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An Elite Particle Swarm Optimization Algorithm Based On Quadratic Approximations For High-Dimension Bilevel Single Objective Programming Problems

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Abstract: *Bilevel programming problem (BLP) is a nested optimization problem that contains one optimization task as a constraint to another optimization task. However, current existing algorithms for BLP often need enormous computational expense, which limit these algorithms to solve BLP only with smaller number of variables. In this paper, an elite particle swarm optimization based on quadratic approximations (PSO-QA) is proposed for solving the BLP, in which the elite strategy can efficiently prevent the premature convergence of the swarm and the quadratic approximations technology can further accelerate the convergence speed. Finally, we use the unconstrained test problems to measure and evaluate the proposed algorithm. The results suggest that the proposed algorithm can reduce the computational expense and improved the convergence speed.*

Keywords: *Bilevel single objective programming; quadratic approximations technology; particle swarm optimization*

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I. Introduction

The bilevel programming problem (BLP) is a nested optimizations problem with two levels in a hierarchy: the upper and lower level decision-makers. The upper level maker makes his decision firstly, followed by the lower level decision maker. The objective function and constraint of the upper level problem not only rely on their own decision variables but also depend on the optimal solution of the lower level problem. The decision maker at the lower level has to optimize his own objective function under the given parameters from the upper level decision maker, who, in return, with complete information on the possible reactions of the lower level, selects the parameters so as to optimize its own objective function. Since many practical problems, such as engineering design, management, economic policy and traffic problems, can be formulated as hierarchical problems, BLP has been studied and received increasing attention in the literatures. During the past decades, some surveys and bibliographic reviews were given by several authors [1–4]. Reference books on bilevel programming and related issues have emerged [5–8].

The bilevel programming problem is a nonconvex problem, which is extremely difficult to solve. As we know, BLP is a NP-Hard problem [9-11]. Vicente et al. [12] also showed that even the search for the local optima to the bilevel linear programming is NP-Hard. Even so, many researchers are devoted to develop the algorithms for solving BLP and propose many efficient algorithms. To date a few algorithms exist to solve BLP, it can be classified into four types: Karus-Kuhn-Tucker approach (KKT) [13-16], Branch-and-bound method [17], penalty function approach [18-21] and descent approach [22, 23]. The properties such as differentiation and continuity are necessary when proposing the traditional algorithms. Unfortunately, the bilevel programming problem is nonconvex. Thus, many researchers tend to propose the heuristic algorithms for solving BLP because of their key characteristics of minimal problem restrictions such as differentiation. Mathieu et al. [24] firstly developed a genetic algorithm (GA) for bilevel linear programming problem because of its good characteristics such as simplicity, minimal problem restrictions, global perspective and implicit parallelism. Motivated by the same reason, other kinds of genetic algorithm for solving bilevel programming were also proposed in [25–28]. Because of the prominent advantage that neural computing can converge to the equilibrium point (optimal solution) rapidly, the neural network approach was used to solve bilevel programming problem in [29–31]. Tabu search [32–34], simulated annealing [35], ant colony optimization [36] and λ -cut and goal-programming-based algorithm [37] are also typical intelligent algorithms for solving bilevel programming problem. Recently, Sinha et al. [38] proposed a nested bilevel evolutionary algorithm for BLP. However, it is worth noting that the most of the existing evolutionary procedures often need enormous computational expense, which limits their utility to solve bilevel optimization problems only with smaller number of variables.

