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Optimal Service Control in a Discrete Time Service Facility System with Inventory

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Abstract: *In this article we considered a service facility system with inventory maintained for service completion. Number of customers arrival and service completion follow a general distributions* $f(\cdot)$ *and* $g(\cdot)$ *respectively. Assume that the arriving customers enter the server are else if it inside join the queue. Demands occurs throughout the period and (0,M) policy is adopted for replenishing of inventory with zero lead time. At each decision epoch, the controller observes the number of customers in the system and selects the service rate from the set of probability distributions with service rate parameter. The problem is formulated as a MDP and policy iteration algorithm is used to get optimal policy. Numerical examples are provided to enhance the insight into the problem domain. And simulations are studied to compare the system with general queuing system.*

Keywords (11Bold) *Discrete time Markov Decision Process , inventory management, , service control, -Service facility system*

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I. INTRODUCTION

The introduction of the paper should explain the nature of the problem, previous work, purpose, and the The control of customers arrival / and service rates in a single queue service facility is well studied in the last decades. Although this assumption is realistic for production/manufacturing industries, it becomes unrealistic for service facilities where inventory is necessary to perform the service. In such systems, inventory is depleted according to the demand rate when there are no customers waiting and according to the service rate when there are customers queued up for service. Examples where inventory is used in the provision of service include installing car tires at car service stations.

Modelling of inventory systems maintained at a service facility has received considerable attention in the last two decades. Berman et al. (1999) [2] introduced the concept of deterministic inventory management system with a service facility using one item of inventory for each service.

Berman and sapna (1998) [5,6] analyzed a problem in a stochastic environment where customers arrive at service facilities according to a Poisson process. The service times are exponentially distributed with mean inter-arrival time assumed to be larger than the mean service time. The optimal policy is derived given that the order quantity is known. A logically related model has been studied by He et al. (1998) [5], who analyzed a Markovian inventory production system.

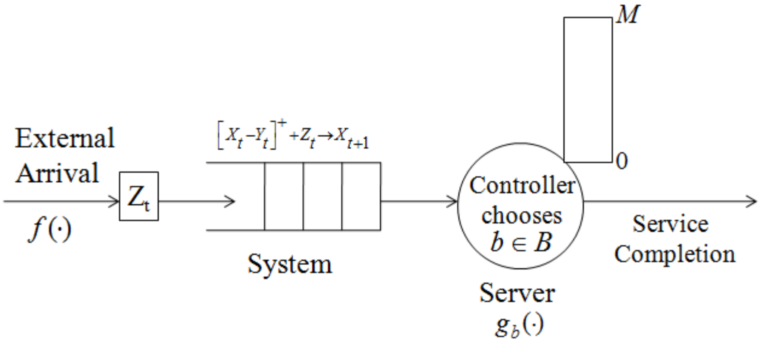
Berman and Kim (1999) [2] analyzed the problem in a stochastic environment where customers arrive at service facilities according to a Poisson process and service times are exponentially distributed. The paper deals with the case of lead time that is Erlang-distributed. The main result of the papers is that, under both the discounted cost case and the average cost case, the optimal policy of both the finite and infinite time horizon problems is a threshold ordering policy. In their papers, optimal policies to are derived given that the order quantity is known. They suggested a simple heuristic to obtain the optimal order quantity. Berman and Sapna (2000, 2001) [5,6] dealt with inventory control problems at service facilities where customers arrive for service according to a Poisson process and there is a finite waiting space having capacity N (an arriving customer who sees N customers ahead is forced to balk). Berman Sapna (2000) [3] considered a system where items in inventory have infinite lifetimes and service times are independently and identically distributed random variables with a general distribution function. The objective of that paper was to determine the optimal stocking level S that minimizes the long-run expected cost function when lead times are zero. The problem of determining the service rates to be employed as a function of the inventory level and number of customers in the system when service times and lead times have exponential distribution and lifetime is infinite is presented in Berman Sapna (2001). Selvakumar. C et al., and Maheswari. P et al., (2017) [6,8] are deals with Discrete MDP problem with Admission and Inventory Control in Service Facility Systems and Optimal Admission and Service Control in a Discrete time Service Facility Systems: MDP Approach and

In their paper, they try to control both the admission and service in a service facility system under periodic review (equally spaced time epochs). The queue before the server is divided into eligible queue and potential queue. Here, we use policy iteration method to optimize the expected total reward. In the last section a numerical example is provided to illustrate the model

In this paper we deal with a system having service facility system with inventory maintained for service completion. We assume that customers arrive according to a general probability distribution to the facility which has unlimited waiting space. There is no lead time for orders and customers are served on an FCFS basis. Our objective is to determine optimal mean service times (or optimal service rates) to be employed each time a customer finishes service for a general service probability distribution. These decisions are based on the number of customers in the system.

The paper is organized as follows. Section 2 presents the model description. Section 3 contains the MDP formulation, Section 4 description of the analysis , Section 5 deals with cost analysis, Section 6 contains cost analysis , Section 7 present the policy iteration algorithm and Section 8 deals with numerical examples and also sensitively analysis and system simulation results.

II. MODEL DESCRIPTION



(i) The system is observed every $\eta > 0$ unit of time and the decision epochs are $0, \eta, 2\eta, ..., U\eta, U < \infty$ (Finite horizon).

(ii) Service is controlled by selecting the service rate from the set of probability mass functions indexed by elements of a set B, depends on the number of customers in the system. ($B = \{b_1, b_2, \cdots b_r\}$).

(iii) Assume that the maximum capacity of inventory is M (finite).

(iv) Arriving customers to service facility system follows a probability mass function $f(\cdot)$ Possible number of service completion follows a probability mass functions $g_b(\cdot)$

(v) All serviced customers take unit item from inventory and depart the system at end of the period.

(vi) The arrivals during a period do not receive service in that period (arrival at period 't') Z(t) get service only a period 't+1'.

(vii) The (0,M) policy is adopted for replenishing inventory instantaneous replenishment when the inventory level 0 at the decision epoch is for the sufficient quantity to pull back the level to M, else don't order.

(viii) Decision to order additional stock is made at the beginning of each period and delivery occurs instantaneously.

