International Journal of Engineering Science Invention
ISSN (Online): 2319 – 6734, ISSN (Print): 2319 – 6726
www.ijesi.org ||Volume 6 Issue 9|| September 2017 || PP. 50-58

Optimal Pricing Policy for Deteriorating Items with Variable Demand Rate and Offering Backorder Discount

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Abstract: *In this paper a single item inventory model is developed for deteriorating items with non-instantaneous deterioration and time varying demand sensitive to the selling price. To compensate for the inconvenience due to stock out and to reduce the lost sales during partial backordering period a price discount is declared on the backordered items. Numerical examples are presented to illustrate the model. Cases are discussed for optimal price, price discount, and optimal profit per unit time. Tables, sensitivity analysis and graphs are formed to depict the effects of changes in various parameters on optimal decisions.*
Kewwords: *Non-instantaneous deterioration, price dependent demand, lost sales, Price discounting, partial backordering*

Date of Submission: 24-08-2017 Date of acceptance: 09-09-2017

I. INTRODUCTION

So many inventory models with constant demand rate have been formulated by researchers in the past. But usually demand may depend on many factors like time, price, stock on hand, advertisement and frequency etc. Firstly Silver and Meal (31) introduced economic order quantity model with assuming time-varying demand rate. After that many researchers formulated inventory models with time varying demand. Panda et al (24), Skouri et al (32) and Karmarkar and Chaudhary (16) worked on inventory problems with time varying demand. In the present situation of competitive market pricing policy has a great importance .Adequate pricing and marketing policies may uplift the companies from bottom-line in such competition. Present time is the time where fashion changes very soon as new products are launched day by day. Therefore, it is essential to make such pricing policy which can ensure sale of the entire stock before the next cycle starts. Thus in the demand function price factor has a great value. Papachristos and Skouri (28), Chang et al (7), He and huang (14) developed inventory models with price dependent demand. Khedlekar et al (17) formulated an inventory model with price and time decreasing demand using preservation technology for deteriorating items. Mashud et al (20) developed an inventory model for deteriorating items rates involving partial backlogging and price and stock dependent demand. In inventory control problems deterioration of many items is a key factor which cannot be ignored. There are many products of real life which decay or deteriorate day by day. Ghare and Schrader (12) were the first who developed an inventory model considering deterioration of an item. Then Covert and Philip (9) extended Ghare and Schrader's model including a two-parameter Weibull distribution deterioration function in their model. Mukhopadhyayet al (21), Shah and Acharya (30), Bhuniaet al (4), Skouri et al (32) and many others developed inventory models for time dependent deteriorating items.
In the existing literature it was assumed that the deterioration starts form the time of arrival of inventory to the stock. But, in real market, most goods would have a span of maintaining quality of original condition and deterioration starts after that span. This feature is known as 'non-instantaneous deterioration. This feature can be observed in fruits, green vegetables, food stuffs and fashionable goods. Many researchers like Castro & Alfa (6), Chang et al. (7) and Bhojak & Gothi (3), Vaish and Garg (35), Garg, Vaish and Gupta (11) and Vaish and Agarwal (34) have worked in this direction. It has been seen many times that stock ends before the arrival of next replenishment and some customers do not want wait up to the next replenishment. This is termed as partial backlogging. Cheng and Dye (8), Dye et al. (10) and Pandey et al (27) developed inventory models with partial backlogging. Further price discount on unit selling price of goods is a factor for customers to attract them to buy more and more. For example, in the market of fashionable goods after some times, some products start to lose their luster, but they can be sold with some discounted price. A supplier also wants to sell more to make large profits. Further to secure orders during the shortage period and avoiding lost sales from royal and patient customers, the inventory manager offers a backorder price discount. Thus price discount is one of the key factors which enhance the demand which in turn increases the total profit per unit time. Ardalan(2),Sana and Chaudhari (29), Hsu and Yu (15), Panda et al. (25),Cardanas-Barron et al (5),Garg, Vaish and Gupta (11),

Vaish, B and Agarwal, D(34), Pan and Hsiao (23) and Lee et al (18) developed inventory models considering the price discount factor. Pal and Chandra (22) formulated an inventory model with permissible delay in payment, stock dependent demand and price discount on back orders.Annadurai and Uthaykumar (1) also considered price discounting on back orders while designing their ordering cost reduction inventory model for defective products. Pandey and Vaish (26) developed an inventory model with seasonal demand and price discounting on back orders.