Particle swarm optimization (PSO) is a relatively novel heuristic algorithm inspired by the choreography of a bird flock, which has been found to be quite successful in a wide variety of optimization tasks

[39]. Due to its high speed of convergence and relative simplicity, the PSO algorithm has been employed for solving BLP problems. For example, Li et al. [40] proposed a hierarchical PSO for solving BLP problem. Kuo and Huang [41] applied the PSO algorithm for solving bilevel linear programming problem. Jiang et al. [42] presented the PSO based on CHKS smoothing function for solving nonlinear bilevel programming problem. Gao et al. [43] presented a method to solve bilevel pricing problems in supply chains using PSO. Zhang et al. [44] presented a new strategic bidding optimization technique which applies bilevel programming and swarm intelligence. In addition, the hybrid algorithms based on PSO are also proposed to solve the bilevel programming problems [45-47]. Though the PSO algorithm has widely applications in optimization problems, the global convergence of the PSO cannot be guaranteed [48].

In this paper, an elite particle swarm optimization based on quadratic approximations (PSO-QA) is proposed for solving the BLP, in which the elite strategy can efficiently prevent the premature convergence of the swarm and the quadratic approximations technology can bring down the computational expense significantly and further accelerate the convergence speed. The rest of this paper is organized as follows. Sect.2 introduces the definitions and properties of bilevel programming problems. Sect.3 proposes the PSO-QA algorithm for BLP. We use the unconstrained test problems from the reference to measure and evaluate the proposed algorithm in Sect.4., while the conclusion is reached in Sect.5.

II. Formulation And Properties Of BLP

Let $x \in R^{n_1}$, $y \in R^{n_2}$, $F, f : R^{n_1} \times R^{n_2} \to R, G, g : R^{n_1} \times R^{n_2} \to R$. The optimistic formulation of BLP can be written as follows:

$$\min_{(x,y)} \ F(x,y)$$

$$s.t. \ \ G(x,y) \le 0$$

where y solves the following problem: (1.1)

$$\min_{y} \ f(x,y)$$

$$s.t. \ \ g(x,y) \le 0,$$

where $F(x,y)$ and $f(x,y)$ are the upper level and the lower level objective functions, respectively. $G(x,y)$ and $g(x,y)$ denote the upper level and the lower level constraints, respectively. $x \in R^{n_1}$ and $y \in R^{n_2}$ are the decision variables under the control of the upper and lower level problems, respectively. The problem (1) can be rewritten as follows:

$$\min_{(x,y)} \ \ F(x,y)$$

$$s.t. \ y \in \arg\min\{f(x,y), g(x,y) \le 0\} \qquad (1.2)$$

$$G(x,y) \le 0,$$

Definition 2.1. A point (x,y) is feasible if $(x,y) \in IR$.

Definition 2.2. A feasible point (x^*, y^*) is an optimal solution if $(x^*, y^*) \in IR$ and $F(x^*, y^*) \le F(\bar{x}, \bar{y})$, $\forall (\bar{x}, \bar{y}) \in IR$.

For problem (1), it is noted that a solution (x^*, y^*) is feasible for the upper level problem if and only if y^* is an optimal solution for the lower level problem with $x = x^*$. In practice, we often make the approximate optimal solutions of the lower level problem as the optimal response feedback to the upper level problem, and this point of view is accepted usually. Based on this fact, the PSO-QA algorithm may have a great potential for solving BLP. In the following, an algorithm based on the PSO-QA is presented for solving problem (1.1).

III. The Algorithm

In this sub-section, the elite PSO algorithm based on quadratic approximation is proposed for BLP. The elite PSO main means that the global optimal particle is selected from the elite set. The quadratic approximation technology used by [49] is employed in this paper. To begin with, the population of upper level is initialized randomly. For each member, the lower level optimization problem is solved using elite PSO and the optimal lower level solutions obtained. Based on the lower level optimal solutions, a quadratic function between the upper level variables and each lower level optimal variable is established. At each generation of the algorithm, a new quadratic function is generated which can improve the approximate optimal solution converges to the true optima. For each iteration, an approximate optimal solution for problem (1.1) is obtained and this procedure is repeated until the accurate optimal solutions of the original problem are found. The details of the proposed algorithm are given as follows:

