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A Mathematical Modeling of Double Exponential Voltage – Waveshape (Impulse Generator) For Power Substations Using Laplace-Transform

[1]s. L. Braide, [2]e.J. Diema
[1]Department of Electrical/Computer Engineering Rivers State University, Port Harcourt, Rivers State Nigeria
[1]Department of Electrical/Computer Engineering Rivers State University, Port Harcourt, Rivers State Nigeria
Corresponding Auther: s. L. Braide

Abstract: *This paper presented a framework of mathematical model of double exponential voltage waveshape for purpose of generation of impulse voltages and currents. An impulse voltage is a unidirectional voltage which, without appreciable oscillations, rises rapidly to a maximum value and falls more or less rapidly to zero. The maximum value is called the peak value of the impulse and the impulse voltage is specified by this value. Small oscillations are tolerated provided that their amplitude is less than 5% of the peak value of the impulse voltage. An impulse voltage develops without causing flash over or puncture, is called a full impulse voltage, but if flash over or puncture occur thus causing sudden collapse of the impulse voltage is called a chopped impulse voltage. A full impulse voltage is characterized by its peak value and its two time interval, the wave – front the wave-tail time intervals. The wave front time of an impulse wave is the time taken by the wave to reach to its maximum value starting from zero value. It is difficult to identify the start and peak points of the wave, hence the wave front time is specified as 1.25 times* $(t_2 – t_1)$*, where* t_2 *is the time for the wave to reach to its 90% of the peak value and* t_1 *is the time to reach 10% of the peak value. Since* $(t_2 – t_1)$*, represents about 80% of the wavefront time, it is multiplied by 1.25 to give total wave front time. Evidently, some basic concept of circuit analysis including Laplace transform principle were used to solve for the double exponential voltage wave-shape, formulated from the circuit analysis. The solutions of the formulated problems represents the model for the exponential growing voltage/currents wave shape representatively. Some specific assumptions were made in analyzing this models. The results from the analysis shows an exponential increase in voltage values with respect to time which represents the growth of the voltage pattern. This result are were represented on graph using matrix-laboratory (Matlab). Most importantly, the protection of power substations against lighting over voltages is critical reliable operation of electrical network since a frequent initiation of lighting surges etc without adequate moderation of the developed overvoltage may serious damage equipment, facilities etc resulting in power supply interruptions. This means that this paper will analyse time the voltage profile with respect to time in is order to access at what point the voltage is critical not adequate for power system equipments.*

Keywords: *Double exponential, impulse generator, transient condition, voltage wave-shape, substation, over-voltage*

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I Introduction

The protection of power substations against lighting over-voltage is a major concern for power system planner and operators. Since natural occurrences of atmospheric surges are liable for causing serious damages of power system equipment, facilities resulting into interruptions[1].

Lighting can cause huge damage to low voltage installations/electronics circuits, especially in medium and high rise building structures and communications lines[2].

Analysis of lighting performance of substations are necessary in order to calculate the expected over-voltages and take the appropriate measures. Considering various factors such as: the installation position of surge arrestors and the length of the underground cables. The analysis of this paper will be useful for practicing engineers and electrical power utilities for the improvement of the lighting performance of existing substations. The basic parts of a typical substations are the power transformer, the incoming overhead transmission lines, the cables, the circuit breakers, the current transformers, the disconnecting switches. External over-voltages can cause several damages to a substations, leading to insulation breakdown, a series of activities of power interruptions and the concern for safety to the personnel are always necessary[3]..

Lighting surges due to natural phenomenon may cause dangerous electronmagnetic interferances problem to low voltage system especially in electro-magnetic intereference problems to low voltage systems and particularly to electronics devices[4]..

Considering the implication of high cost of electrical installations. Importantly, lighting surges that impinge on the overhead transmission lines that are connected with the substations are the main cause of overvoltage stresses of the insulation of the substation's equipment. [5]. If lighting is directly on the phase conductors of the transmission lines, two traveling waves may appears, the magnitude of which depends on the lighting peak current and the surge impendance of the conductors.

The overvoltage wave will arrives at the entrance of the substation and can result in several serious damages of the components of the substations[6]..

The analysis and effect of over voltage due to lighting

When the activities of lighting (surge) hits on tower or ground wire of an overhead transmission line, the magnitude of developed over-voltages will seriously depends on the rise time of the injected light current; the induction of tower, tower footing resistance[8]..

Evidently, if the initiation of over-voltage exceed the insulation level of the line then a problem of black flash over will occur, and this may result into propagation of surges to the connected substation resulting into serious mal-operations of the system, thereby destroying the power equipments and facilities in substations. [9].

Back flashover as a result of mismatches between installed insulation level of the line and the developed overvoltage on transmission line is a common substation equipment faults occurrence, which need to be moderated with power electronic controller to take care of excessive overvoltage stresses at all time.

Analysis for improving the developed lighting surges (over-voltages)

The activities of lighting performance of the system can be improved by:

- Installing surge arrestors in parallel with the insulators and the primary side of the transformer.

Because surge arrestor will divert the current of the lighting strike to the earth and restrain the voltage at the terminals of the equipment. This is in line with developed overvoltage across the insulators of the transmission line exceeding their dielectric strength as compared to the flashover. [11].

Directly lighting hit due overvoltage

Substation, are adequately protected against direct lighting hits, in accordance to electromagnetic and electrogeometrical models and equations[12]..

That is lighting surges that hits on overhead transmission lines that are current with the substation are the main problems of overvoltage (voltage stresses) of the insulation of the substation equipments.

For examples, lighting hits is directly on phase conductors of transmission lines, two travelling wave appear: the magnitude of which depends on the lighting peak current and surge impendance of the conductors[7]..

Critical factors that determine the efficiency of lighting protection system

The initiation of voltage wave will arrive at the entrance of the substations, and this may result into several damages of the substation components, if not moderated immediately. That is, the developed 'overvoltage' is not depended by protection measure, tower footing resistance only.

Evidently, the critical factors to be considered includes: the lenght of cable, the position of lighting hit, the position of the installation of the arrestors, grounding resistance, the characteristics (behaviour) of the surge arrestor implemented. These actually determine the efficiency of the lighting protection system and affect the lighting performance of the substations. [10].

Similarly, if surge arrestor is installed between phase and earth of the transmission-line part of the lighting current will be diverted to the grounding system, depending on the nature of grounding resistance. The ground resistance low values ensure that almost the total currents will pass through the arrestor, and the developed over-voltages will not exceed the insulation levels of the system.

Indirect lighting hit due to over voltage

When a lighting strike, hits on the ground wire or the times of an overhead transmission line, the magnitude of the developed over-voltages depends on the time footing resistance, the induction of the time and the rise-time of the lighting current[13]..

Eventually, if the overvoltage exceeds the insulation level of the line, a back flashover is occurred. This may results into surge propagation to the connected substation and may lead to serious mal-operation of the power system.

Therefore, the lighting performance can be improved by installing surge arrestors in parallel with the insulators at the primary side of the transformer. Surge arrestor which will divert the currents of the lighting strike to earth and taken the voltage at the terminals of the equipment.

When lighting of various degree (35KA, 40KA..... etc) hits a phase conductor of a 132KV overhead transmission line. Then the resulting developed voltage surge will travels through the conductor and cable to the substation's transformer (132/33KV). The developed overvoltage at the beginning (position -1) and at the end (position - 2) of the cable can be determined using E-tap (Electrical transient analyzer) tool. [14]

Eventually, as far as the length of cable of (position 1&2) is concerns, the longer the cable will result into reduction of the 'developed over-voltages'. [15].

This also means that position of the arrestor to be installed is a great necessity for adequate and sufficient lighting performance of the substations. Hence, increasing the distance between arrestor and transformer will reduces the effectiveness of the installed arrestors. [16].

II Materials And Methods

Analysis Of Impulse Generator

The analysis and evaluation of impulse circuits elements are shown in figure 1.

