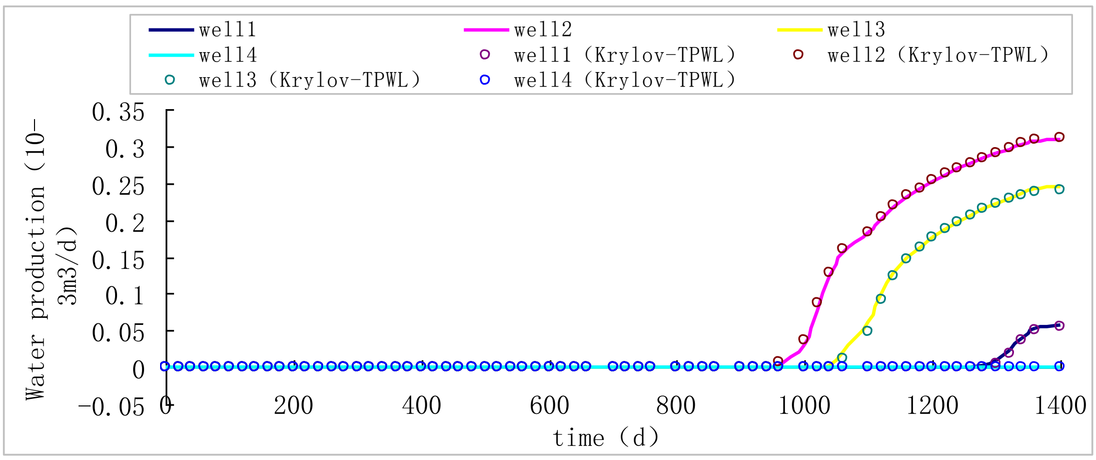

Fig.4 Water production contrast of four production wells (training process)

The above results indicate that in the training process, oil production and water production of four production wells of reduce order and full order simulator are almost identicall, but the simulation time is increased nearly 9 times, the running time of the full order simulator is 35.917s, and the running time of reduction simulator is 4.012s.

(2) Forecasting process

At this time, the bottom hole pressure of production wells is changed to 26.5MPa, and the flow rate at the bottom of the injection well remains unchanged. The comparison between the full order simulator and the reduced order simulator is shown in figure 5, 6.

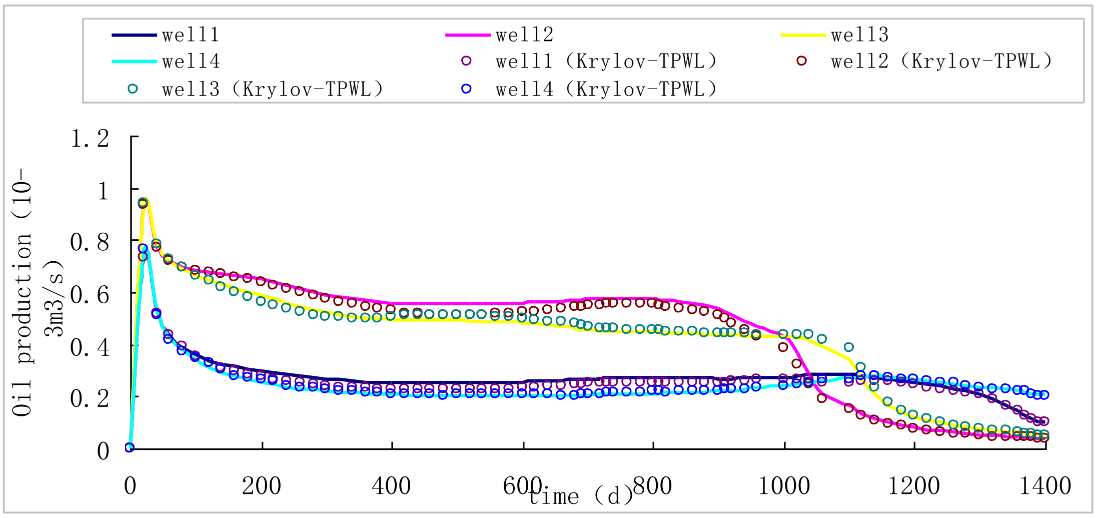

Fig. 5 Oil production contrast of four production wells (prediction process)

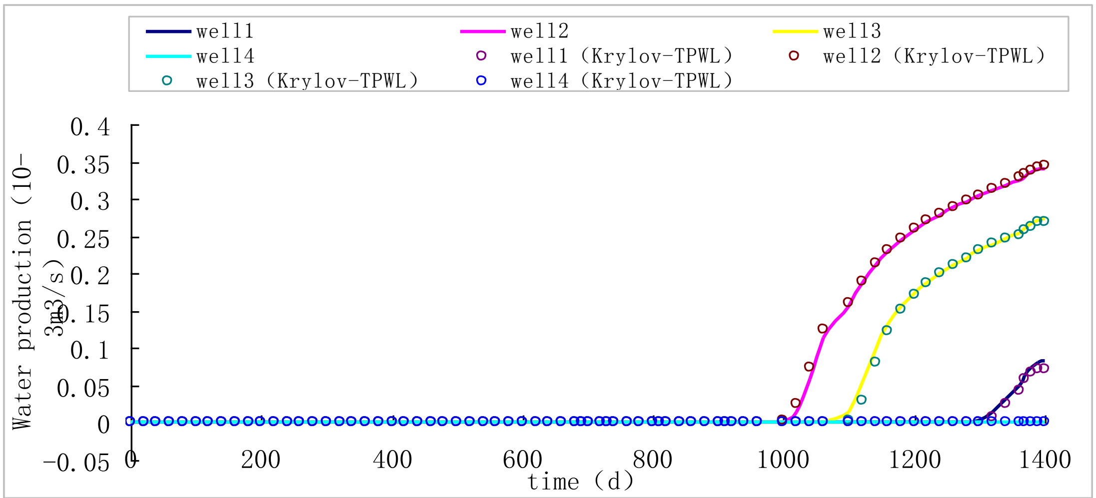

Fig. 6 Water production contrast of four production wells (prediction process)

In the forecasting process, the results show that when the production schedule of forecasting process and training process are different, oil production and water production of four production wells of reduce order and full order simulator are also almost identicall. The full order simulator runs for 35.946s, and the running time of the reduced order simulator is 4.101s.

V. Conclusion

1) The application of Krylov-TPWL model reduced order method to reservoir simulator can greatly reduce the dimension of reservoir model, and improve the operation speed of the simulator by nearly 9 times.
2) When the production schedule of the training and forecasting process is different, oil production and water production of four production wells of reduce order and full order simulator are also almost identicall.
3) The improvement of the operation speed of the reservoir simulator provides an important solution for the practical application of the reservoir production optimization and history matching.

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