III. MDP FORMULATION

We consider the problem as MDP having five components (tuples) $(T, S, A_s, p_t(\cdot|), r_t(\ ))$

Decision Epochs:

$$T = \{0, \eta, 2\eta, ..., U\eta\},\ U < \infty.$$

State space:

S_1 - the number of customers in the system
S_2 - the number of items in stock
S_3- possible service rate.
State space - $S = \{0,1,2,...\} \times \{0,1,2,...,M\} \times B = S_1 \times S_2 \times S_3,$

Actions: $A_{(s_1,s_2,s_3)} = B = \{b_1, b_2, \cdots, b_r\}$

Transition Probability:

$p_t\left((\mathrm{s},\mathrm{a}') \mid (\mathrm{s},\mathrm{a}),\mathrm{a}'\right)=\sum_{n=1}^{s_1-1}\mathrm{g}_{a'}(\mathrm{n})\,\mathrm{g}\left(s_1'-s_1+n\right)+\left[\sum_{n=s_1}^{\infty}\mathrm{g}_{a'}(\mathrm{n})\right]\mathrm{f}(s_1').$ where $s=(s_1,s_2,s_3),\quad s'=(s_1',s_2',s_3').$

Cost:

$c_t(s,a')=l(s_1)+h(s_2)+c(a,a'),\quad a'\in A=\bigcup_{s\in S}A_s,\quad s=(s_1,s_2,s_3).$

$$\mathrm{c}(\mathrm{a}',\mathrm{a})=\begin{cases}K+d(\mathrm{b}') & b'\neq b\\ d(\mathrm{b}) & b'=b\end{cases}.$$

The stationary cost structure consist of three components: a waiting cost $l\left(s_1\right)$ per period when there are s_1 customers in the system and an expected holding cost $h\left(s_2\right)$ per period when there are s_2 items in inventory and fixed cost K for changing the service rate and service cost d(b) for current server $b\in B$ per period.

IV. ANALYSIS

Let X_t denote the number of customer in the system immediately prior to the decision epoch t and Z_t is the number of customers arrive in the time t are placed in the system. Let B denote the set of possible service parameter values. Let Y_t denote the number of "possible service completions" during period t. Let I_t denote the number of items in stock at time epoch t. The random variable Y_t assume non-negative integer values and follows a time invariant probability mass function $g_b(n)=\Pr\left\{Y_t=n\right\},\ t=0,1,2,...,\mathrm{U}$ and Z_t assumes a non-negative values which follows a time invariant probability distribution $\mathrm{f}\,(n)=\Pr\left\{Z_t=n\right\},\ t=0,1,2,...,\mathrm{U}$.

The number of customers in the system $X_{t+1}=\left[X_t-Y_t\right]^+ +Z_t$

The one step costs are given by, $c_t\left(s,a\right),\ s=\left(s_1,s_2,s_3\right)$.

Let $\left(X_t,\mathrm{I}_t,B_t\right)$ denote the state of the system of decision epoch t (beginning of t^{th} period). Assume the stationary policy R and hence the transition probability

$p_t\left(s' \mid \mathrm{s},a\right)=P\mathrm{r}\left\{\left(X_{t+1},\mathrm{I}_{t+1},B_{t+1}\right)=\mathrm{s}' \mid \left(X_t,\mathrm{I}_t,B_t\right)\ =s,a\right\},\quad s'=\left(s_1',s_2',s_3'\right),\ s=\left(s_1,s_2,s_3\right).$

regardless the past history of the system up to time epoch t.

Then $\left\{\left(X_t,I_t,B_t\right):t\geq 0\right\}$ is a Markov chain with discrete state space $S=S_1\times S_2\times S_3$ The t - step transition probabilities of the Markov chain under policy R is given by

$p_t\left(s' \mid \mathrm{s}\right)\left(R\right)=\Pr\left\{\left(X_t,\mathrm{I}_t,B_t\right)=s' \mid \left(X_0,\mathrm{I}_0,B_0\right)\ =s\right\},\quad s'=\left(s_1',s_2',s_3'\right),\ s=\left(s_1,s_2,s_3\right).$

Define $V_t\left(s,R\right),\ s=\left(s_1,s_2,s_3\right)$ denote the total expected cost over the first t decision epochs with initial state $\left(s_1,s_2,s_3\right)$ and policy R is adopted.

Then $V_t\left(\mathrm{s},\mathrm{R}\right)=\sum_{k=0}^{t-1}\sum_{s'\in S}p^{(k)}\left(s,s'\right)\left(R\right)c_{s'}\left(R_{s'}\right),\quad s'=\left(s_1',s_2',s_3'\right),\ s=\left(s_1,s_2,s_3\right)$

where , $C_s\left(R\right)=$ waiting cost of customer/period + holding cost of inventory/period + service cost per period.

$=\ l\times\overline{L}+h\times\overline{I}+c_1\cdot\overline{\alpha}$

Where

l - waiting cost per customer per period

$\overline{L}$ - the number of customers in the system + 1 in service counter

h – holding cost unit item per period

$\overline{I}$ - average inventory in stock during the t^{th} period

c_1 – service rate cost using server $b\in B$.

$\overline{\alpha}$ - the current service rate.

V. COST ANALYSIS

AThe average cost function $q_s(R)$ is given by $q_s(R)=\lim_{t\to\infty}\frac{1}{t}V_t(\mathrm{s,R}),\ (s_1,s_2,s_3)\in S$. The elements of the above average cost function is due to the Theorem (Puterman (1994) & Tijims (2003)).

Theorem 5.1

For all $s'=(s_1',s_2',s_3'),\ s=(s_1,s_2,s_3)\in S,\ \lim_{t\to\infty}\frac{1}{t}\sum_{k=1}^{t}p_t^{(k)}(s'|s)(R)$ always exists and for any $s'=(s_1',s_2',s_3')\in \mathrm{S}$.

$$\lim_{t\to\infty}\frac{1}{t}\sum_{k=1}^{t}p^{(k)}(s'|s)=\begin{cases}\dfrac{1}{\mu_{s'}} & \textit{if state } s' \textit{ is recurrent}\\ 0 & \textit{if state } s' \textit{ is transient.}\end{cases}$$

Where μ_s' denote the mean recurrent time from state (s_1',s_2',s_3') to itself.