- The simplified form of practical impulse generator given as:

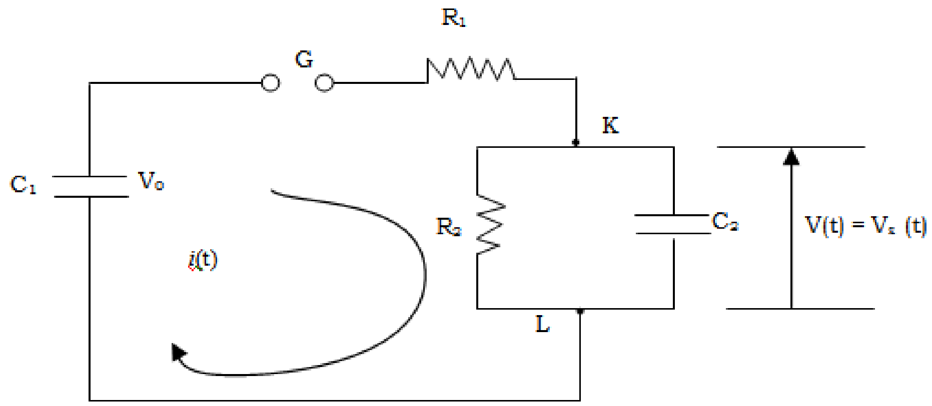


Fig. 1: Simple Circuit of an Impulse generator

The impedance (z) of the circuit can be analyzed using Laplace transform, then the impedance of the circuit can be written in the form of:

$$Z = R_1 + \frac{1}{C_1} + \left(R_2 // C_2\right) \qquad (1)$$

or

$$Z = R_1 + \frac{1}{C_1} + \frac{1}{R_2} + C_2 \qquad (2)$$

$$Z = R_1 + \frac{1}{C_1} + \frac{1}{Z_1} \qquad (3)$$

Where:

$$Z_1 = \left({}^{1}\!/_{R_2} + C_2 \right) = {}^{R_2}\!/_{1 + R_2 C_2} \qquad (4)$$

$$Z = R_1 + \frac{1}{C_1} + {}^{R_2}\!/_{1 + R_2 C_2} \qquad (5)$$

Taking Laplace's transform of equation (5):

Remember, Laplace's rely strongly on time vary function

$$Z(s) = \frac{R_1}{1} + \frac{1}{C_1(s)} + \frac{R_2}{R_2 C_2(s) + 1} \qquad (6)$$

Simplifying the equation further, we have:

Z(s) = $\frac{R_1}{1} + \frac{1}{C_1(s)} + \frac{R_2}{R_2C_2(s)+1}$ = $\frac{R_1C_1(s)(R_2C_2(s)+1)+R_2C_2(s)+1+R_2C_1(s)}{C_1(s)(R_2C_2(s)+1)}$ (7)

or

Z(s) = $\frac{R_1R_2C_1C_2S^2+R_1C_1(s)+R_2C_2(s)+1+R_2C_1(s)}{C_1(s)(R_2C_2(s)+1)}$ (8)

But, V_0 = i(t) z (9)

Taking equation (9) into Laplace's transform

V_0 = I(s) Z(s) (10)

- Since, currents (I) is a time varying functions
- Impedance (z), is a time-varying functions because it depends on: resistor R_1, capacitors, c and Inductor L
- Therefore to enable Prof. Laplace to operate on time varying function (t) or variable into or transform to frequency or complex domain(s)

We have:

$$I(s) = \frac{V_o}{S} \times \frac{1}{Z(s)} \quad (11)$$

Substituting equation (8): Z(s); into equation (11) as:

Z(s) = $\frac{R_1R_2C_1C_2S^2+R_1C_1(s)+R_2C_2(s)+1+R_2C_1(s)}{C_1(s)(R_2C_2(s)+1)}$

Then,

I(s) = $\frac{V_o}{S} \times \frac{1}{\frac{R_1R_2C_1C_2S^2+R_1C_1(s)+R_2C_2(s)+1+R_2C_1(s)}{C_1(s)(R_2C_2(s)+1)}}$ (12) or

I(s) = $\frac{V_o}{S} \times \frac{1}{\frac{R_1R_2C_1C_2S^2+(R_1C_1+R_2C_2+R_2C_1)S+1}{C_1(s)(R_2(s)+1)}}$ (13)

or

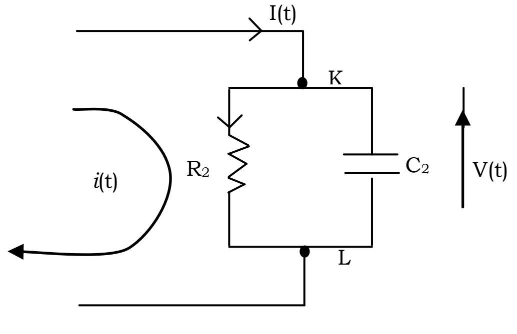
I(s) = $\frac{V_o}{S} \times \frac{C_1(s)(R_2(s)+1)}{R_1R_2C_1C_2S^2+(R_1C_1+R_2C_1)S+1}$ (14)

or

I(s)= $\frac{V_o}{SZ(s)} = \frac{V_o}{S} \times$

$$\frac{C_1(s)(R_2C_2(s)+1)}{R_1R_2C_1C_2S^2+(R_1C_1+R_2C_2+R_2C_1)S+1} \quad (15)$$

Considering the word section, KL,



$$\left.\begin{aligned} V &= I \times Zeq \quad &- \quad (a)\\ Zeq &= R_2 // C_2 \quad &- \quad (b)\\ Zeq &= \frac{R_2}{R_2C_2+1} \quad &- \quad (c)\end{aligned}\right\} \qquad (16)$$

This means, that equation (16):

$$V(t) = I_{(t)} \times Zeq \qquad (16)\ a$$

or

$$V(t) = I(t)\ \frac{R_2}{R_2C_2+1} \qquad (16)\ b$$

Taking equation (16), to Laplace's transform:

$$V(s) = I(S) \times \frac{R_2}{R_2C_2(s)+1} \qquad (17)$$

- Now substituting I(s) in equation (14) into equation (17) of V(s):

Where, $I(s) = \frac{V_o}{SZ(s)} = \frac{V_o}{S} \times$

$$\frac{C_1S(R_2C_2S+1)}{R_1R_2C_1C_2S^2+(R_1C_1+R_2C_1+R_2C_1)+1} \qquad (14)$$

This evidently becomes:

$$V(s) = \frac{R_2V_0C_1(R_2C_2S+1)}{R_2C_2(s)+1}$$

$$\frac{1}{R_1R_2C_1C_2S^2+(R_1C_1+R_2C_2R_2C_1)S+1} \qquad (18)$$

$$Vs = V_0R_2C_1 \times$$

$$\frac{1}{R_1R_2C_1C_2S^2+(R_1C_1+R_2C_2+R_2C_1)S+1} \qquad (19)$$

- Divide through by the coefficient of S^2

$$V_s = \frac{V_oR_2C_1}{(R_1R_2C_1C_2)S^2+} \times$$

$$\frac{1}{\left(\frac{R_1C_1+R_2C_2+R_2C_1}{R_1R_2C_1C_2}\right)\frac{S+1}{R_1R_2C_1C_2}} \qquad (20)$$

- Representing equation (20) into general quadratic expression as:

$$X = ax^2 + bx + C \qquad (21)$$

$$X = -b \pm \frac{\sqrt{b^2-4ac}}{2a} \qquad (22)$$

That is; $Vs = \frac{VoR_2C_1}{\underbrace{R_1R_2C_1C_2S^2}_{a}} \times$

$$\frac{1}{+\underbrace{\frac{R_1C_1+R_2C_2+R_2C_1}{R_1R_2C_1C_2}}_{b}S+\underbrace{\frac{1}{R_1R_2C_1C_2}}_{c}} \qquad (20')$$

Where:

Coefficient of x^2 = a = $\frac{V_oR_2C_1}{R_1R_2C_1C_2}$ (23)

Coefficient of x = b =

$$\frac{R_2C_1+R_2C_2+R_2C_1}{R_1R_2C_1C_2} \qquad (24)$$

Constants = C = $\frac{1}{R_1R_2C_1C_2}$ (25)

- The roots of the expression to the denominator are:

$$V_s=\left[-\frac{(R_1C_1+R_2C_2+R_2C_1)}{R_1R_2C_1C_2}\pm\right.$$

$$\frac{\sqrt{\left(\frac{R_1C_1+R_2C_2+R_2C_1}{R_1R_2C_1C_2}\right)^2-4ac}}{2a} \qquad (26)$$

or

$$V_s=½\left[\frac{-(R_1C_1+R_2C_2+R_2C_1)}{R_1R_2C_1C_2}\pm\right. \qquad \sqrt{\left(\frac{R_1C_1+R_2C_2+R_2C_1}{R_1R_2C_1C_2}\right)^2-4c} \quad (27)$$

or

$$V_s=½\left[\frac{-(R_1C_1+R_2C_2+R_2C_1)}{R_1R_2C_1C_2}\pm\right. \qquad \sqrt{\left(\frac{R_1C_1+R_2C_2+R_2C_1}{R_1R_2C_1C_2}\right)^2-\frac{4}{R_1R_2C_1C_2}} \qquad (28)$$

or

$$V_s=½\left[\frac{-(R_1C_1+R_2C_2+R_2C_1)}{R_1R_2C_1C_2}\pm\sqrt{\left(\frac{R_1C_1+R_2C_2+R_2C_1}{R_1R_2C_1C_2}\right)^2-\frac{4}{R_1R_2C_1C_2}}\right. \qquad (29)$$

$$V_s=½\left[\frac{-(R_1C_1+R_2C_2+R_2C_1)}{R_1R_2C_1C_2}\pm\right. \qquad (30)$$