In the present paper an inventory model is developed by considering price sensitive and linearly increasing demand. Deterioration is non- instantaneous and is described by two parameter weibull function. In the model it is assumed that deteriorated items are in a condition to be sold with some price reduction. Shortages are allowed and are partially backlogged. A fraction of demand is backordered which depends on waiting time up to the next replenishment. Practical experience of market tells that the sales are increased significantly if discount is offered on unit selling price. The present paper deals with a declaration of price discount on unit selling price of backordered quantity when stock out period starts to enhance the demand and simultaneously to reduce the lost sales. Further, in the existing literature most of the inventory models are developed for determining minimum total cost per unit time. Very few researchers have developed models to obtain maximum profit per unit time. In the present model profit maximization technique is used to solve the model. Numerical illustrations, tables, graphs and sensitivity analysis are presented in the model to explain the various factors involved in the model.

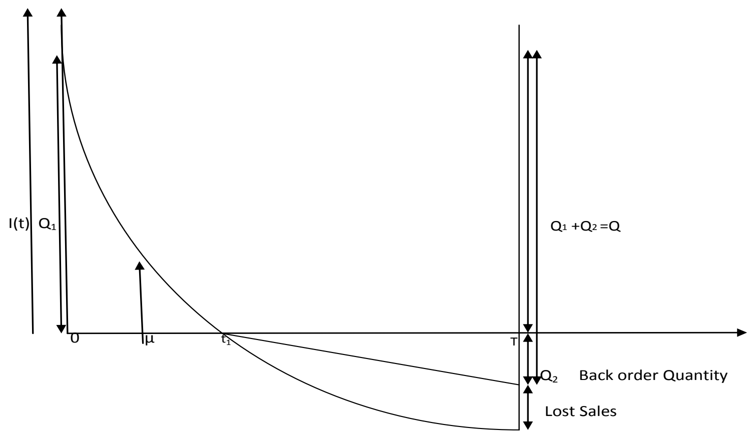
II. ASSUMPTIONS AND NOTATIONS:

1. Demand is price sensitive and time dependent and it follows the pattern D (t) = $(\frac{a}{p^{\beta}} + bt)$,where p is selling price per unit, a > 0 is a scaling factor, b>0 and β>1is the index of price elasticity.
2. Shortages are allowed and are partial backlogged. The backlogged rate is described as decreasing function of the waiting time $\frac{1}{1+\delta(T-t)}$ where ($\delta > 0$).Thus a fraction of the demand is backlogged
3. d_1 ($0 \le d_1 \le 1$)is the percentage discount offer on unit selling price on backordered quantity declared at the start of the stock out period. $\alpha = (1-d_1)^{-n}$ ($n \in$ R, the set of real numbers and $n \ge 1$) is the positive effect of discounted selling price on demand during stock out period, when $d_1 \to 0, \alpha \to 1$ i.e. the demand during stocked period will not be increased.
4. d_2 is the percentage discount offered on unit selling price of deteriorated quantity.
5. The deterioration is non-instantaneous and follows Weibull distribution function.
Therefore deterioration rate W (t) = $\lambda \phi t^{\phi-1}$ where, λ>0,λ<<1 and φ is the shape parameter (φ>0).
6. μ is the time at which deterioration starts.
7. Delivery lead time is zero and cycle length of the inventory model is finite as well as infinite in different cases considered separately.

c Purchasing cost per unit
T Cycle length
t_1 The time at which inventory level becomes zero
Q_1 Initial inventory level at the beginning of each cycle and
Q_2 Backordered quantity
Q Ordered Quantity (Q_1+Q_2)
DQ Deteriorated quantity
h Holding cost per unit per unit time
s Shortage cost per unit per unit time
l Lost sale cost per unit per unit time
O Ordering cost per order
δ Rate of backlogging
SR Sales revenue per replenishment cycle
I(t) The inventory level at time t.
F(t_1, T)Profit per unit time
t_1*, T^*, F^*(t_1, T), Q^* represents the optimal values of t_1, T, F (t_1, T), Q,

MODEL FORMULATION and ANALYSIS:

The behavior of the inventory level during cycle T is depicted in figure1.



