Bölcsföldi-Dömötör prime numbers

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Abstract: *After defining, Bölcsföldi-Dömötör prime numbers will be presented from 23 to 5533553. How many Bölcsföldi-Dömötör prime numbers are there in the interval $(10^{p-1}, 10^p)$, where p is a prime number? On the one hand, it has been counted by computer among the prime numbers with up to 13 digits. On the other hand, the function (1) gives the approximate number of Bölcsföldi-Dömötör prime numbers in the interval $(10^{p-1}, 10^p)$. Near-proof reasoning has emerged from the conformity of Mills' prime numbers with Bölcsföldi-Dömötör prime numbers. The set of Bölcsföldi-Dömötör prime numbers is probably infinite.*

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I. Introduction

The sets of special prime numbers within the set of prime numbers are well-known. For instance, the Erdős-primes (the sum of the digits is prime) [8], Fibonacci-primes ($F_0=0$, $F_1=1$, $F_n=F_{n-1}+F_{n-2}$), Gauss-primes (in the form 4n+3), Leyland-primes (in the form x^y+y^x, where $1\leq x\leq y$), Pell-primes ($P_0=0$, $P_1=1$, $P_n=2P_{n-1}+P_{n-2}$), Bölcsföl di-Birkás-Ferenczi primes (all digits are prime and the number of digits is prime), etc. Question: Which further sets of special prime numbers are there within the set of prime numbers? We have found a further set of special prime numbers within the set of prime numbers. It is the set of Bölcsföldi-Dömötör prime numbers.

II. Bölcsföldi-Dömötör prime numbers [3], [9], [10], [11], [12].

Definition: a positive integer number is a Bölcsföldi-Dömötör prime number, if
a/ the positive integer number is prime, b/ all digits are (2 or 3 or 5),
c/ the number of digits is prime, d/ the sum of digits is prime.

The set of prime numbers meeting the conditions a/ and c/ is well-known: it is the set of prime-long prime numbers [3], [9]. Positive integer numbers meeting all the four conditions (a/, b/, c/, d/) at the same time are Bölcsföldi-Dömötör prime numbers (Fig.1, Fig.2).
Bölcsföldi-Dömötör prime number p has the following sum form:

$$p=\sum_{j=0}^{k(p)} e_j(p) \cdot 10^j \text{ where } e_j(p) \in \{2, 3, 5\} \text{ and } k(p)+1 \text{ is prime and } e_0(p) \in \{3\} \text{ and } \sum_{j=0}^{k(p)} e_j(p) \text{ is prime.}$$

The Bölcsföldi-Dömötör prime numbers are as follows (the last digit can only be 3):