Also $\lim_{t\to\infty}\frac{1}{t}\sum_{k=1}^{t}p^{(k)}(s'|s)=f_{(s)}^{(s')}\lim_{t\to\infty}\frac{1}{t}\sum_{k=1}^{t}p_t^{(k)}(s'),\quad s=(s_1,s_2,s_3)\ ,\ s'=(s_1',s_2',s_3').$

Since the Markov Chain $\{(X_t,\mathrm{I}_t,B_t):t=0,1,2,...,\mathrm{U}\}$ is a unichain which is irreducible, all its states are ergodic and have a unique equilibrium distribution.

Thus, $\pi_{(s')}(R)=\lim_{t\to\infty}\frac{1}{t}\sum_{k=1}^{t}p^{(k)}(s'|s)(R),\quad s=(s_1,s_2,s_3),\ s'=(s_1',s_2',s_3'),$ exist and is independent of initial state , such that $\pi P=\pi$ and $\sum_{s\in S}\pi_{(s)}=1.$

VI. OPTIMAL POLICY

A stationary policy R^* is said to be an average cost optimal policy if $q_{s_1,s_2}(R^*)\le q_{s_1,s_2}(R)$ for each stationary policy R uniformly in the initial state (s_1,s_2,s_3).

The relative value associated with a given policy R provides a tool for constructing a new policy R^* whose average cost is more than that of the current policy R .

The objective is to improve the given policy R whose average cost is $q(R)$ and relative value $v_{(s_1,s_2,s_3)}(R),\ (s_1,s_2,s_3)\in S.$

By constructing a new policy R such that for each $(s_1,s_2,s_3)\in S$,

$c_{(s)}(R_s^*)-q(R)+\sum_{s'\in S}p_{(s,s')}(R_s^*)v_{s'}\le v_s$.................(1)

Where $s=(s_1,s_2,s_3)$ and $s'=(s_1',s_2',s_3')$.

We obtain an improved rule R^* with $q(R^*)\le q(R)$. We have to find the optimal policy R_s^* satisfying (1) is to minimize the cost functions $c_i(a)-\mathrm{q}(R)+\sum_{s'\in S}p_t(s'|\mathrm{s,a})v_{s'}(R)$ over all actions $a\in A(s)$.

VII. ALGORITHM

a **Step 0: (Initialization)**

Choose a stationary policy R for the periodic review based admission control in service facility system maintaining inventory.

Step 1: (Value determination step)

For the current policy R , compute the unique solution $(q(R),\mathrm{v}_s(R))$ to the following linear equations $v_s=c_s(R_s)-q+\sum_{s'\in S}p_t(s'|\mathrm{s})(R_s)v_s',\quad s=(s_1,s_2,s_3)\in S,$

$v_s=0,\quad \textit{where } s=(s_1,\mathrm{s}_2,s_3)\ \textit{is arbitarily chosen state in } S$

Step 2:(Policy Improvement)

For each state $s=(s_1,s_2,s_3)\in S$ determine the actions yielding

$$\arg\min_{a\in A_s}\left\{c_s(a)-q+\sum_{s'\in S}p_t(s'|s,a)v_s'(R)\right\}$$

The new stationary policy R^* is obtained by choosing $R_s^*=a_s$.

Step 3:(Convergence test)

If the new policy $R^*=R$, the old one. Then the process of searching stops with policy R. Otherwise go to **Step 1** with R replaced by new R^*.

VIII. NUMERICAL EXAMPLE

For the given service facility system with inventory we assume, $M=3, K=2.$ We have infinite capacity of the system, truncating we take N=3, η=1 The state space become

$$\begin{aligned}S=\{&(3,3,b_1),(3,3,b_2),(3,2,b_1),(3,2,b_2),(3,1,b_1),(3,1,b_2),(3,0,b_1),(3,0,b_2),\\&(2,3,b_1),(2,3,b_2),(2,2,b_1),(2,2,b_2),(2,1,b_1),(2,1,b_2),(2,0,b_1),(2,0,b_2),\\&(1,3,b_1),(1,3,b_2),(1,2,b_1),(1,2,b_2),(1,1,b_1),(1,1,b_2),(1,0,b_1),(1,0,b_2),\\&(0,3,b_1),(0,3,b_2),(0,2,b_1),(0,2,b_2),(0,1,b_1),(0,1,b_2),(0,0,b_1),(0,0,b_2)\}.\end{aligned}$$

u	0	1	2	3
c(u)- Server b_1	0	0.3	0.6	0.9
c(u)- Server b_2	0	0.5	0.8	1.1

Action set at $(s_1,s_2,s_3)\in S$ is $A_{(s_1,s_2,s_3)}=\{b_1,b_2\}$. Assume the service cost $c=1$ for each service completion per customer, waiting cost $l=0.3$ per customer/ period, holding cost $h=0.35$ per item/ period and a fixed cost $K=0.25$ per period for changing service rate .

Computational Procedure:

For any given policy R, the policy improvement quantity is given by

$$T_s(a,R)=c_s(a)-q(R)+\sum_{s'\in S}p_t(s'|s)(a)\,v_{s'}(a)\quad where\ T_s(a,R)=v_s(R)\quad for\ a=R_s.$$

/s'	(3,3,b1)	(3,3,b2)	(3,2,b1)	(3,2,b2)	(3,1,b1)	(3,1,b2)	(3,0,b1)	(3,0,b2)	(2,3,b1)	(2,3,b2)	(2,2,b1)	(2,2,b2)	(2,1,b1)	(2,1,b2)	(2,0,b1)	(2,0,b2)	(1,3,b1)	(1,3,b2)	(1,2,b1)	(1,2,b2)	(1,1,b1)	(1,1,b2)	(1,0,b1)	(1,0,b2)	(0,3,b1)	(0,3,b2)	(0,2,b1)	(0,2,b2)	(0,1,b1)	(0,1,b2)	(0,0,b1)	(0,0,b2)
(3,3,b1)	0.25										0.35										0.22										0.18	
(3,3,b2)		0.17										0.24										0.38										0.21

(3,2,b1)			0.25										0.45										0.3									
(3,2,b2)				0.2										0.35										0.45								
(3,1,b1)					0.45										0.55																	
(3,1,b2)						0.4										0.6																
(3,0,b1)	0.3											0.28									0.25										0.17	
(3,0,b2)		0.31											0.2									0.3										0.19
(2,3,b1)	0.27								0.2										0.33										0.2			

Iteration 1:

Policy iteration algorithm is initialized with

$R^{(1)} = \left(2,2,1,2,1,2,2,2,1,2,1,2,1,2,1,2,1,2,1,2,1,2,1,2,1,2,1,2,1,2,1,2\right)$, which prescribes the adjust must in service rate at states $\left(3,3,b_1\right)$ and $\left(3,0,b_1\right)$. Solving the system of linear equations connecting the average reward $q\left(R\right)^{(1)}$ by assuming v_{0ij}, $i=0,1,2,3,\ j=1,2$.