(Fig- 1)

The differential equations governing the fluctuation of inventory with time t are shown as below:

$$\frac{dI(t)}{dt} = -(\frac{a}{p^{\beta}} + bt) \qquad 0 \le t \le \mu \qquad (1)$$

$$\frac{dI(t)}{dt} + \lambda \phi t^{\phi-1} I(t) = -(\frac{a}{p^{\beta}} + bt) \qquad \mu \le t \le t_1 \qquad (2)$$

$$\frac{dI(t)}{dt} = \frac{-\alpha(\frac{a}{p^{\beta}} + bt)}{1 + \delta(T - \mathrm{t})} \qquad t_1 \le t \le T \qquad (3)$$

With boundary condition:

$$I(\mathrm{t}_1) = 0, I(0) = Q_1 \qquad (4)$$

the solutions of above equations are given by:

$$I(t) = -(\frac{at}{p^{\beta}} + \frac{bt^2}{2}) + Q_1 \qquad 0 \le t \le \mu \qquad (5)$$

$$I(t) = X - \lambda t^{\phi}(\frac{at_1}{p^{\beta}} + \frac{bt_1^2}{2}) - \frac{at}{p^{\beta}} - \frac{bt^2}{2} + \frac{a\lambda\phi t^{\phi+1}}{p^{\beta}(\phi+1)} + \frac{b\lambda\phi t^{\phi+2}}{2(\phi+2)} \qquad \mu \le t \le t_1 \qquad (6)$$

where $X = (\frac{at_1}{p^{\beta}} + \frac{bt_1^2}{2} + \frac{a\lambda t^{\phi+1}}{p^{\beta}(\phi+1)} + \frac{b\lambda t_1^{\phi+2}}{(\phi+2)})$

$$I(t) = \frac{\alpha b}{\delta}(\mathrm{t} - \mathrm{t}_1) + \frac{\alpha}{\delta}(\frac{a}{p^{\beta}} + \frac{b(1+\delta T)}{\delta})(\log[1 + \delta(\mathrm{T} - t)] - \log[1 + \delta(\mathrm{T} - t_1)]) \qquad t_1 \le t \le T \qquad (7)$$

the value of Q_1 and Q_2 are obtained as

$$Q_1 = X - \lambda\mu^{\phi}(\frac{at_1}{p^{\beta}} + \frac{bt_1^2}{2}) + \frac{a\lambda\phi\mu^{\phi+1}}{p^{\beta}(\phi+1)} + \frac{b\lambda\phi\mu^{\phi+2}}{2(\phi+2)} \qquad (8)$$

$$Q_2 = \frac{\alpha}{\delta}(\frac{a}{p^{\beta}} + \frac{b(1+\delta T)}{\delta})\log[1 + \delta(\mathrm{T} - t_1)] - \frac{\alpha b}{\delta}(\mathrm{T} - t_1) \qquad (9)$$

Deterioration Quantity DQ

$$D\,\mathrm{Q} = \mathrm{Q}_1 - \int_0^{t_1}(\frac{a}{p^{\beta}} + bt)dt = \mathrm{Q}_1 - (\frac{at_1}{p^{\beta}} + \frac{bt_1^2}{2}) \qquad (10)$$

Sales Revenue SR

SR= SR from Demand (0, t_1) + SR from Deterioration Quantity + SR from Back Ordered Quantity

$$SR = p\{d_2(\frac{at_1}{p^\beta}+\frac{bt_1^2}{2})+(1-d_2)Q_1+(1-d_1)Q_2\}$$

$$SR = p\{d_2(\frac{at_1}{p^\beta}+\frac{bt_1^2}{2})+(1-d_2)(X-\lambda\mu^\phi(\frac{at_1}{p^\beta}+\frac{bt_1^2}{2})+\frac{a\lambda\phi\mu^{\phi+1}}{p^\beta(\phi+1)}+\frac{b\lambda\phi\mu^{\phi+2}}{2(\phi+2)} \quad (11)$$

$$+(1-d_1)(\frac{\alpha}{\delta}(\frac{a}{p^\beta}+\frac{b(1+\delta T)}{\delta})\log[1+\delta(\mathrm{T}-t_1)]-\frac{\alpha b}{\delta}(\mathrm{T}-t_1)\}$$