Step 1. Initialization scheme. Initialize a random population (N_u) of the upper level variables. For each upper level member, perform a lower level optimization procedure to determine the corresponding optimal lower level variables using the elite PSO and evaluate the fitness value of the complete upper level solutions based on the upper level function and constraints.
Step 2. The upper level members that have undergone a successful lower level optimization run are assigned a number 1 and others as 0. Copy the members tagged 1 in an elite set A_t .
Step 3. Update the the upper level decision varia1bles using the simulated binary crossover operator (SBX) and the polynomial variation method (PM).
Step 4. If the numbers in the elite set is greater than $\frac{(\dim(x_u)+1)(\dim(x_u)+2)}{2}+\dim(x_u)$, and then selects all the members to construct quadratic functions $q_t(x_u)$, $t\in\{1,2,\cdots,\dim(x_l)\}$ to represent lower level optimal variables as a function of upper level variables. Otherwise, the quadratic approximation is not performed.
Step 5. If a quadratic approximation was performed in the previous step, find the lower level optimum for the offspring using the quadratic functions. And the mean squared error is less than ε , the offsprings are tagged as 1, otherwise they are tagged as 0. If a quadratic approximation was not performed in the previous step, execute lower level optimization runs use elite PSO for each offspring. Tag the offspring as 1 for which a successful lower level optimization is performed.
Step 6. Copy the tag 1 offsprings from the previous step to the elite set. After finding the lower level variables for the offsprings, choose r members from the parent population. A pool of chosen r members and offsprings is formed. The best r members from the pool replace the chosen r members from the population.
Step 7. Perform a termination check. If the termination check is false, go to step 3.

IV. Numerical Experiment

In this section, the parameters are set as follows: The PSO parameters are set as follows: $r_1, r_2 \in random(0,1),$ the inertia weight $w=0.7298$ and acceleration coefficients with. All results presented in this paper have been obtained on a personal computer (CPU:AMD Phenon(tm) ⅡX6 1055T 2.80GHz; RAM:3.25GB) using a $C^{\#}$ implementation of the proposed algorithm.
We performed 11 runs for 10-dimension problems. For the10-dimensional problems, the problems 1-5 we choose $p=5$, $q=5$ and $r=2$. Table 1 gives these problems and Table 2 provides the function evaluations at upper and lower levels. The accuracy of both levels and the number of lower level calls for 11 runs, as well as the average lower level function evaluations required per lower level call are reported in Table 3.

Table 1. Unconstrained test problems

No.	Problem	Best solutions
1	$\min\ F(x,y)=\sum_{i=1}^{p}x_i^2+\sum_{i=r+1}^{q}y_i^2+\sum_{i=1}^{r}(x_i-\tan y_i)^2$ $\min\ f(x,y)=\sum_{i=r+1}^{p}x_i^2+\sum_{i=r+1}^{q}y_i^2+\sum_{i=1}^{r}(x_i-\tan y_i)^2$ $x_i\in[-5,10]\quad i=1,2,\cdots,p$. $y_i\in[-\pi/2,\pi/2]\ i=1,2,\cdots,r$; $y_i\in[-5,10]\quad i=r+1,r+2,\cdots,q$	$F=225.0$ $f=100.0$
2	$\min\ F(x,y)=\sum_{i=1}^{p}x_i^2-\sum_{i=r+1}^{q}y_i^2-\sum_{i=1}^{r}(x_i-\log y_i)^2$ $\min\ f(x,y)=\sum_{i=r+1}^{p}x_i^2+\sum_{i=r+1}^{q}y_i^2+\sum_{i=1}^{r}(x_i-\log y_i)^2$ $x_i\in[-5,1]\ i=1,2,\cdots,r$; $x_i\in[-5,10]\quad i=r+1,r+2,\cdots,p$. $y_i\in(0,e]\ i=1,2,\cdots,r$; $y_i\in[-5,10]\quad i=r+1,r+2,\cdots,q$	$F=225.0$ $f=100.0$