$C_s(a,R^{(1)})$		
s\a	b_1	b_2
$(3,3,b_1)$	3.351502286	3.30
$(3,3,b_2)$	3.10	3.084276679
$(3,2,b_1)$	2.542491232	2.65
$(3,2,b_2)$	2.45	2.400842879
$(3,1,b_1)$	1.780958981	2.00
$(3,1,b_2)$	1.80	1.701506139
$(3,0,b_1)$	3.396588325	3.30
$(3,0,b_2)$	3.10	3.002413557
$(2,3,b_1)$	2.400454858	2.70
$(2,3,b_2)$	2.50	2.460476430
$(2,2,b_1)$	2.194519635	2.35
$(2,2,b_2)$	2.15	2.033691383
$(2,1,b_1)$	1.537179235	1.70
$(2,1,b_2)$	1.50	1.333691383
$(2,0,b_1)$	2.766875577	3.00
$(2,0,b_2)$	2.80	2.695904824
$(1,3,b_1)$	1.811481561	2.10
$(1,3,b_2)$	1.90	1.7009043695
$(1,2,b_1)$	1.500953582	1.75
$(1,2,b_2)$	1.55	1.506352846
$(1,1,b_1)$	1.059369225	1.40
$(1,1,b_2)$	1.20	1.106352846
$(1,0,b_1)$	2.045976416	2.10
$(1,0,b_2)$	1.90	2.074673268

The new policy will be $R^{(2)} = \left(2,2,1,2,1,2,2,2,1,2,1,2,1,2,1,2,1,2,1,2,1,2,1,1,1,2,1,2,1,2,1,2\right)$. Since the new policy $R^{(2)}$ is different from the initial policy $R^{(1)}$ the searching process continues.

Iteration 2:

For the policy $R^{(2)}$, Solving the system of linear equations connecting the average reward $q\left(R\right)^{(2)}$ by assuming v_{0ij}, $i=0,1,2,3,\ j=1,2$.

$C_s(a,R^{(2)})$		
s\a	b_1	b_2
$(3,3,b_1)$	3.389267505	3.30
$(3,3,b_2)$	3.10	3.075673894
$(3,2,b_1)$	2.438972767	2.65
$(3,2,b_2)$	2.45	2.387627326
$(3,1,b_1)$	1.894774765	2.00
$(3,1,b_2)$	1.80	1.687988933
$(3,0,b_1)$	3.468223901	3.30
$(3,0,b_2)$	3.10	2.903675821
$(2,3,b_1)$	2.568738723	2.70
$(2,3,b_2)$	2.50	2.289837437
$(2,2,b_1)$	2.297875348	2.35
$(2,2,b_2)$	2.15	1.995343366
$(2,1,b_1)$	1.468787292	1.70
$(2,1,b_2)$	1.50	1.283774387
$(2,0,b_1)$	2.837645747	3.00
$(2,0,b_2)$	2.80	2.557887328
$(1,3,b_1)$	1.783643676	2.10
$(1,3,b_2)$	1.90	1.797837837
$(1,2,b_1)$	1.693493093	1.75
$(1,2,b_2)$	1.55	1.458384939
$(1,1,b_1)$	1.239998839	1.40
$(1,1,b_2)$	1.20	1.007374837
$(1,0,b_1)$	1.997743873	2.10
$(1,0,b_2)$	1.90	2.286674677

Since the new policy will be $R^{(3)} = \left(2,2,1,2,1,2,2,2,1,2,1,2,1,2,1,2,1,2,1,2,1,2,1,1,1,2,1,2,1,2,1,2\right)$. which is identical with the previous $R^{(2)}$ policy, the searching process stops here. After two iterations we obtained the optimal policy $R^{*} = \left(2,2,1,2,1,2,2,2,1,2,1,2,1,2,1,2,1,2,1,2,1,2,1,1,1,2,1,2,1,2,1,2\right)$.

Simulation Result:

Total Expected cost		(Mean waiting cost, Holding cost, Mean service cost)	Service rate									
			1	2	3	4	5	6	7	8	9	10
Arrival rate	5	(0.1, 0.3, 0.1)	19.008	14.986	10.887	7.216	**6.471**	6.965	7.670	8.512	9.434	10.376
		(0.1, 0.6, 0.1)	31.860	24.254	16.515	9.510	**7.511**	7.635	8.120	8.844	9.706	10.604
		(0.1, 0.9, 0.1)	44.711	33.522	22.143	11.804	8.551	**8.304**	8.571	9.177	9.979	10.832
		(0.1, 1.2, 0.1)	57.563	42.789	27.770	14.098	9.591	**8.973**	9.022	9.510	10.251	11.060
		(0.1, 1.5, 0.1)	70.415	52.057	33.398	16.392	10.632	9.643	**9.473**	9.842	10.524	11.288
		(0.1, 1.8, 0.1)	83.267	61.324	39.026	18.686	11.672	10.312	**9.924**	10.175	10.796	11.516
	6	(0.1, 0.3, 0.1)	19.484	16.591	13.213	10.159	7.460	**7.359**	7.922	8.649	9.466	10.379
		(0.1, 0.6, 0.1)	32.854	27.160	20.633	14.662	9.301	**8.399**	8.639	9.163	9.843	10.690
		(0.1, 0.9, 0.1)	46.224	37.730	28.052	19.164	11.141	9.439	**9.356**	9.677	10.219	11.001
		(0.1, 1.2, 0.1)	59.594	48.299	35.472	23.666	12.981	10.480	**10.072**	10.191	10.596	11.311
		(0.1, 1.5, 0.1)	72.965	58.868	42.891	28.169	14.821	11.520	10.789	**10.704**	10.973	11.622
		(0.1, 1.8, 0.1)	86.335	69.438	50.311	32.671	16.661	12.560	11.506	**11.218**	11.350	11.932
	7	(0.1, 0.3, 0.1)	19.729	17.698	14.949	12.342	10.031	**8.227**	8.350	8.964	9.712	10.529
		(0.1, 0.6, 0.1)	33.355	29.130	23.667	18.451	13.743	9.907	**9.391**	9.717	10.275	10.955
		(0.1, 0.9, 0.1)	46.982	40.562	32.386	24.560	17.455	11.588	**10.431**	10.470	10.839	11.382
		(0.1, 1.2, 0.1)	60.608	51.995	41.105	30.669	21.168	13.268	11.471	**11.222**	11.403	11.808
		(0.1, 1.5, 0.1)	74.234	63.427	49.823	36.778	24.880	14.949	12.512	11.975	**11.967**	12.235
		(0.1, 1.8, 0.1)	87.860	74.859	58.542	42.886	28.592	16.630	13.552	12.728	**12.531**	12.661
	8	(0.1, 0.3, 0.1)	19.854	18.504	16.324	14.108	12.078	10.195	**9.064**	9.344	9.997	10.763
		(0.1, 0.6, 0.1)	33.630	30.573	26.080	21.528	17.295	13.316	10.632	**10.384**	10.778	11.367
		(0.1, 0.9, 0.1)	47.407	42.642	35.837	28.947	22.512	16.437	12.201	**11.425**	11.559	11.972
		(0.1, 1.2, 0.1)	61.183	54.711	45.593	36.366	27.729	19.558	13.769	12.465	**12.339**	12.577
		(0.1, 1.5, 0.1)	74.960	66.779	55.350	43.786	32.945	22.680	15.338	13.505	**13.120**	13.182
		(0.1, 1.8, 0.1)	88.737	78.848	65.106	51.205	38.162	25.801	16.907	14.546	13.901	**13.787**
	9	(0.1, 0.3, 0.1)	19.942	19.043	17.392	15.432	13.662	12.074	10.547	**9.949**	10.339	11.023
		(0.1, 0.6, 0.1)	33.830	31.547	27.961	23.843	20.052	16.577	13.205	11.438	**11.380**	11.826
		(0.1, 0.9, 0.1)	47.718	44.050	38.530	32.254	26.442	21.079	15.863	12.928	**12.420**	12.630
		(0.1, 1.2, 0.1)	61.606	56.554	49.099	40.664	32.832	25.581	18.522	14.417	13.460	**13.433**
		(0.1, 1.5, 0.1)	75.494	69.057	59.668	49.075	39.222	30.084	21.180	15.907	14.501	**14.236**
		(0.1, 1.8, 0.1)	89.382	81.560	70.237	57.485	45.612	34.586	23.839	17.396	15.541	**15.039**
	10	(0.1, 0.3, 0.1)	20.019	19.472	18.191	16.572	15.045	13.596	12.289	11.038	**10.871**	11.335
		(0.1, 0.6, 0.1)	34.002	32.323	29.373	25.839	22.465	19.224	16.230	13.332	**12.307**	12.376
		(0.1, 0.9, 0.1)	47.985	45.174	40.556	35.106	29.884	24.851	20.171	15.626	13.743	**13.416**
		(0.1, 1.2, 0.1)	61.968	58.025	51.739	44.374	37.303	30.479	24.111	17.920	15.179	**14.456**
		(0.1, 1.5, 0.1)	75.951	70.877	62.921	53.641	44.722	36.106	28.052	20.214	16.615	**15.497**
		(0.1, 1.8, 0.1)	89.934	83.728	74.104	62.908	52.142	41.734	31.993	22.508	18.051	**16.537**

Total Expected cost		(Mean waiting cost, Holding cost, Mean service cost)	Service rate									
			1	2	3	4	5	6	7	8	9	10
Arrival rate	5	(0.5, 0.3, 0.5)	39.633	29.861	19.923	10.904	8.192	**8.149**	8.545	9.228	10.078	10.968
		(0.5, 0.6, 0.5)	52.485	39.129	25.551	13.198	9.232	**8.819**	8.995	9.561	10.350	11.196
		(0.5, 0.9, 0.5)	65.336	48.396	31.178	15.492	10.272	9.488	**9.446**	9.893	10.623	11.424