Purchasing cost PC

$$PC = c\{Q_1+Q_2\}$$

$$PC = c\{X-\lambda\mu^\phi(\frac{at_1}{p^\beta}+\frac{bt_1^2}{2}) \quad (12)$$

$$+\frac{a\lambda\phi\mu^{\phi+1}}{p^\beta(\phi+1)}+\frac{b\lambda\phi\mu^{\phi+2}}{2(\phi+2)}+\frac{\alpha}{\delta}(\frac{a}{p^\beta}+\frac{b(1+\delta T)}{\delta})\log[1+\delta(\mathrm{T}-t_1)]-\frac{\alpha b}{\delta}(\mathrm{T}-t_1)\}$$

Holding cost HC

$$HC = h\{\int_0^\mu I(t)dt+\int_\mu^{t_1} I(t)dt\}$$

$$HC = h\{\frac{at_1^2}{2p^\beta}+\frac{bt_1^3}{3}+\frac{a\lambda t_1^{\phi+1}}{p^\beta(\phi+1)}+\frac{a\lambda\phi t_1^{\phi+2}}{p^\beta(\phi+1)(\phi+2)}+\frac{b\lambda\phi t_1^{\phi+3}}{(\phi+1)(\phi+3)}$$

$$-(\frac{at_1}{p^\beta}+\frac{bt_1^2}{2})\frac{\lambda\phi\mu^{\phi+1}}{(\phi+1)}+\frac{a\lambda\phi\mu^{\phi+2}}{p^\beta(\phi+2)}+\frac{b\lambda\phi\mu^{\phi+3}}{2(\phi+3)}\}$$

(13)

Shortage cost SC

$$SC = -s\int_{t_1}^{T} I(t)dt$$

$$SC = -s\int_{t_1}^{T}\frac{\alpha b}{\delta}(\mathrm{t}-\mathrm{t}_1)+\frac{\alpha}{\delta}(\frac{a}{p^\beta}+\frac{b(1+\delta T)}{\delta})(\log[1+\delta(\mathrm{T}-t)]-\log[1+\delta(\mathrm{T}-t_1)])dt \quad (14)$$

$$SC = s\{-\frac{\alpha b}{2\delta}(\mathrm{T}-\mathrm{t}_1)^2+\frac{\alpha}{\delta^2}(\frac{a}{p^\beta}+\frac{b(1+\delta T)}{\delta})(\delta(\mathrm{T}-t_1)-\log[1+\delta(\mathrm{T}-t_1)])\}$$

Lost sale cost LSC

$$LSC = l\int_{t_1}^{T}(\frac{a}{p^\beta}+bt)-\frac{\alpha(\frac{a}{p^\beta}+bt)}{1+\delta(T-t)})dt \quad (15)$$

$$LSC = l\{(\frac{a}{p^\beta}+\frac{\alpha b}{\delta})(\mathrm{T}-t_1)+\frac{b}{2}(\mathrm{T}^2-\mathrm{t}_1^2)-\frac{\alpha}{\delta}(\frac{a}{p^\beta}+\frac{b(1+\delta T)}{\delta})\log[1+\delta(\mathrm{T}-t_1)]\}$$

Ordering cost OC

$$OC = O$$

(16)

Profit function per unit time for the system

$$F(t_1,T)=\frac{1}{T}[SR-PC-HC-SC-LSC-OC]$$

$$F(t_1,T)=\frac{1}{T}[p\{d_2(\frac{at_1}{p^\beta}+\frac{bt_1^2}{2})+(1-d_2)(X-\lambda\mu^\phi(\frac{at_1}{p^\beta}+\frac{bt_1^2}{2})+\frac{a\lambda\phi\mu^{\phi+1}}{p^\beta(\phi+1)}+\frac{b\lambda\phi\mu^{\phi+2}}{2(\phi+2)}$$