Let, α = ½ $\frac{R_1C_1+R_2C_2+R_2C_1}{2R_1R_2C_1C_2}$

$$\sqrt{\left(\frac{R_1C_1+R_2C_2+R_2C_1}{R_1R_2C_1C_2}\right)^2-\frac{4}{R_1R_2C_1C_2}} \qquad (31)$$

and

Let, = β = ½ $\sqrt{\left(\frac{R_1C_1 + R_2C_2 + R_2C_1}{R_1R_2C_1C_2}\right)^2 \qquad - \frac{4}{R_1R_2C_1C_2}}$ (32)

From the expression of equation (30), the roots of the equation becomes:

$(-\alpha \pm \beta)$ or $(-\alpha \pm \beta)$ (32)

- Therefore, the roots are:
  (–α + β) and (–α – β), which can be expressed in terms of voltage (v);

Recalling equation (20)

$$Vs = \frac{VoR_2C_1}{R_1R_2C_1C_2S^2} \times \frac{1}{+\frac{R_1C_1 + R_2C_2 + R_2C_1}{R_1R_2C_1C_2} + \frac{1}{R_1R_2C_1C_2}} \qquad (20)$$

or

$$V_s = \frac{V_o}{R_1C_2} S^2 \times$$

$$\frac{1}{+\left(\frac{R_1C_1 + R_2C_2 + R_2C_1}{R_1R_2C_1C_2}\right)s + \frac{1}{R_1R_2C_1C_2}}$$

or

Similarly,

$$V_s = \frac{V_o}{R_1C_2} \times \frac{1}{(S + \alpha - \beta)(S + \alpha + \beta)} \qquad (34)$$

- Finding the partial fraction of the expression above:

$$\frac{1}{(S + \alpha - \beta)(S + \alpha + \beta)} = \frac{A}{(S + \alpha - \beta)} + \frac{B}{(S + \alpha + \beta)} \qquad (35)$$

$$\frac{1}{(S + \alpha - \beta)(S + \alpha + \beta)} = \frac{A(S + \alpha + \beta) + B(S + \alpha + \beta)}{(S + \alpha - \beta)(S + \alpha + \beta)} \qquad (36)$$

- Equating equations on the LHS and RHS, we have:

1 ≡ A(S + α + β) + B(S + α - β) (37)

Expanding we have:

1 ≡ AS +Aα + Aβ + BS + Bα - Bβ (38)

or

1 ≡ AS +BS + Aα + Bα - Aβ - Bβ (39)

or

1 ≡ (A +B)S + (A + B) α + (A - B)β (40)

- Equating coefficients:

Therefore the coefficient of [S] to LHS and RHS:

O = A + B (41)

or

A = - B (42)

Equating coefficient of constants terms; to LHS and RHS;

1 = (A + B)α + (A - β)β (40)

or

1 = (A + A)α + (A - A)β (43)

or

$1 = A\alpha - A\alpha + 2A\beta$ (44)

$1 = 2 A\beta$ (45)

or

$A = \dfrac{1}{2\beta}$ (46)

But A = -B or B = −A

or

$$\boxed{B = -\frac{1}{2\beta}} \quad (47)$$

Substituting these values of A and B, we have:

From (35):

$$Vs = \frac{V_o}{R_1C_2} \times \frac{1}{2\beta}\left(\frac{1}{S+\alpha-\beta}\right) - \frac{1}{2\beta}\left(S+\alpha-\beta\right)$$

or

$$Vs = \frac{V_0}{R_1C_2} \times \frac{1}{2\beta}\left[\frac{1}{S+(\alpha-\beta)} - \frac{1}{S+(\alpha-\beta)}\right] \quad (48)$$

- Since, the expression is in inverse, therefore we can take the inverse Laplace transform of equation (48), we have:

$$V(s) = \frac{V_0}{2\beta R_1C_2}\left[\frac{1}{S+(\alpha-\beta)} - \frac{1}{S+(\alpha-\beta)}\right] \quad (49)$$

Let, $V_n = \dfrac{V_0}{2\beta R_1 C_2}$ (50)

Then, equation (49), becomes:

$$V(t) = V_n\left[\ell^{-(\alpha-\beta)t} - \ell^{-(\alpha+\beta)t}\right] \quad (51)$$

Evidently, equation (51), represents the double – exponentials – voltage wave shape equation.

This is an expression of the difference between two exponentials equations.

- Let t_1 be the voltage wave front time,
- Let t_2 represents the wave tail time.
- This nears that, the characteristic roots of the voltage wave form (α and β) must have unique values, irrespective of the particular circuit being used.
- At time t_1, the slope of the voltage wave is zero, (0) this evidently means that, the mathematical relation, $\dfrac{dv}{dt}(t) = 0$
- From (51):

$$V(t) = V_n\left[\ell^{-(\alpha-\beta)t} - \ell^{-(\alpha+\beta)t}\right] \quad (51)$$

Then,

$$\frac{dv}{dt}(t) = V_n\left[-(\alpha-\beta)\ell^{-(\alpha-\beta)t} + \ell^{-(\alpha+\beta)t}\right] = 0 \quad (52)$$

$$\text{Slope} = \frac{dv}{dt}(t) = 0 \quad (52)$$

Therefore, equation (52) becomes:

$$V_n\left[-(\alpha-\beta)\ell^{-(\alpha-\beta)t} + (\alpha+\beta)\ell^{-(\alpha+\beta)t}\right] = 0 \quad (52')$$

or

$$-V_n(\alpha-\beta)\ell^{-(\alpha-\beta)t} = -V_n(\alpha+\beta)\ell^{-(\alpha+\beta)t} \quad (53)$$

or

Simplifying eqn. (53):

$$(\alpha-\beta)\ell^{-(\alpha-\beta)t} = (\alpha+\beta)\ell^{-(\alpha+\beta)t} \quad (54)$$

or

$$(\alpha+\beta)\ell^{-(\alpha+\beta)t} = (\alpha-\beta)\ell^{-(\alpha-\beta)t} \qquad (55)$$

Simplifying further:

$$\frac{(\alpha+\beta)}{(\alpha-\beta)} = \frac{\ell^{-(\alpha-\beta)t}}{\ell^{-(\alpha+\beta)t}} \qquad (56)$$

or

This means that;

$$\frac{(\alpha+\beta)}{(\alpha-\beta)} = \ell^{-\alpha t+\beta t+\alpha t+\beta t} \qquad (57)$$

or

$$\frac{(\alpha+\beta)}{(\alpha-\beta)} = \ell^{2\beta t} \qquad (58)$$

Taking natural log to both side of equation (58):

$$\ln\frac{(\alpha+\beta)}{(\alpha-\beta)} = \ln\left(\ell^{2\beta t}\right) \qquad (59)$$

or

$$\ln\frac{(\alpha+\beta)}{(\alpha-\beta)} = 2\beta t \qquad (60)$$

or

$$2\beta t = \ln\frac{(\alpha+\beta)}{(\alpha-\beta)} \qquad (61)$$

For t = t_1, we have:

$$2\beta t_1 = \ln\frac{(\alpha+\beta)}{(\alpha-\beta)} \qquad (62)$$

Then,

$$t_1 = \frac{1}{2\beta} \ln\frac{(\alpha+\beta)}{(\alpha-\beta)} \qquad (63)$$

Hence, the peak value of the voltage in equation (51) given as:

$$V_{m(t)} = V_n\left[\ell^{-(\alpha-\beta)t_1} - \ell^{-(\alpha+\beta)t_1}\right] \qquad (64)$$

Similarly, we can also find value of t_2, since t_1 is already known.

Let t = t_2

Assume that, the voltage wave form (v) is half of what, it is when $t = t_1$

This implies that the voltage waveform at $t = t_2$ is equal to half the voltage waveform at $t = t_1$

Then

$$V_n\left[\ell^{-(\alpha-\beta)t_2} - \ell^{-(\alpha+\beta)t_2}\right] = \frac{1}{2}\times V_n\left[\ell^{-(\alpha-\beta)t_1} - \ell^{-(\alpha+\beta)t_1}\right]$$

- It is realized that from equation (63) and (65), it is possible to obtain the desired values of α and β for the required voltage – wave shape (which means, that the shape of voltage depends on α and β).