		(0.5, 1.2, 0.5)	78.188	57.664	36.806	17.786	11.313	10.157	**9.897**	10.226	10.896	11.652
		(0.5, 1.5, 0.5)	91.040	66.932	42.434	20.080	12.353	10.827	**10.348**	10.559	11.168	11.880
		(0.5, 1.8, 0.5)	103.892	76.199	48.062	22.374	13.393	11.496	**10.799**	10.891	11.441	12.108
	6	(0.5, 0.3, 0.5)	39.937	32.677	24.388	16.786	9.941	**8.633**	8.742	9.189	9.821	10.655
		(0.5, 0.6, 0.5)	53.307	43.246	31.808	21.289	11.781	9.673	**9.458**	9.703	10.198	10.966
		(0.5, 0.9, 0.5)	66.677	53.816	39.227	25.791	13.621	10.714	**10.175**	10.217	10.575	11.276
		(0.5, 1.2, 0.5)	80.047	64.385	46.647	30.293	15.462	11.754	10.892	**10.731**	10.952	11.587
		(0.5, 1.5, 0.5)	93.418	74.954	54.066	34.796	17.302	12.794	11.609	**11.245**	11.329	11.897
		(0.5, 1.8, 0.5)	106.788	85.524	61.486	39.298	19.142	13.835	12.326	11.759	**11.706**	12.208
	7	(0.5, 0.3, 0.5)	40.142	34.760	27.868	21.277	15.306	10.411	**9.591**	9.809	10.302	10.936
		(0.5, 0.6, 0.5)	53.768	46.193	36.587	27.386	19.019	12.092	10.632	**10.562**	10.866	11.363
		(0.5, 0.9, 0.5)	67.394	57.625	45.305	33.494	22.731	13.772	11.672	**11.315**	11.430	11.789
		(0.5, 1.2, 0.5)	81.020	69.057	54.024	39.603	26.443	15.453	12.712	12.068	**11.994**	12.216
		(0.5, 1.5, 0.5)	94.646	80.489	62.743	45.712	30.155	17.133	13.752	12.820	**12.558**	12.643
		(0.5, 1.8, 0.5)	108.272	91.922	71.461	51.821	33.867	18.814	14.793	13.573	13.122	**13.069**
	8	(0.5, 0.3, 0.5)	40.162	36.247	30.592	24.864	19.525	14.493	11.044	**10.560**	10.861	11.393
		(0.5, 0.6, 0.5)	53.939	48.315	40.348	32.284	24.741	17.614	12.612	**11.600**	11.642	11.998
		(0.5, 0.9, 0.5)	67.716	60.384	50.105	39.703	29.958	20.735	14.181	12.640	**12.423**	12.603
		(0.5, 1.2, 0.5)	81.492	72.453	59.861	47.122	35.175	23.856	15.750	13.681	**13.204**	13.208
		(0.5, 1.5, 0.5)	95.269	84.522	69.618	54.542	40.392	26.977	17.318	14.721	13.985	**13.813**
		(0.5, 1.8, 0.5)	109.045	96.590	79.374	61.961	45.608	30.098	18.887	15.761	14.765	**14.418**
	9	(0.5, 0.3,	40.157	37.204	32.682	27.520	22.751	18.362	14.099	11.786	**11.535**	11.902

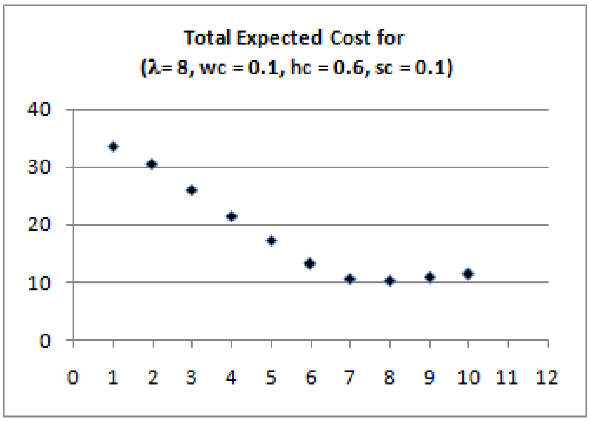
		0.5)										
		(0.5, 0.6, 0.5)	54.045	49.707	43.251	35.930	29.141	22.865	16.757	13.275	**12.576**	12.705
		(0.5, 0.9, 0.5)	67.933	62.211	53.820	44.341	35.531	27.367	19.416	14.765	13.616	**13.509**
		(0.5, 1.2, 0.5)	81.821	74.714	64.389	52.751	41.921	31.869	22.074	16.254	14.656	**14.312**
		(0.5, 1.5, 0.5)	95.709	87.217	74.958	61.162	48.311	36.372	24.733	17.744	15.697	**15.115**
		(0.5, 1.8, 0.5)	109.597	99.721	85.527	69.572	54.701	40.874	27.391	19.233	16.737	**15.918**
	10	(0.5, 0.3, 0.5)	40.165	37.955	34.223	29.791	25.550	21.469	17.685	14.015	12.610	**12.516**
		(0.5, 0.6, 0.5)	54.148	50.806	45.406	39.058	32.969	27.097	21.625	16.309	14.046	**13.556**
		(0.5, 0.9, 0.5)	68.131	63.657	56.588	48.325	40.388	32.724	25.566	18.603	15.482	**14.596**
		(0.5, 1.2, 0.5)	82.114	76.508	67.771	57.593	47.808	38.352	29.507	20.897	16.917	**15.637**
		(0.5, 1.5, 0.5)	96.096	89.360	78.954	66.860	55.227	43.980	33.447	23.191	18.353	**16.677**
		(0.5, 1.8, 0.5)	110.079	102.211	90.136	76.127	62.646	49.607	37.388	25.485	19.789	**17.717**