{{23}, {223, 353}, {25253, 25523, 32233, 32323, 33223, 33353, 33533, 35353, 35533, 52253, 53353, 55333}, {2222333, 2222533, 2223233, 2223253, 2225233, 2225323, 2232323, 2232523, 2233223, 2235353, 2252233, 2253353, 2255333, 2322253, 2332333, 2333323, 2335253, 2335523, 2352223, 2352353, 2352533, 2355233, 2523223, 2523533, 2532223, 2535233, 2553233, 3222253, 3223223, 3223333, 3232553, 3233323, 3252533, 3253253, 3255233, 3325253, 3332233, 3353333, 3355553, 3532253, 3532523, 3552233, 3553223, 3553553, 3555353, 5222323, 5225333, 5232223, 5233523, 5235233, 5235323, 5252333, 5322353, 5322533, 5325323, 5353223, 5353553, 5533223, 5533553}}, etc.
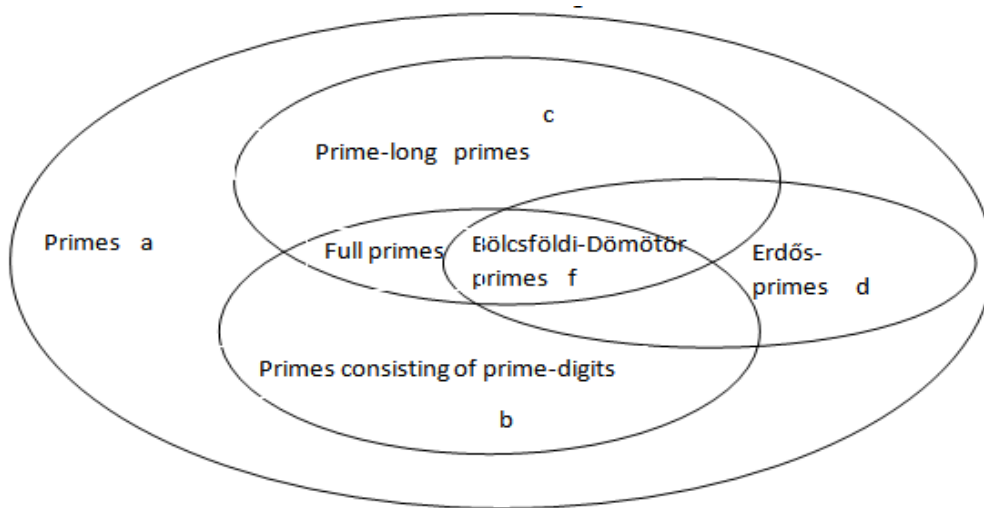
B(p) is the factual frequency of Bölcsföldi-Dömötör prime numbers in the interval $(10^{p-1}, 10^p)$. B(2)=1,B(3)=2,B(5)=12,B(7)=59,B(11)=2302,B(13)=20780,etc.
S(p) function gives the number of Bölcsföldi-Dömötör prime numbers in the interval $(10^{p-1},10^p)$. We think that

S(p)=0,9475x2,3^{p-1}, where p is prime. (1)

The factual number of Bölcsföldi-Dömötör primes and the number of Bölcsföldi-Dömötör primes calculated according to function (1) are as follows:

Number of digits p	The factual number of Bölcsföldi-Dömötör primes in the interval $(10^{p-1}, 10^p)$ B(p)	The number of Bölcsföldi-Dömötör primes according to function S(p)=0,9475x2,3^{p-1}	B(p)/S(p)
2	1	2,18	0.46
3	2	5,01	0,40
5	12	26,51	0,45
7	59	140,26	0,42
11	2302	3925,16	0,59
13	20780	20764,11	1,00

III. Number of the elements of the set of Bölcsföldi-Dömötör prime numbers [3], [9],[10], [11], [12].

Let's take the set of Mills' prime numbers!

Definition: The number m=[M ad 3^n] is a prime number, where M=1,3063778838630806904468614492602 is the Mills' constant, and n=1,2,3,… is an arbitrary positive integer number. It is already known that the number of the elements of the set of Mills' prime numbers is infinite. The Mills' prime numbers are the following: m=2, 11, 1361, 2521008887,…

The connection n→m is the following:1→2, 2→11, 3→1361, 4→2521008887,… The Mills' prime number m=[M ad 3^n] corresponds with the interval $(10^{m-1}, 10^m)$ and vice versa. For instance:2→(10, 10^2), 11→(10^{10},10^{11}), 1361→(10^{1360},10^{1361}), etc. and vice versa. The number of the elements of the set of Mills' prime numbers is infinite. As a consequence, the number of the intervals $(10^{m-1}, 10^m)$ that contain at least one Mills' prime number is infinite. The number of Bölcsföldi235 primes in the interval $(10^{m-1},10^m)$ is S(m)=0,9475x2,3^{m-1}. The number of Bölcsföldi-Dömötör prime numbers is probably infinite: lim B(p)=∞ is probably where p is prime. p→∞

IV. Conclusion

Countless different sets of special prime numbers have been known. We have found the following set of special prime numbers within the set of prime numbers. There may be further sets of special prime numbers that we do not know yet. Finding them will be task of researchers of the future.

Acknowledgements

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