3	$\min\ F(x,y)=\sum_{i=1}^{p}x_i^2-\sum_{i=r+1}^{q}y_i^2+\sum_{i=1}^{r}(x_i^2-\tan y_i)^2$ min $f(x,y)=\sum_{i=r+1}^{p}x_i^2+\sum_{i=r+1}^{q}(y_i^2-\cos 2\pi y_i)+\sum_{i=1}^{r}(x_i^2-\tan y_i)^2+q$ $x_i\in[-5,10]\quad i=1,2,\cdots,p$. $y_i\in[-\pi/2,\pi/2]\quad i=1,2,\cdots,r$; $y_i\in[-5,10]$ $i=r+1,r+2,\cdots,q$	$F=225.0$ $f=100.0$
4	$\min\ F(x,y)=\sum_{i=1}^{p}x_i^2-\sum_{i=r+1}^{q}y_i^2-\sum_{i=1}^{r}(\lvert x_i\rvert-\log(1+y_i))^2$ $\min\ f(x,y)=\sum_{i=r+1}^{p}x_i^2+\sum_{i=r+1}^{q}(y_i^2-\cos 2\pi y_i)$ $+\sum_{i=1}^{r}(\lvert x_i\rvert-\log(1+y_i))^2+q$ $x_i\in[-1,1]\ i=1,2,\cdots,r$; $x_i\in[-5,10]\ \ i=r+1,r+2,\cdots,p$. $y_i\in(0,e]\quad i=1,2,\cdots,r$; $y_i\in[-5,10]$ $i=r+1,r+2,\cdots,q$.	$F=225.0$ $f=100.0$
5	min $F(x,y)=\sum_{i=1}^{p}x_i^2-\sum_{i=r+1}^{q}((y_{i+1}-y_i^2)+(y_i-1)^2)-\sum_{i=1}^{r}(\lvert x_i\rvert-y_i^2))^2$ min $f(x,y)=\sum_{i=r+1}^{p}x_i^2+\sum_{i=r+1}^{q}((y_{i+1}-y_i^2)+(y_i-1)^2)+\sum_{i=1}^{r}(\lvert x_i\rvert-y_i^2))^2$ $x_i\in[-5,10]\quad i=1,2,\cdots,p$. $y_i\in[-5,10]$ $i=1,2,\cdots,q$.	$F=225.0$ $f=100.0$

Table 2.Tthe function evaluations for test problems from 11 runs

No.	The best FE LL	UL	The median FE LL	UL	The worst FE LL	UL
1	500687	837	427332(3.81)	2078 (1.19)	1630227	2606
2	300953	847	356467(4.21)	1082(2.07)	1176568	1673
3	191195	762	299216(4.72)	832(2.68)	1053279	1279
4	330158	511	478956(2.31)	878(1.86)	936204	1392
5	294595	873	324012(6.01)	949(3.03)	1392755	2170

Table 3. The accuracy for test problems from 11 runs

No.	The median UL accuracy	LL accuracy	The median LL calls	The average FE LL calls
1	0.003541	0.000661	1726	382.32
2	0.001050	0.000300	1515	386.45
3	0.006090	0.001060	1408	389.75
4	0.005460	0.001620	1271	415.42
5	0.000840	0.001920	23588	445.42

The fifth column and sixth column in Table 2 provides the median function evaluations required at the lower and upper levels respectively. From the resultS, it can be seen that the global convergence of the proposed algorithm is greatly improved. At lower level, it can bring down the computational expense significantly and further accelerate the convergence speed.

V. Conclusion

In this paper, the PSO-QA is proposed to solve the bilevel programming problem (BLP) in which the elite strategy can efficiently prevent the premature convergence of the swarm and the quadratic approximations technology can further accelerate the convergence speed. We use the unconstrained test problems to measure and evaluate the proposed algorithm. The results suggest that the proposed algorithm can reduce the computational expense and improved the convergence speed.

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