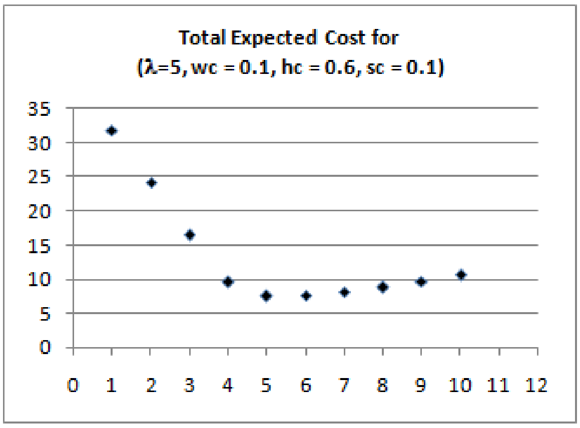
Total Expected cost		(Mean waiting cost, Holding cost, Mean service cost)	Service rate									
			1	2	3	4	5	6	7	8	9	10
Arrival rate	5	(0.3, 0.2, 0.2)	24.166	18.707	13.148	8.140	**6.914**	7.289	7.926	8.736	9.646	10.580
		(0.6, 0.2, 0.2)	37.023	27.980	18.781	10.440	**7.994**	8.041	8.491	9.206	10.074	10.978
		(0.9, 0.2, 0.2)	49.880	37.253	24.414	12.739	9.073	**8.793**	9.055	9.676	10.502	11.376
		(1.2, 0.2, 0.2)	62.738	46.526	30.047	15.039	10.153	**9.545**	9.620	10.146	10.930	11.773
		(1.5, 0.2, 0.2)	75.595	55.799	35.681	17.339	11.232	10.297	**10.185**	10.615	11.357	12.171
		(1.8, 0.2, 0.2)	88.452	65.072	41.314	19.639	12.312	11.049	**10.749**	11.085	11.785	12.569
	6	(0.3, 0.2, 0.2)	24.499	20.514	15.909	11.718	7.983	**7.591**	8.052	8.719	9.497	10.396
		(0.6, 0.2, 0.2)	37.574	30.789	23.034	15.926	9.532	**8.370**	8.546	9.037	9.700	10.550
		(0.9, 0.2, 0.2)	50.650	41.064	30.159	20.133	11.081	9.150	**9.039**	9.356	9.903	10.705
		(1.2, 0.2, 0.2)	63.726	51.338	37.283	24.341	12.630	9.929	**9.533**	9.674	10.106	10.859
		(1.5, 0.2, 0.2)	76.802	61.613	44.408	28.549	14.179	10.709	10.026	**9.992**	10.309	11.013
		(1.8, 0.2, 0.2)	89.877	71.888	51.533	32.757	15.728	11.488	10.519	**10.311**	10.512	11.167
	7	(0.3, 0.2, 0.2)	24.734	21.865	18.080	14.478	11.252	8.677	**8.574**	9.099	9.792	10.570
		(0.6, 0.2, 0.2)	38.066	33.003	26.504	20.292	14.669	10.072	**9.353**	9.624	10.153	10.814
		(0.9, 0.2, 0.2)	51.397	44.141	34.928	26.106	18.087	11.466	**10.133**	10.149	10.514	11.058
		(1.2, 0.2, 0.2)	64.729	55.278	43.353	31.921	21.505	12.860	10.913	**10.673**	10.875	11.302
		(1.5, 0.2, 0.2)	78.060	66.416	51.777	37.735	24.922	14.255	11.692	**11.198**	11.236	11.546
		(1.8, 0.2, 0.2)	91.392	77.554	60.201	43.549	28.340	15.649	12.472	11.723	**11.597**	11.790
	8	(0.3, 0.2, 0.2)	24.833	22.842	19.792	16.699	13.842	11.172	**9.465**	9.561	10.136	10.851
		(0.6, 0.2, 0.2)	38.315	34.616	29.254	23.824	18.764	13.998	10.751	**10.341**	10.685	11.247
		(0.9, 0.2, 0.2)	51.797	46.390	38.716	30.949	23.686	16.825	12.036	**11.120**	11.234	11.643
		(1.2, 0.2, 0.2)	65.279	58.164	48.178	38.074	28.608	19.652	13.322	11.900	**11.783**	12.040
		(1.5, 0.2, 0.2)	78.761	69.938	57.640	45.198	33.530	22.478	14.608	12.680	**12.332**	12.436
		(1.8, 0.2, 0.2)	92.243	81.713	67.102	52.323	38.453	25.305	15.894	13.459	12.881	**12.832**
	9	(0.3, 0.2, 0.2)	24.897	23.485	21.116	18.356	15.836	13.548	11.337	**10.315**	10.551	11.165
		(0.6, 0.2, 0.2)	38.491	35.694	31.391	26.472	21.932	17.756	13.701	11.524	**11.331**	11.733
		(0.9, 0.2, 0.2)	52.084	47.903	41.665	34.588	28.027	21.964	16.065	12.734	**12.111**	12.302

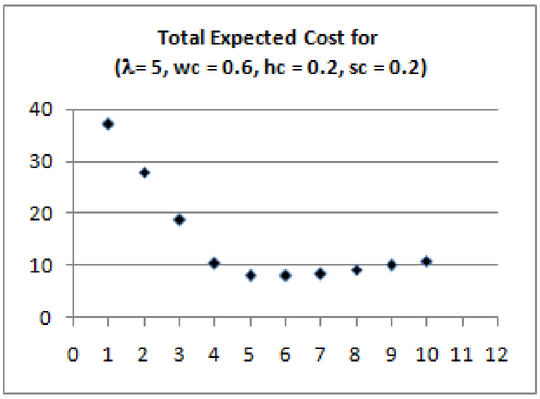
		(1.2, 0.2, 0.2)	65.678	60.112	51.940	42.704	34.123	26.171	18.429	13.944	12.890	**12.871**
		(1.5, 0.2, 0.2)	79.271	72.321	62.214	50.820	40.218	30.379	20.793	15.153	13.670	**13.439**
		(1.8, 0.2, 0.2)	92.865	84.529	72.489	58.936	46.314	34.587	23.158	16.363	14.449	**14.008**
	10	(0.3, 0.2, 0.2)	24.957	23.994	22.101	19.779	17.573	15.466	13.540	11.684	**11.213**	11.544
		(0.6, 0.2, 0.2)	38.646	36.551	32.989	28.751	24.698	20.799	17.186	13.684	12.371	**12.323**
		(0.9, 0.2, 0.2)	52.334	49.108	43.877	37.724	31.823	26.132	20.832	15.684	13.529	**13.103**
		(1.2, 0.2, 0.2)	66.023	61.664	54.765	46.697	38.947	31.465	24.478	17.684	14.688	**13.882**
		(1.5, 0.2, 0.2)	79.711	74.221	65.653	55.669	46.072	36.798	28.124	19.684	15.846	**14.662**
		(1.8, 0.2, 0.2)	93.399	86.778	76.541	64.642	53.197	42.131	31.770	21.683	17.004	**15.442**