Assumption:

For purpose of simplification and calculation, the second term on the LHS of equation (65) is much smaller, than the first-term of the LHS – equation, therefore, we can neglect it, therefore:

$$V_n\ell^{-(\alpha-\beta)t_2} = \frac{1}{2}V_n \times \left[\ell^{-(\alpha-\beta)t_1} - \ell^{-(\alpha+\beta)t_1}\right] \qquad (66)$$

- Simplifying equation (66):

$$2\ell^{-(\alpha-\beta)t_2} = \ell^{-(\alpha-\beta)t_1} - \ell^{-(\alpha+\beta)t_1}$$

- From equation (65), we can relate t_2 and t_1, hence it can also be stated mathematically as:

$t_2 \;\alpha\; t_1$ 68)

or

$t_2 = kt_1$ (69)

- Substitute, $t_2 = kt_1$ into equation (66), we have:

$$\ell^{-(\alpha-\beta)kt_1} = \frac{1}{2}\left[\ell^{-(\alpha-\beta)t_1} - \ell^{-(\alpha+\beta)t_1}\right] \qquad (70)$$

or $2\ell^{-(\alpha-\beta)kt_1} = \ell^{-(\alpha-\beta)t_1} - \ell^{-(\alpha+\beta)t_1}\Big]$(71)

- Multiply both side of the equation (71) by: $\ell^{(\alpha-\beta)t_1}$

$$2\ell^{-(\alpha-\beta)kt_1} \times \ell^{(\alpha-\beta)t_1} =$$

$$\left[\ell^{-(\alpha-\beta)t_1} - \ell^{(\alpha+\beta)t_1}\right] \times \ell^{(\alpha-\beta)t_1} \qquad (72)$$

or

$$2\ell^{-(\alpha-\beta)kt_1} \times \ell^{(\alpha-\beta)t_1} = \ell^{-(\alpha-\beta)t_1} \times \ell^{(\alpha-\beta)t_1} - \ell^{-(\alpha+\beta)t_1} \times \ell^{(\alpha-\beta)t_1} \qquad (73)$$

or

$$2\ell^{-(\alpha-\beta)t_1(k-1)} = \ell^{-(\alpha-\beta)t_1+(\alpha-\beta)t_1} - \ell^{-(\alpha+\beta)t_1+(\alpha+\beta)t_1} \qquad (74)$$

or

$$2\ell^{-(\alpha-\beta)t_1(k-1)} = \ell^{0} - \ell^{(-\alpha t_1-\beta t_1+\alpha t_1-\beta t_1)} \qquad (75)$$

This implies that, $2\ell^{-(\alpha-\beta)(k-1)t_1 =}$

Divide both side by: $1-\ell^{-2\beta t_1}$

This implies that,

$$\frac{2\ell^{-(\alpha-\beta)(k-1)t_1}}{1-\ell^{-2\beta t_1}} = \frac{1-\ell^{-2\beta t_1}}{1-\ell^{-2\beta t_1}} \qquad (76)$$

or

$$\frac{2}{1-\ell^{-2\beta t_1}} \times \ell^{-(\alpha-\beta)(k-1)t_1} = 1 \qquad (77)$$

Similarly divide both of the equation by $\ell^{-(\alpha-\beta)(k-1)t_1}$ to both side:

This implies,

$$\frac{2}{1-\ell^{-2\beta t_1}} \times \frac{\ell^{-(\alpha-\beta)(k-1)t_1}}{\ell^{-(\alpha-\beta)(k-1)t_1}} = \frac{1}{\ell^{-(\alpha-\beta)(k-1)t_1}} \qquad (78)$$

or

$$\frac{2}{1-\ell^{-2\beta t_1}} = \frac{1}{\ell^{-(\alpha-\beta)(k-1)t_1}} \qquad (79)$$

- Taking natural logarithms to both of equation (79):

$$\ln\left(\frac{2}{1-\ell^{-2\beta t_1}}\right) = \ln\left(\frac{1}{\ell^{-(\alpha-\beta)(k-1)t_1}}\right) \qquad (80)$$

or

$$\ln\left(\frac{2}{1-\ell^{-2\beta t_1}}\right) = +(\alpha-\beta)(k-1)t_1 \qquad (81)$$

or

$$(\alpha-\beta)(k-1)t_1 = \ln\left(\frac{2}{1-\ell^{-2\beta t_1}}\right) \qquad (82)$$

- Divide both side of the equation by $(k-1)t_1$ of equation (82):

$$\frac{(\alpha-\beta)(k-1)t}{(k-1)t_1}\ _1 = \frac{1}{(k-1)t_1} \times I_n\left(\frac{2}{1-\ell^{-2\beta t_1}}\right)$$

or

$$(\alpha-\beta) = \frac{1}{t_1(k-1)} I_n\left(\frac{2}{1-\ell^{-2\beta t_1}}\right) \qquad (84)$$

Condition 1: for determination of (α - β)

If the term: $2\beta t_1$ in the expression of equation (84) is greater than four (or $2\beta t_1 > 4$), then this mean that $2\beta t_1 = 5$ or more

then,

$$\alpha-\beta = \frac{1}{t_1(k-1)} \ln\left(\frac{2}{1-\ell^5}\right) \qquad (85)$$

or

$$\alpha-\beta = \frac{1}{t_1(k-1)} \times \ln\left(\frac{2}{1-2.718^5}\right) \qquad (86)$$

or

$$\alpha-\beta = \frac{1}{t_1(k-1)} \times$$

$$\ln\left(\frac{2}{1-0.006741441\times10^3}\right) \qquad (87)$$

or

$$\alpha-\beta = \frac{1}{t_1(k-1)} \times \ln\left(\frac{2}{0.99325856}\right) \qquad (88)$$

or

$$\alpha-\beta = \frac{1}{t_1(k-1)} \ln\left(2.013574391\right) \qquad (89)$$

$$\alpha-\beta = \frac{1}{t_1(k-1)} \times 0.699 \qquad (90)$$

or

$$\alpha-\beta = \frac{0.7}{t_1(k-1)} \qquad (91)$$

- With this analysis and model,
- We can use equation (65) and equation (91) to determine, or estimates the approximate values of α and β respectively.

Recalled previous equations for analysis:

$$\left.\begin{array}{l} V_n\left[\ell^{-(\alpha-\beta)t_2} - \ell^{-(\alpha+\beta)t_2}\right] \\ \frac{1}{2}V_n\left[\ell^{-(\alpha-\beta)t_1} - \ell^{-(\alpha+\beta)t_1}\right] \\ \qquad \text{or} \\ V_n\left[\ell^{-(\alpha-\beta)kt_1} - \ell^{-(\alpha+\beta)kt_1}\right] = \\ \frac{1}{2}\times V_n\left[\ell^{-(\alpha-\beta)t_1} - \ell^{-(\alpha+\beta)t_1}\right] \end{array}\right\} \qquad (65)$$

When:

$$(\alpha-\beta) = \frac{0.7}{t_1(k-1)} \qquad (91)$$

Generation/simulation of voltage wave-form/shape with respect to different values of time in microseconds.

- Recalling equation (51): 'the double – exponential voltage waveform equations', we can generate different voltage values for voltage wave shape .