$$+(1-d_1)(\frac{\alpha}{\delta}(\frac{a}{p^\beta}+\frac{b(1+\delta T)}{\delta})\log[1+\delta(T-t_1)]-\frac{\alpha b}{\delta}(T-t_1)\}-c\{X-\lambda\mu^\phi(\frac{at_1}{p^\beta}+\frac{bt_1^2}{2})$$

$$+\frac{a\lambda\phi\mu^{\phi+1}}{p^\beta(\phi+1)}+\frac{b\lambda\phi\mu^{\phi+2}}{2(\phi+2)}+\frac{\alpha}{\delta}(\frac{a}{p^\beta}+\frac{b(1+\delta T)}{\delta})\log[1+\delta(T-t_1)]-\frac{\alpha b}{\delta}(T-t_1)\}$$

$$-h\{\frac{at_1^2}{2p^\beta}+\frac{bt_1^3}{3}+\frac{a\lambda t_1^{\phi+1}}{p^\beta(\phi+1)}+\frac{a\lambda\phi t_1^{\phi+2}}{p^\beta(\phi+1)(\phi+2)}+\frac{b\lambda\phi t_1^{\phi+3}}{(\phi+1)(\phi+3)}-(\frac{at_1}{p^\beta}+\frac{bt_1^2}{2})\frac{\lambda\phi\mu^{\phi+1}}{(\phi+1)}+\frac{a\lambda\phi\mu^{\phi+2}}{p^\beta(\phi+2)}+\frac{b\lambda\phi\mu^{\phi+3}}{2(\phi+3)}\}$$

$$-s\{-\frac{\alpha b}{2\delta}(T-t_1)^2+\frac{\alpha}{\delta^2}(\frac{a}{p^\beta}+\frac{b(1+\delta T)}{\delta})(\delta(T-t_1)-\log[1+\delta(T-t_1)])\}$$

$$-l\{(\frac{a}{p^\beta}+\frac{\alpha b}{\delta})(T-t_1)+\frac{b}{2}(T^2-t_1^2)-\frac{\alpha}{\delta}(\frac{a}{p^\beta}+\frac{b(1+\delta T)}{\delta})\log[1+\delta(T-t_1)\}-O]$$

(17)

Now unit time profit is considered as a function of two variables t_1 and T. To find out the optimal solution the optimal values of t_1 and T are obtained by solving the following equations simultaneously

$$\frac{\partial F(t_1,T)}{\partial t_1}=0,\ \frac{\partial F(t_1,T)}{\partial T}=0$$

(18)

Provided $\frac{\partial^2 F(t_1,T)}{\partial t_1^2}\cdot\frac{\partial^2 F(t_1,T)}{\partial T^2}-\left(\frac{\partial^2 F(t_1,T)}{\partial t_1\partial T}\right)>0$

(19)

Numerical Illustration-1:

λ=0.009, δ =2.6 units, p=830rs, s=0.5rs/unit/time, l=1.2rs/unit/time, a=100, O =50rs/ order h=1.25rs/unit/time, n=1.5, b=2.8, c=170rs, β=1.16, d_1=0.30, d_2=0.15,φ=3, μ=0.30 weeks

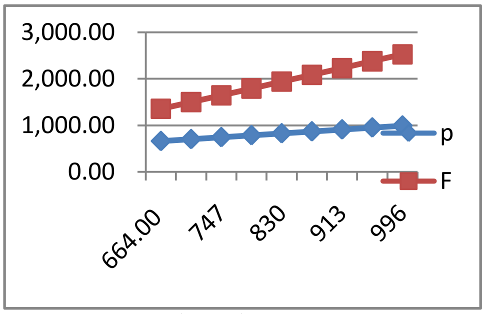
Applying the solution procedure described above the optimal values obtained is as follows:

t_1*= 1.82198weeks, T^* =3.62586weeks,$F^*(t_1, T)$ =1939.32rs, Q^* =8.82552 units

Effects of parameter"p" on Total Profit per Unit Time

%change in p	p	t_1	T	$F(t_1,T)$
-20%	664.00	1.71835	3.29895	1,356
-15%	705.5	1.7494	3.3958	1500.95
-10%	747	1.7766	3.48141	1646.7
-5%	788.5	1.80062	3.55761	1792.85
0	830	1.82198	3.62586	1939.32
5%	871.5	1.8411	3.68735	2086.06
10%	913	1.8583	3.74303	2233.02
15%	954.5	1.87387	3.79369	2380.18
20%	996	1.88802	3.83997	2527.5