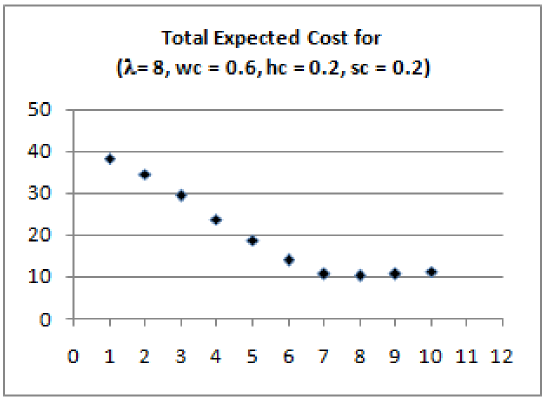
Total Expected cost		(Mean waiting cost, Holding cost, Mean service cost)	Service rate									
			1	2	3	4	5	6	7	8	9	10
Arrival rate	5	(0.3, 0.4, 0.4)	34.475	26.141	17.662	9.980	**7.748**	7.826	8.288	9.003	9.865	10.763
		(0.6, 0.4, 0.4)	47.332	35.414	23.295	12.280	8.828	**8.578**	8.852	9.473	10.293	11.161
		(0.9, 0.4, 0.4)	60.189	44.687	28.928	14.580	9.908	**9.330**	9.417	9.943	10.721	11.559
		(1.2, 0.4, 0.4)	73.046	53.960	34.562	16.879	10.987	10.082	**9.982**	10.412	11.148	11.956
		(1.5, 0.4, 0.4)	85.904	63.233	40.195	19.179	12.067	10.834	**10.546**	10.882	11.576	12.354
		(1.8, 0.4, 0.4)	98.761	72.506	45.828	21.479	13.146	11.586	**11.111**	11.352	12.004	12.751
	6	(0.3, 0.4, 0.4)	34.921	28.754	21.693	15.228	9.418	**8.401**	8.611	9.120	9.790	10.638
		(0.6, 0.4, 0.4)	47.997	39.028	28.818	19.435	10.967	9.181	**9.105**	9.438	9.993	10.793
		(0.9, 0.4, 0.4)	61.073	49.303	35.942	23.643	12.516	9.961	**9.598**	9.756	10.196	10.947
		(1.2, 0.4, 0.4)	74.149	59.578	43.067	27.851	14.064	10.740	10.091	**10.075**	10.400	11.101
		(1.5, 0.4, 0.4)	87.224	69.853	50.192	32.059	15.613	11.520	10.585	**10.393**	10.603	11.255
		(1.8, 0.4, 0.4)	100.300	80.127	57.317	36.267	17.162	12.299	11.078	**10.711**	10.806	11.409
	7	(0.3, 0.4, 0.4)	35.137	30.593	24.736	19.141	14.086	9.960	**9.368**	9.674	10.222	10.895
		(0.6, 0.4, 0.4)	48.468	41.731	33.160	24.956	17.503	11.355	**10.148**	10.199	10.583	11.139
		(0.9, 0.4,	61.800	52.868	41.584	30.770	20.921	12.749	10.927	**10.723**	10.944	11.383

		0.4)										
		(1.2, 0.4, 0.4)	75.131	64.006	50.009	36.584	24.339	14.143	11.707	**11.248**	11.306	11.627
		(1.5, 0.4, 0.4)	88.463	75.144	58.433	42.398	27.756	15.538	12.486	11.773	**11.667**	11.871
		(1.8, 0.4, 0.4)	101.795	86.281	66.857	48.213	31.174	16.932	13.266	12.297	**12.028**	12.115
	8	(0.3, 0.4, 0.4)	35.183	31.909	27.123	22.274	17.761	13.517	10.643	**10.343**	10.722	11.305
		(0.6, 0.4, 0.4)	48.665	43.684	36.585	29.398	22.683	16.343	11.929	**11.122**	11.271	11.701
		(0.9, 0.4, 0.4)	62.147	55.458	46.047	36.523	27.606	19.170	13.215	11.902	**11.821**	12.098
		(1.2, 0.4, 0.4)	75.629	67.232	55.509	43.648	32.528	21.997	14.501	12.682	**12.370**	12.494
		(1.5, 0.4, 0.4)	89.111	79.006	64.971	50.773	37.450	24.823	15.787	13.461	12.919	**12.890**
		(1.8, 0.4, 0.4)	102.593	90.780	74.433	57.898	42.372	27.650	17.073	14.241	13.468	**13.287**
	9	(0.3, 0.4, 0.4)	35.201	32.762	28.957	24.596	20.577	16.889	13.309	11.420	**11.323**	11.761
		(0.6, 0.4, 0.4)	48.795	44.971	39.232	32.712	26.673	21.096	15.673	12.629	**12.103**	12.329
		(0.9, 0.4, 0.4)	62.388	57.180	49.506	40.828	32.768	25.304	18.037	13.839	**12.882**	12.898
		(1.2, 0.4, 0.4)	75.982	69.388	59.781	48.944	38.864	29.512	20.402	15.049	13.662	**13.466**
		(1.5, 0.4, 0.4)	89.575	81.597	70.056	57.060	44.959	33.720	22.766	16.258	14.442	**14.035**
		(1.8, 0.4, 0.4)	103.168	93.806	80.330	65.176	51.054	37.927	25.130	17.468	15.221	**14.604**
	10	(0.3, 0.4, 0.4)	35.226	33.432	30.313	26.584	23.022	19.599	16.434	13.369	**12.267**	12.308
		(0.6, 0.4, 0.4)	48.915	45.989	41.201	35.557	30.147	24.932	20.080	15.369	13.426	**13.087**
		(0.9, 0.4, 0.4)	62.603	58.546	52.089	44.530	37.271	30.265	23.726	17.369	14.584	**13.867**
		(1.2, 0.4, 0.4)	76.292	71.102	62.977	53.502	44.396	35.598	27.372	19.368	15.742	**14.646**
		(1.5, 0.4, 0.4)	89.980	83.659	73.865	62.475	51.521	40.931	31.018	21.368	16.900	**15.426**
		(1.8, 0.4, 0.4)	103.668	96.215	84.754	71.448	58.645	46.264	34.664	23.368	18.059	**16.206**









Optimal service rates are 5,8 for arrival rates 5 and 8 respectively in which $\frac{\lambda}{\mu}=1$, the optimal cost exists. Where $\mu<\lambda$ or $\frac{\lambda}{\mu}>1$ the cost is high and conjunction in queue occurs.

IX. Conclusion

When $\lambda<\mu$, also the service cost increasing and hence the total cost becomes high. From the plots we have $\frac{\lambda}{\mu}=1$ is only the $\lambda=\mu$ (arrival rate is equal to service rate), the cost is minimum and the by policy iteration method we found the solution $R^*=(2,2,1,2,1,2,2,2,1,2,1,2,1,2,1,2,1,2,1,2,1,2,1,2,1,1,1,2,1,2,1,2,1,2)$. whenever the service rates and arrival rates are almost equal the long run total cost minimum. In future the authors plan to work as the discrete MDP systems with perishable inventory.

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