V(t) $= V_n\left[\ell^{-(\alpha-\beta)t} - \ell^{-(\alpha+\beta)^t}\right]$ - (51)

- Given, the standard test – data from (International electro-technical commission for values of α and β corresponding to some typical waveshape as forms are:

Waveform	α	β
0.5/5	4.080	3.922
1/5	1.557	1.366
1/10	2.040	1.961
1/5/40	1.776	1.757
1/50	3.044	3.029

Case 1:

- From table 1, we can generate, values for 1/50 microsecond – impulse waveform using – the double – exponential equation (50):

V(t) $= V_n\left[\ell^{-(\alpha-\beta)t} - \ell^{-(\alpha+\beta)^t}\right]$ - (51)

At 1/50 microsecond impulse wave correspond to: α = 3.044, β = 3.029, let V_n = 1.01749

V(t) = 1.01749 $\left[\ell^{-(3.044-3.029)t} - (3.044-3.029)t\right]$

V(t) = 1.01749 $\left[\ell^{-0.015t} - \ell^{-6.073t}\right]$

V(t) = 1.01749 $\left[\ell^{-0.015t} - \ell^{-6.075t}\right]$

- For 0.1 microsecond, we have:

$V(0.1_{ms})$ = 1.01749 $\left[2.719^{-0.015\times0.1} - 2.719 -^{6.073} \times 0.1\right]$

$V(0.1_{ms})$ = 1.01749 $\left[2.719^{-0.0015} - 2.719 -^{0.6.073}\right]$

$V(0.1_{ms})$ = 1.01749 $\left[0.998500 - 0.54473\right]$

$V(0.1_{ms})$ = 1.01749 $\left[0.45377\right]$

$V(0.1_{ms})$ = 0.4617064

Similarly for (1.0 microsecond),

We have:

V(1.0ms) = 1.0174 $\left[2.719^{-0.015\times1..0} - 2.719^{-6.073\times1.0}\right]$

V(0.1ms) = 1.01749 $\left[0.9851080 - 0.00230055\right]$

V(0.1ms) = 1.01749 [0.98280745]

= 0.99990 ≈ 1.00

At t = 1.0ms, the voltage (*V)* is maximum

Then,

At *t* = 50 micro second

or

V(t) = 1.0174 $\left[2.719^{-0.015t} - 2.719^{-6.073t}\right]$

V(50) = 1.0174 $\left[2.719^{-0.015\times50} - 2.719^{-6.073\times50}\right]$

or

V(50) = 1.0174 $\left[2.719^{-0.75} - 2.719^{-30365}\right]$

or

V(50) = 1.0174 $\left[0.4722729 - 0\right]$

or

V(50) = 1.0174 $\left[0.4722729\right]$

or

V(50) = 0.480490

At t = 50 microsecond, the voltage is 0.480490, which represent: $\frac{48}{100}$ of the maximum

Value (1.00) gives = $\frac{48}{100} \times 1.00 = 0.480$

Analysis of Determining the Parameter For Voltage Wave Shape Behaviour:

- Condition consideration:

- For certain impulse generator, the impulse capacitance C_1, is farad while, the load capacitance C_2, will vary depending upon the equipment to be tested.
- The ratio of these capacitance, that is (C_1/C_2) plays, the design role for impulse generator.
- Recalling from our previous equations, of characteristics roots (α and β); equation (29) and (32):

$$\alpha = \frac{1}{2}\frac{R_1C_1 + R_2C_2 + R_2C_1}{R_1R_2C_1C_2} \quad (29)$$

or

$$\alpha = \frac{1}{2}\ \frac{R_1C_1}{R_1R_2C_1C_2} + \frac{R_2C_1}{R_1R_2C_1C_2}$$

$$+\frac{R_2C_1}{R_1R_2C_1C_2} \quad (92)$$

$$\alpha = \frac{1}{2}\left[\frac{1}{R_2C_2} + \frac{1}{R_1C_1} + \frac{1}{R_1C_2}\right] \quad (93)$$

or

$$\alpha = \frac{1}{2}\left[\frac{1}{R_1C_1} + \frac{1}{R_2C_2} + \frac{1}{R_1C_2}\right] \quad (94)$$

- Taking square to both side, we have:

$$\alpha^2 = \frac{1}{4}\left(\frac{1}{R_1C_1} + \frac{1}{R_2C_2} + \frac{1}{R_1C_2}\right)^2 \quad (95)$$

- Similarly, we can also recalled equ. 32 as:

$$\beta = \frac{1}{2}\sqrt{\left(\frac{R_1C_1 + R_2C_2 + R_2C_1}{R_1R_2C_1C_2}\right)^2 \frac{4}{R_1R_2C_1C_2}} \quad (32)$$

- Squatting both side of the equation, we have:

$$\beta^2 = \frac{1}{4}\left[\left(\frac{R_1C_1 + R_2C_2 + R_2C_1}{R_1R_2C_1C_2}\right)^2 - \frac{4}{R_1R_2C_1C_2}\right]$$

or

$$\beta^2 = \frac{1}{4}\left[\left(\frac{R_1C_1 + R_2C_2 + R_2C_1}{R_1R_2C_1C_2}\right)^2 - \frac{1}{4}\left(\frac{4}{R_1R_2C_1C_2}\right)\right]$$

or

$$\beta^2 = \left(\frac{R_1C_1 + R_2C_2 + R_2C_1}{R_1R_2C_1C_2}\right)^2 - \left(\frac{1}{R_1R_2C_1C_2}\right) \quad (97)$$

or

- This means that the roots of the equation $(\alpha^2 - \beta^2)$ becomes:

$$X^2 - \beta^2 = \frac{1}{4}\left(\frac{1}{R_1C_1} + \frac{1}{R_2C_2} + \frac{1}{R_1C_2}\right) - \frac{1}{4}$$

$$\left(\frac{R_1C_1 + R_2C_2 + R_2C_1}{R_1R_2C_1C_2}\right)^2 + \frac{1}{R_1R_2C_1C_2} \quad (99)$$

- Simplifying further, we have:

$$\alpha^2 - \beta^2 = \frac{1}{R_1 R_2 C_1 C_2} \qquad (100)$$

or

$$\frac{1}{R_1 R_2 C_1 C_2} = \alpha^2 - \beta^2 \qquad (101)$$

- Simplifying further;

$$\frac{1}{R_1 C_2} = R_2 C_1 \left(\alpha^2 - \beta^2\right) \qquad (102)$$

From our previous equation 49

$$V(s) = \frac{V_o}{2\beta R_1 C_2}\left[\ell^{-(\alpha-\beta)t} - \ell^{-(\alpha+\beta)}t\right] \qquad (49)$$

or

$$V(s) = \frac{1}{R_1 C_2} \bullet \frac{V_o}{2\beta}\left[\ell^{-(\alpha-\beta)t} - \ell^{-(\alpha+\beta)}t\right] \qquad (103)$$

From equa: (102), substitute, $\frac{1}{R_1 C_2} = R_2 C_1\left(\alpha^2 - \beta^2\right)$ V(s) =

$$R_2 C_1 \bullet \frac{V_o}{2\beta}\left(\alpha^2 - \beta^2\right)\left[\ell^{-(\alpha-\beta)t} - \ell^{-(\alpha+\beta)}t\right] \qquad (104)$$

or

$$V(s) = V_o R_2 C_1 \frac{\left(\alpha^2 - \beta^2\right)}{2\beta}\left[\ell^{-(\alpha-\beta)t} - \ell^{-(\alpha+\beta)}t\right]$$

- From Previous equations: (94), (100), (102), (102)

$$\alpha = \frac{1}{2}\left[\frac{1}{R_1 C_1} + \frac{1}{R_2 C_2} + \frac{1}{R_1 C_2}\right] \qquad (94)$$

$$\alpha^2 - \beta^2 = \frac{1}{R_1 R_2 C_1 C_2} \qquad (100)$$

$$\frac{1}{R_1 C_2} = R_2 C_1\left(\alpha^2 - \beta^2\right) \qquad (102)$$

or

$$\frac{1}{R_2 C_2} = R_1 C_1\left(\alpha^2 - \beta^2\right) \qquad (102)a$$

$$\frac{1}{R_1 C_1} = R_2 C_2\left(\alpha^2 - \beta^2\right) \qquad (102)b$$

Substituting equation (100), (102), (102), (102)a, (102) b into equation (94):

$$\alpha = \frac{1}{2}\left[R_2 C_2\left(\alpha^2 - \beta^2\right) + \frac{1}{R_2 C_2} + R_2 C_1\left(\alpha^2 - \beta^2\right)\right] \qquad (106)$$

or

But, $= \frac{C_1}{C_2}$ or $C_2 = \frac{C_1}{x}$ (107)

$$\alpha = \frac{1}{2}\left[\left(\alpha^2 - \beta^2\right)R_2 C_2 + \frac{1}{R_2 C_2} + R_2 C_1\left(\alpha^2 - \beta^2\right)\right] (107)$$

or

$$\alpha = \frac{1}{2}\left[\left(\alpha^2 - \beta^2\right)R_2\frac{C_1}{x} + \frac{1}{R_2 \bullet \frac{C_2}{x}} + R_2C_1\left(\alpha^2 - \beta^2\right)\right] \text{(108)}$$

or

$$\alpha = \frac{1}{2}\left[\left(\alpha^2 - \beta^2\right)\frac{R_2C_1}{x} + \frac{x}{R_2C_1} + R_2C_1\left(\alpha^2 - \beta^2\right)\right] \text{(109)}$$

or

$$\alpha = \frac{1}{2}\left[\left(\alpha^2 - \beta^2\right)R_2C_1 \bullet \frac{1}{x} + R_2C_1\left(\alpha^2 - \beta^2\right) + \frac{1}{R_2C_1}\right] \text{(110)}$$

or

$$\alpha = \frac{1}{2}\left[\left(\alpha^2 - \beta^2\right)R_2C_1\left[\left(\frac{1}{x} + 1\right)\right] + \frac{x}{R_2C_1}\right] \text{(111)}$$

or

$$\alpha = \frac{1}{2}\left[\left(\alpha^2 - \beta^2\right)R_2C_1\left(\frac{x+1}{x}\right) + \frac{x}{R_2C_1}\right] \text{(112)}$$

or

$$\alpha = \frac{1}{2}\left[\left(\frac{x+1}{x}\right)\left(\alpha^2 - \beta^2\right)R_2C_1 + \frac{x}{R_2C_1}\right] \text{(113)}$$