(Table -1)

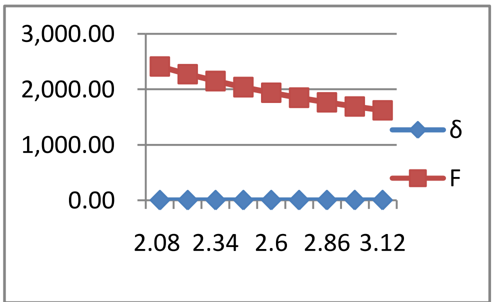


(Fig-2)

Effects of parameter"δ"on Total Profit per Unit Time

%change inδ	δ	t_1	T	$F(t_1,T)$
-20%	2.08	2.23664	4.50151	2,409
-15%	2.21	2.11693	4.24585	2271.44
-10%	2.34	2.00887	4.0172	2148.84
-5%	2.47	1.91097	3.81161	2038.72
0	2.6	1.82198	3.62586	1939.32
5%	2.73	1.74081	3.4573	1849.16
10%	2.86	1.66655	3.30371	1767.03
15%	2.99	1.5984	3.16324	1691.93
20%	3.12	1.53567	3.03431	1623

(Table 2)

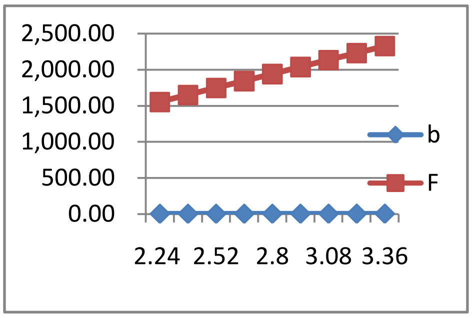


(Fig-3)

Effects of parameter"b" on Total Profit per Unit Time

%change in b	b	t_1	T	$F(t_1,T)$
-20%	2.24	1.82185	3.62573	1,552
-15%	2.38	1.82189	3.62577	1649.15
-10%	2.52	1.82192	3.62581	1745.87
-5%	2.66	1.82195	3.62584	1842.6
0	2.8	1.82198	3.62586	1939.32
5%	2.94	1.82201	3.62589	2036.04
10%	3.08	1.82203	3.62591	2132.77
15%	3.22	1.82205	3.62593	2229.49
20%	3.36	1.82207	3.62595	2326.22

(Table 3)

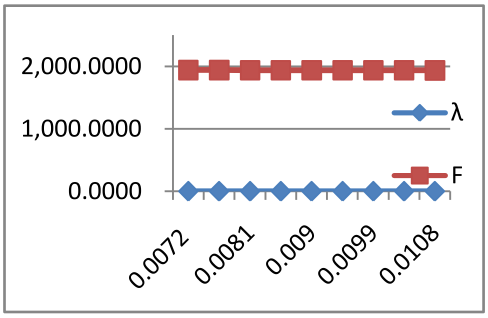


(Fig-4)

Effects of parameter"λ" on Total Profit per Unit Time

% change in λ	λ	t_1	T	$F(t_1,T)$
-20%	0.0072	1.82859	3.63262	1,942
-15%	0.00765	1.82692	3.63091	1941.02
-10%	0.0081	1.82526	3.62922	1940.45
-5%	0.00855	1.82362	3.62753	1939.88
0	0.009	1.82198	3.62586	1939.32
5%	0.00945	1.82036	3.62421	1938.76
10%	0.0099	1.81875	3.62256	1938.21
15%	0.01035	1.81715	3.62093	1937.66
20%	0.0108	1.81556	3.61931	1937.12

(Table 4)

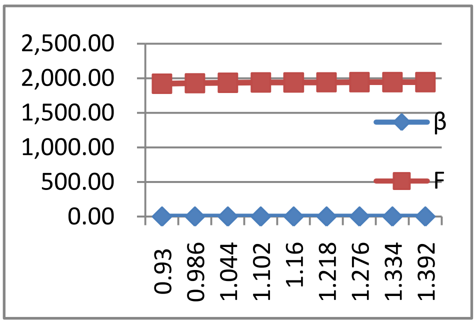


(Fig-5)