Multiply equa. (113) by: $\frac{R_2C_1}{x}$

$$\alpha = \frac{1}{2}\left[\left(\frac{x+1}{x}\right)\left(\alpha^2 - \beta^2\right)R_2C_1 + \frac{x}{R_2C_1}\right]$$

or

$$\frac{R_2C_1}{x} \times \alpha = \frac{1}{2} \times \frac{R_2C_1}{x}$$

$$\left[\left(\frac{x+1}{x}\right)\left(\alpha^2 - \beta^2\right)R_2C_1 + \frac{x}{R_2C_1}\right] \qquad \text{(114)}$$

or

$$\alpha \times \frac{R_2C_1}{x} = \frac{1}{2} \times \frac{R_2C_1}{x} \times \frac{1}{x}(x+1)\left(\alpha^2 - \beta^2\right)$$

$$R_2C_1 + \frac{x}{R_2C_1} \times \frac{R_2C_1}{x} \qquad \text{(115)}$$

or

$$\frac{R_2C_1}{x} \times \frac{1}{2} = \frac{R_2C_1}{x^2}(x+1)\left(\alpha^2 - \beta^2\right)R_2C_1 + 1 \text{ (116)}$$

or

$$\alpha \times \frac{R_2C_1}{x} = \frac{1}{2}\left(\frac{R_2C_1}{x^2}\right)R_2C_1$$

$$(x+1)\left(\alpha^2 - \beta^2\right) + 1 \qquad \text{(117)}$$

$$\alpha \times \frac{R_2C_1}{x} = \frac{1}{2}\left(\frac{R_2C_1}{x^2}\right)^2(x+1)\left(\alpha^2 - \beta^2\right) + 1$$

or

$$2\alpha \times \frac{R_2C_1}{x} = \left(\frac{R_2C_1}{x^2}\right)^2 (x+1)(\alpha^2 - \beta^2) + 1$$

or

$$\left(\frac{R_2C_1}{x^2}\right)^2 (x+1)(\alpha^2 - \beta^2)$$

$$+ 2\alpha \frac{R_2C_1}{x} + 1 = 0 \qquad (120)$$

Divide through by: $(x+1)(\alpha^2 - \beta^2)$ to both side of equation (120).

$$\left(\frac{R_2C_1}{x}\right)^2 \frac{(x+1)(\alpha^2 - \beta^2)}{(x+1)(\alpha^2 - \beta^2)} - 2\alpha \frac{R_2C_1}{x}$$

$$\bullet \frac{1}{(x+1)(\alpha^2 - \beta^2)} +$$

$$\frac{1}{(x+1)(\alpha^2 - \beta^2)} \qquad (131)$$

or

$$\left(\frac{R_2C_1}{x}\right)^2 - 2\alpha \frac{R_2C_1}{x} \bullet \frac{1}{(x+1)(\alpha^2 - \beta^2)} + \frac{1}{(x+1)(\alpha^2 - \beta^2)} = 0$$

or

$$\left(\frac{R_2C_1}{x}\right)^2 = \frac{2\alpha}{(x+1)(\alpha^2 - \beta^2)} \bullet \frac{R_2C_1}{x} + \frac{1}{(x+1)(\alpha^2 - \beta^2)} = 0 \qquad (123)$$

$$\frac{2\alpha}{(x+1)(\alpha^2 - \beta^2)} = \frac{\alpha}{(x+1)(\alpha^2 - \beta^2)} + \frac{\alpha}{(x+1)(\alpha^2 - \beta^2)} = 0 \qquad (124)$$

Simplifying further; equa. (123):

$$\left(\frac{R_2C_1}{x}\right)^2 = \frac{2\alpha}{(x+1)(\alpha^2 - \beta^2)} \bullet \frac{R_2C_1}{x} + \frac{1}{(x+1)(\alpha^2 - \beta^2)} = 0$$

$$ax^2 \uparrow \quad + \quad b\uparrow \quad \bullet \quad x \quad + \quad \uparrow c$$

$$= 0 \qquad (123)$$

We can represent equ. (123) in the form general quadratic equation as:

$ax^2 + bx + C = 0$

or

$$x = - \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

Where:

a $= 1,\ x^2 = \left(\frac{R_2C_1}{x}\right)^2,\ b = \frac{-2\alpha}{(x+1)(\alpha^2 - \beta^2)}$

x $= x^2 = \frac{R_2C_1}{x},\qquad C = \frac{-2\alpha}{(x+1)(\alpha^2 - \beta^2)}$

Substituting the coefficient into the quadratic equation (125).

We have:

$$x = - \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = R_2C_1 = -\left(-\frac{2\alpha}{(x+1)(\alpha^2 - \beta^2)}\right) \pm$$

$$\sqrt[2]{\frac{\left(\frac{-2\alpha}{(x+1)(\alpha^2-\beta^2)}\right)^2-\frac{1}{(x+1)(\alpha^2-\beta^2)}}{2\times 1}}$$

or

$$\frac{R_2C_1}{x}=\frac{2\alpha}{(x+1)(\alpha^2-\beta^2)}\pm\frac{2\alpha}{(x+1)(\alpha^2-\beta^2)}$$

$$-\frac{2}{2}\left(\frac{2x1}{(x+1)(\alpha^2-\beta^2)}\right)\frac{1}{2} \qquad (127)$$

$$\frac{L_2C_1}{x}=\frac{2\alpha}{(x+1)(\alpha^2-\beta^2)}\pm\frac{2\alpha}{(x+1)(\alpha^2-\beta^2)}-$$

$$\left(\frac{1}{(x+1)(\alpha^2-\beta^2)}\right)\frac{1}{2} \qquad (128)$$

or

$$\frac{L_2C_1}{x}=\frac{\alpha}{(x+1)(\alpha^2-\beta^2)}\pm\frac{\alpha}{(x+1)(\alpha^2-\beta^2)}- \qquad \left(\frac{1}{(x+1)(\alpha^2-\beta^2)}\right)\frac{1}{2} \qquad (129)$$

or

$$\frac{L_2C_1}{x}=\frac{2\alpha}{(x+1)(\alpha^2-\beta^2)}\pm$$

$$\left[\left(\frac{\alpha}{(x+1)(\alpha^2-\beta^2)}\right)^2-\frac{1}{(x+1)(\alpha^2-\beta^2)}\right]\frac{1}{2}$$

or

Simplifying further, we have;

$$\frac{R_2C_1}{x}=\frac{\alpha}{(x+1)(\alpha^2-\beta^2)}\pm\left[\frac{\alpha^2}{(x+1)(\alpha^2-\beta^2)}-\frac{1}{(x+1)(\alpha^2-\beta^2)}\right]\frac{1}{2} \qquad (131)$$

or

$$\frac{R_2C_1}{x}=\frac{\alpha}{(x+1)(\alpha^2-\beta^2)}\pm\left[\frac{\alpha^2-((x+1)(\alpha^2-\beta^2))}{(x+1)(\alpha^2-\beta^2)}\right]\frac{1}{2} \qquad (132)$$

or

$$\frac{R_2C_1}{x}=\frac{\alpha}{(x+1)(\alpha^2-\beta^2)}\pm\left[\frac{\alpha^2-(x+1)(\alpha^2-\beta^2)}{(x+1)(\alpha^2-\beta^2)}\right]\frac{1}{2} \qquad (133)$$

- Similarly, simplifying again:

$$\frac{R_2C_1}{x}=\pm\frac{\left[\alpha^2-(x+1)(\alpha^2-\beta^2)\right]^{\frac{1}{2}}}{(x+1)(\alpha^2-\beta^2)}$$

- Dividing through equa 134 by: $\frac{C_1}{x}$ both side of the equation:

$$\frac{R_2C_1}{x}=\frac{\frac{\alpha\pm\left[\alpha^2-(x+1)(\alpha^2-\beta^2)\right]^{\frac{1}{2}}}{(x+1)(\alpha^2-\beta^2)}}{\frac{C_1}{x}}$$

or

$$\frac{R_2 C_1}{x} = \frac{x}{C_1} = \frac{\alpha \pm \left[\alpha^2 - (x+1)(\alpha^2 - \beta^2)\right]^{\frac{1}{2}}}{(x+1)(\alpha^2 - \beta^2)} \times \frac{x}{C_1}$$

or

$$R_2 = \frac{1}{C}\left[\frac{\alpha \pm \sqrt{\alpha^2 - (x+1)(\alpha^2 - \beta^2)}}{(x+1)(\alpha^2 - \beta^2)}\right] \quad (136)$$

or

$$R_2 = \frac{1}{C}\left[\frac{\alpha \pm \sqrt{\alpha^2 - (x+1)(\alpha^2 - \beta^2)}}{\frac{(x+1)}{x}(\alpha^2 - \beta^2)}\right] \quad (137)$$

- Similarly, the value of R_1 can also be obtained as:

$$R_2 = \frac{1}{C}\left[\frac{(x+1)}{\alpha \pm \sqrt{\alpha^2 (x+1)(\alpha^2 - \beta^2)}}\right] \quad (138)$$

- It is evident, that R_1 and R_2 can be real sable, only if:

$$\alpha^2 - (x+1)(\alpha^2 - \beta^2) \geq 0 \quad (139)$$

or

$$\alpha^2 \geq (x+1)(\alpha^2 - \beta^2) \quad (140)$$

or

$$\frac{\alpha^2}{\alpha^2 - \beta^2} \leq x+1 \quad (141)$$

or

$$\frac{\beta^2}{\alpha^2 - \beta^2} \geq x \quad (142)$$

or

$$x \leq \frac{\beta^2}{\alpha^2 - \beta^2} \quad (143)$$

- This means that, in order to release, a given shape in of voltage work, certain values of R_1 and R_2 are required and these values will depend on the ratio of $\frac{C_1}{C_2}$ which must not exceeds certain value.

- These values can be evaluated for different wave shaped by choosing the typical values of α and β, from a give standard table: (IEC)

Table 1.2:

Wave	Max Value
0.515	12.168
1/5	3.342
1/40	12.167
1.5/40	45.988
1/50	100.717

- The standard impulse wave shapes lighting

Impulse wave shapes for voltage are:

$\frac{1.2}{50}$ years and the standard lighting impulse wave shape for currents 80/20 years.

- The switching, lighting impulse withstand test voltage

The flash over strength V_{FO} (KV) is determined by voltage time characteristics of the insulation strings, as:

$$V_{FO} = \left(400 + \frac{710}{t^{0.75}} \right) \times D \tag{144}$$

Where;

D : insulator string length in (m),

t : Elasped time after lighting strike in (μs)

Similarly, the lighting current is given by the double exponential equation as:

$$i(t) = I \times \left(\ell^{-\alpha t} - \ell^{-\beta t} \right) \tag{145}$$

Where;

α , β are constants depending on the lighting current wave shape that is, this is the parameter determine the behaviour of the lighting wave pattern.

Case 1: Analysis 1 Result and Discussion

When, t = 0ms, α = 3.044, β = 3.029

Then,

$$V_{(t)} = V_n \left[\ell^{-(\alpha-\beta)t} - \ell^{-(\alpha-\beta)t} \right]$$
$$V_{(oms)} = V_n \left[2.719^{-(0.015)\times 0} - 2.719^{-(0.06073)\times 0} \right]$$

or

$$V_{(oms)} = V_n [1-1] = 0$$

In case 1, of analysis 1,

That is, the time taken before initiation of the voltage at profile t = 0ms obviously make the voltage profile also to be zero (0). This means at the instant of initiation both the voltage profile and time, t are zero.

Case 2: Analysis 2

When, t = 0.1ms, α = 3.044, β = 3.029

Then,

$$V_{(t)} = V_n \left[\ell^{-(\alpha-\beta)t} - \ell^{-(\alpha-\beta)t} \right] \tag{51}$$
$$V_{(0.1ms)} = V_n \left[2.719^{-(3.044-3.029)\times 0.1} - 2.719^{-(3.044-3.029)\times 0.1} \right]$$
$$V_{(0.1ms)} = V_n \left[2.719^{-(0.015)\times 0.1} - 2.719^{-(3.044-3.029)\times 0.1} \right]$$
$$V_{(0.1ms)} = V_n \left[2.719^{-0.0015} - 2.719^{-0.6073} \right]$$

or

$$V_{(0.1ms)} = V_n [0.998500 - 0.5 \times 473]$$

or

$$V_{(0.1ms)} = V_n (0.45377)$$

In case 2, of analysis 2,

Thus, the time taken, *t = 0.1*ms is the time it will take to develop voltage profile to reach about 45% of its voltage magnitude of V_n. In order to determine the check whether the developed voltage magnitude not matching the installed substations level voltage.

Case 3: Analysis 3

When, t = 0.5, α = 3.044, β = 3.029

Then,

$$V_{(t)} = V_n \left[\ell^{-(\alpha-\beta)t} - \ell^{-(\alpha-\beta)t} \right]$$
$$V_{(0.5ms)} = V_n \left[2.719^{-(3.044-3.029)\times 0.5} - 2.719^{-(3.044+3.029)\times 0.5} \right]$$

or

$$V_{(0.5ms)} = V_n \left[2.719^{-(0.015)\times 0.5} - 2.719^{-6.073\times 0.5} \right]$$

or

$$V_{(0.5ms)} = V_n \left[2.719^{-0.0075} - 2.719^{-3.0365} \right]$$

or

$$V_{(0.5ms)} = V_n [0.9925260 - 0.0479641]$$

$$V_{(0.5ms)} = V_n[0.94456]$$

Thus, in case 3 of analysis 3 the time taken, t = 0.5ms is the time it will take to developed the voltage profile (?) to reach about 95% of its voltage magnitude of V_n.

Case 4: Analysis 4

When, t = 1.0, α = 3.044, β = 3.029

Then,

$$V_{(t)} = V_n\left[\ell^{-(\alpha-\beta)t} - \ell^{-(\alpha-\beta)t}\right]$$

$$V_{(1.0ms)} = V_n\left[2.719^{-(0.015)\times 1} - 2.719^{-(6.073)\times 1.0}\right]$$

$$V_{(1.0ms)} = V_n[0.985100 - 0.00230055]$$

$$V_{(1.0ms)} = V_n[0.982799]$$

In case 4 of analysis 4

Thus, the time taken, $t = 1.0$ms is the time it will take to developed the voltage profile to reach about 98.2% of its voltage magnitude of V_n.

Case 5: Analysis 5

When, t = 1.5, α = 3.044, β = 3.029

Then,

$$V_{(t)} = V_n\left[\ell^{-(\alpha-\beta)t} - \ell^{-(\alpha-\beta)t}\right]$$

$$V_{(1.5ms)} = V_n\left[2.719^{-(0.015)\times 1.5} - 2.719^{-(6.073)\times 1.5}\right]$$

or

$$V_{(1.5)} = V_n\left[2.719^{-0.0225} - 2.719^{-9.1095}\right]$$

or

$$V_{(1.5)} = V_n[0.977745 - 0.000110344]$$

or

$$V_{(1.5)} = V_n[0.977745 - 0.000110344]$$

$$V_{(1.5)} = V_n[0.97763]$$

In case 5 of analysis 5 thus, the time taken, t = 1.5ms is the time it will take to developed the voltage profile to reach about 97% of its voltage magnitude of V_n.

Case 6: Analysis 6

When, t = 2.0, α = 3.044, β = 3.029

Then,

$$V_{(t)} = V_n\left[\ell^{-(\alpha-\beta)t} - \ell^{-(\alpha-\beta)t}\right]$$

$$V_{(2.0ms)} = V_n\left[2.719^{-(0.015)\times 2.0} - 2.719^{-(6.073)\times 2.0}\right]$$

$$V_{(2.0ms)} = V_n\left[2.719^{-0.03} - 2.719^{-12.146}\right]$$

$$V_{(2.0ms)} = V_n[0.9704378 - 0.000052925]$$

or

$$V_{(2.0ms)} = V_n(0.97043)$$

In case 6 of analysis 6 thus, the time taken, $t = 2.0$ms is the time it will take to developed the voltage profile to reach about 97.04% of its voltage magnitude of V_n.