Effects of parameter"β" on Total Profit per Unit Time

% changein β	β	t_1	T	$F(t_1,T)$
-20%	0.93	1.70876	3.51020	1,919
-15%	0.986	1.75385	3.55627	1927.07
-10%	1.044	1.78574	3.58884	1932.81
-5%	1.102	1.80734	3.61091	1936.69
0	1.16	1.82198	3.62586	1939.32
5%	1.218	1.83189	3.63599	1941.1
10%	1.276	1.83861	3.64285	1942.31
15%	1.334	1.84315	3.64749	1943.12
20%	1.392	1.84623	3.65063	1943.68

(Table 5)



(Fig-6)

Sensitivity Analysis

Parameter	% Change	% Change t_1	% Change T	% Change F
p	-20	-0.05687	-0.09016	-0.30078
	-10	-0.02489	-0.03983	-0.150887
	10	0.019934	0.032315	0.151444
	20	0.036246	0.05905	0.303219
δ	-20	0.2275875	0.241501	0.242187
	-10	0.102575	0.10793	0.1080378
	10	-0.085308	-0.088847	-0.08884
	20	-0.157142	-0.163147	-0.1631087
b	-20	-0.000071	-0.000035	-0.199794
	-10	-0.000032	-0.000013	-0.1496246
	10	0.000027	0.0000137	0.0997514
	20	0.000049	0.0000248	0.1995029
λ	-20	0.003627	0.001864	0.001381
	-10	0.0018	0.000926	0.0005826
	10	-0.001772	-0.0013596	-0.00057
	20	-0.00352	-0.001806	-0.0011344
β	-20	-0.062141	-0.008797	-0.010477
	-10	-0.01989	-0.01209	-0.003356
	10	0.0091274	0.0046857	0.001548
	20	0.0133096	0.006831	0.00224821

(Table 6)

Observations

1. Table (1) reveals that as the selling price (p) increases, the unit time profit of the system also increases.

2. From table (2) it is observed that as the rate of backlogging (δ)decreases, the unit time profit of the system increases.
3. Table (3) reveals that as (b) increases, the unit time profit of the system also increases.
4. From table (4) it is observed that as (λ) decreases the unit time profit of the system increases.
5. Table (5) reveals that as (β) increases, the unit time profit of the system also increases.
6. From sensitivity table (6) it has been observed (λ)&(β)are negligible sensitive to t_1,T & F(t_1,T) .(b) is negligible sensitive to t_1,T and it is moderate sensitive to F(t_1,T). (δ)shows moderate sensitivity to t_1,T & F(t_1,T). (p) is moderate sensitive to t_1,and T fairly sensitive to F(t_1,T).

Special Cases of the Modal

Case-1: To find optimal price

In this case unit time profit is a function of two variables t_1 and p. To find out the optimal solution

$$\frac{\partial F(t_1,p)}{\partial t_1}=0,\frac{\partial F(t_1,p)}{\partial p}=0 \quad (20)$$

And the optimal values of t_1 and p are obtained by solving these equations simultaneously provided

$$\frac{\partial^2 F(t_1,p)}{\partial t_1^2}.\frac{\partial^2 F(t_1,p)}{\partial p^2}-\left(\frac{\partial^2 F(t_1,p)}{\partial t_1 \partial p}\right)>0 \quad (21)$$

Numerical Illustration-2:

λ =0.009, δ =35.6 units, s=0.2rs/unit/time, l=1.2rs/unit/time, a=50000, O =50rs/ order
h=1.25rs/unit/time, n=1.5, b=.90, c=40rs, β=1.85, d_1=0.20, d_2=0.15, φ=2, μ=2 weeks, T=2.8 weeks.Applying the solution procedure described above the optimal values obtained are as follows:t_1*= 2.20304 weeks, p^* =112.209rs, $F^*(t_1,p)$=622.025rs,Q^* = 27.053units

If price discount is not offered on backordered quantity then the optimal values obtained from above parameters are as follows: t_1*= 1.69318 weeks, p^* =3.36376rs, $F^*(t_1,p)$=1772.4rs,Q^* =9.1556 unit. These results show that sometimes price discount offered on backordered quantity is profitable.