Case 7: Analysis 7

When, t = 2.5, α = 3.044, β = 3.029

Then,

$$V_{(t)} = V_n\left[\ell^{-(\alpha-\beta)t} - \ell^{-(\alpha-\beta)t}\right]$$

$$V_{(2.5ms)} = V_n\left[2.719^{-(0.015)\times 2.5} - 2.719^{-(6.073)\times 2.5}\right]$$

or

$$V_{(2.5ms)} = V_n\left[2.719^{-0.0375} - 2.719^{-15.1825}\right]$$

$V_{(2.5ms)} = V_n[0.9631848 - 0.00000025385]$

or

$V_{(2.5ms)} = V_n(0.96318)$

In case 7 of analysis 7 hence, the time taken, *t = 2.5*ms is the time it will take to developed the voltage profile to reach about 96.31% of its voltage magnitude of V_n.

Case 8: Analysis 8

When, t = 3.0, α = 3.044, β = 3.029

Then,

$$V_{(t)} = V_n\left[\ell^{-(\alpha-\beta)t} - \ell^{-(\alpha-\beta)t}\right]$$

$$V_{(3.0ms)} = V_n\left[2.719^{-(0.015)\times 3} - 2.719^{-(6.073)\times 3}\right]$$

$V_{(3.0ms)} = V_n\left[2.719^{-0.045} - 2.719^{-18.219}\right]$

$V_{(3.0ms)} = V_n[0.955986 - 0.0000000021175]$

$V_{(3.0ms)} = V_n(0.9559)$

In case 8 of analysis 8 thus, the time taken, *t = 3.0*ms is the time it will take to developed the voltage profile to reach about 95.56% of its voltage magnitude of V_n.

Case 9: Analysis 9

When, t = 3.5, α = 3.044, β = 3.029

Then,

$$V_{(t)} = V_n\left[\ell^{-(\alpha-\beta)t} - \ell^{-(\alpha-\beta)t}\right]$$

$$V_{(3.5ms)} = V_n\left[2.719^{-(0.015)\times 3.5} - 2.719^{-(6.073)\times 3.5}\right]$$

$V_{(3.5ms)} = V_n\left[2.719^{-(0.0525)} - 2.719^{-(21.2555)}\right]$

$V_{(3.5ms)} = V_n[0.94884 - 0.00000000058400]$

or

$V_{(3.5ms)} = V_n(0.9488)$

In case 9 of analysis 9, this shows that the time taken, *t =3.5*ms is the time it will take to developed the voltage profile to reach about 94.88% of its voltage magnitude of V_n.

Case 10: Analysis 10

When, t = 4.0, α = 3.044, β = 3.029

Then,

$$V_{(t)} = V_n\left[\ell^{-(\alpha-\beta)t} - \ell^{-(\alpha-\beta)t}\right]$$

$$V_{(4.0ms)} = V_n\left[2.719^{-(0.015)\times 4} - 2.719^{-(6.073)\times 4}\right]$$

$V_{(4.0ms)} = V_n\left[2.719^{-0.06} - 2.719^{-24.292}\right]$

or

$V_{(4.0ms)} = V_n[0.94174 - 0.00000000000280]$

or

$V_{(4.0ms)} = V_n(0.94174)$

In case 10 of analysis 10 evidently, this means the time taken, *t = 4.0*ms is the time it will take to developed the voltage profile to reach about 94.17% of its voltage magnitude of V_n.

Case 11: Analysis 11

When, t = 4.5, α = 3.044, β = 3.029

Then,

$$V_{(t)} = V_n\left[\ell^{-(\alpha-\beta)t} - \ell^{-(\alpha-\beta)t}\right]$$

$$V_{(4.5ms)} = V_n\left[2.719^{-(0.015)\times 4.5} - 2.719^{-(6.073)\times 4.5}\right]$$

$V_{(4.5ms)} = V_n\left[2.719^{-0.0675} - 2.719^{-(27.3285)}\right]$

or

$$V_{(4.5ms)} = V_n\left[0.9347110 - 0.000000000001343\right]$$

$$V_{(4.5ms)} = V_n\left(0.934711\right)$$

In case 11 of analysis 11, essentially, the time taken, *t =4.5*ms is the time it will take to developed the voltage profile to reach about 93.47% of its voltage magnitude of V_n.

Case 12: Analysis 12

When, t = 5.0, α = 3.044, β = 3.029

Then,

$$V_{(t)} = V_n\left[\ell^{-(\alpha-\beta)t} - \ell^{-(\alpha-\beta)t}\right]$$

$$V_{(5.0ms)} = V_n\left[2.719^{-(0.015)\times 5} - 2.719^{-(6.073)\times 5}\right]$$

$$V_{(5.0ms)} = V_n\left[2.719^{-0.075} - 2.719^{-30.365}\right]$$

or

$$V_{(5.0ms)} = V_n\left[0.927725 - 0.0000000000000644\right] \text{or}$$

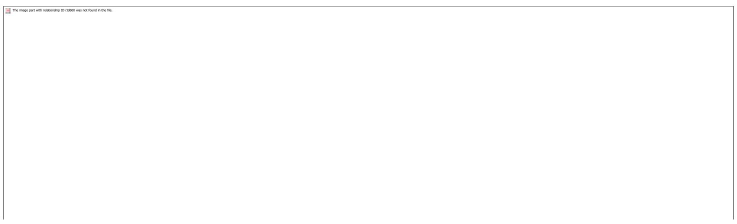
$$V_{(5.0ms)} = V_n\left(0.927725\right)$$

In case 12 of analysis 12, significantly, the time taken, *t = 5.0*ms is the time it will take to developed the voltage profile to reach about 92.77% of its voltage magnitude of V_n.

III Results

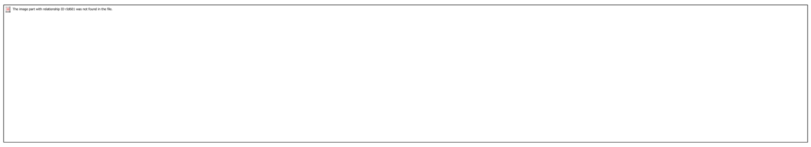
Evidently, the time taken by the initiation of voltage magnitude is plotted in matlab platform, the time required to develop voltage magnitude increase periodically with time while the voltage magnitude profile also increase and decrease down to zero. This show that the impulse voltage wave form, which is without appreciable oscillation rises rapidly to a maximum value and fall or collapse back rapidly to zero. The maximum value is the peak value of the impulse voltage and it is specified by this value.

```
% calculation of Double exponential voltage
%clc;
strt=0.00;
step =0.10;
End  =1.00;
 t   =strt:step:End;
 t;
 x1=-0.015*t;
 x2 =-6.075*t;
 vn =1.01749;
 k  =exp(x1)- exp(x2);
 vt=vn * k;
 vt;
 plot(t,vt);
 box on;
 grid on;
 axis on;
 xlabel('(t)');
 ylabel(' v(t) ');
 title( 'graph of t vs  v(t)');% display  chart title
```



```
% calculation of Double exponential voltage
%clc;
```

```
strt=0;
step =0.5;
End  =5;
 t   =strt:step:End;
 t;
 x1=-0.015*t;
 x2 =-6.075*t;
 vn =1.01749;
 k  =exp(x1)- exp(x2);
 vt=vn * k;
 vt;
 plot(t,vt);
 box on;
 grid on;
 axis on;
 xlabel('(t)');
 ylabel(' v(t) ');
 title( 'graph of t vs  v(t)');% display  chart title
```



IV Conclusion

This paper presents a mathematical model describing the behaviour of a double exponential voltage waveshape of an impulse generator using Laplace transform techniques. An impulse voltage is a unidirectional voltage which without appreciable oscillations rises rapidly to a maximum value and falls more or less rapidly to zero. If an impulse voltage develops without causing flash over or puncture, it is called "full impulse voltage"; if a flash over in puncture occur, thus causing a sudden collapse of the impulse voltage, it is called the 'chopped impulse voltage'. The developed model can be used to study the performance of the lighting overvoltage of either high voltage or middle voltage in transmission stations/substations etc. In order to predict worst or effective lighting protection at all time.

Because, it is very pertinent to consider many variables including the grounding resistance, the installation position of surge arrestors, the length of underground cables etc. That is all this factors influence significantly the lighting performance in substation (transmission line etc.).

Evidently, effective surge protection involves the integration of several ideas such as: eliminate earth loop and differential by creating an equipotenital earth system under transients conditions and protects equipments from surges and transient on in coming power lines, telecommunications and signals line etc. This paper provide the model equations to monitor the activities of exponential voltage wave-shape which can be very useful to practicing engineers and electric power utilities for an effective improvement of lighting performance in substations.

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