Case-2: To find the optimal backordering discount

In this case unit time profit is a function of two variables t_1 and d_1. To find out the optimal solution

$$\frac{\partial F(t_1,d_1)}{\partial t_1}=0\,,\frac{\partial F(t_1,d_1)}{\partial d_1}=0 \quad (22)$$

And the optimal values of t_1 and $d_1$are obtained by solving these equations simultaneously provided

$$\frac{\partial^2 F(t_1,d_1)}{\partial t_1^2}.\frac{\partial^2 F(t_1,d_1)}{\partial d_1^2}-\left(\frac{\partial^2 F(t_1,d_1)}{\partial t_1 \partial d_1}\right)>0 \quad (23)$$

Numerical Illustration-3:

λ =0.009, δ =8.6 units, s=0.5 rs/unit/time, l=1.2 rs/unit/time, a=200, O =50rs/ order
h=1.25rs/unit/time, n=2, b=0.8, c=170rs, β=2.16, d_2=0.15, φ=2, μ=20 days, T=90 days,p=630rs
Applying the solution procedure described above the optimal values obtained are as follows:
t_1*= 73.433 days, d_1^* =0.45932, $F^*(t_1, d_1)$ =164903rs, Q^* = 47158.189units

Case-3: To find optimal values considering profit function F(t_1)

In this case the profit function per unit time is considered for single variable t_1.
The optimal value of t_1 is obtained by solving the equation

$$\frac{dF(t_1)}{dt_1}=0 \quad (24)$$

Provided $\frac{d^2F(t_1)}{dt_1^2}<0$ (25)

Numerical Illustration-4:

T=90 days, λ=0.009, δ=8.6units, p=630rs, s=0.5rs/unit/time, l=1.2rs/unit/time, a=100,

… ..

h=1.25rs/unit/time, n=1, μ=20 days, c=170rs, ϕ=2, o=50rs/order, β=2.16, b=0.8, d_1=0.25, d_2=0.15, Applying the solution procedure described above the optimal values obtained are as follows:

t_1*= 74.2445 days, $F^*(t_1)$ =172292rs, Q^*=49301, $\frac{d^2F^*(t_1)}{dt_1^2} = -1.01123$

If price discount is not offered on backordered quantity then the optimal values obtained from above parameters are as follows:

t_1*= 73.9488 days, $F^*(t_1)$ =169567rs,Q^*=48480.8, $\frac{d^2F^*(t_1)}{dt_1^2} = -0.956615$

These results show that sometimes price discount offered on backordered quantity is profitable.

III. CONCLUSION

The present paper is designed with realistic features of price sensitive demand time dependent demand, non-instantaneous deterioration and partial backlogging. In the market of fashionable goods after some times, some products start to lose their luster, but they can be sold with some discounted price. In the model it is assumed that deteriorated items are in a condition to be sold with some price reduction. Therefore a d_2 percentage reduction in price is offered on each deteriorated unit. Further, to secure orders during the shortage period and avoiding lost sales from royal and patient customers, the inventory manager offers a backorder price discount. The most important feature of the model is the declaration of price discount at the start of shortage period so that demand is boosted in this period and more customers will be willing to wait for the next replenishment. Numerical illustrations are given to describe the model. Special cases for optimal price and optimal discount are presented. Numerical illustrations show that in most of the cases optimal price discount on backorders is profitable. Effects of some parameters involved in the problem on some factors have been discussed through tables and graphs and sensitivity analysis. Results noticed in tables, graphs and sensitivity analysis are suitable to real situations. The model could be useful in retail business of fashionable goods where partial backlogging occurs and deteriorated items can be sold on discounted price. The present study can be further extended for some different factors useful for inventory systems.

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International Journal of Engineering Science Invention (IJESI) is UGC approved Journal with Sl. No. 3822, Journal no. 43302.

Bindu Vaish. “Optimal Pricing Policy for Deteriorating Items with Variable Demand Rate and Offering Backorder Discount.” International Journal of Engineering Research and Applications (IJERA) , vol. 6, no. 9, 2017, pp. 50–58.