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Static Spherically Symmetric VacuumSolution in 5-Dimensions

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Abstract
We present static spherically symmetric vacuum solution in 5- dimensions near a gravitating body. The solution is obtained in a weak gravitational limit.

I. Introduction:

At present, it is believed that Einstein's general theory of relativitybe the most accurate theory of gravity. The standard cosmological modelmay explain the observational data up to an extremely high accuracy byusing a handful set of parameters. However, it demands that more than 95% of the universe may be made of dark matter and dark energy, a hypothetical form of matter and energy. However, even after several attempts to search for dark matter in the past few decades, there is not asingle evidence for dark matter. This shows that there might be some fundamental problem in the basic theory. Attempts were made foralternate cosmological theories such as, Milgrom (1983), Moffat (2006) etc. The theories like Sotiron and Faraoni (2008), though motivated from string theory, requires to select the function *f(R)* in a random manner. Das (2012) presented 5-dimensional Machian gravity, to explain galactic velocity profile data and the accelerated expansion of the universe without demanding any form of dark matter or dark energy. Inthis paper we have presented static spherically symmetric vacuumsolution in 5-dimensions.

II. The Metric and Field Equation:

The five dimensional coordinate system which is composed of one time, 3-spatial and one background component i.e. (t, x^i, ξ). The matric reads

$$ds^2 = \bar{g}_{AB} dx^A dx^B, \tag{1}$$

where $\bar{g}_{AB}$ is the 5-dimensional metric. The indices A,B,C .....run from 0 to 4. The indices α, β, γ.......... run from 0 to 3 and indices i, j, k .... i.e. spatial indices run from 1 to 3. The fifth dimension ξas a space-like coordinate. If there is no external force acting on a particle, then particle will follow a geodesic path. The geodesic equation reads

$$\frac{d^2x^A}{ds^2} - {}_{BC}\frac{dx^B}{ds}\frac{dx^C}{ds} \quad 0. \tag{2}$$

Let us put

$$\frac{dx^2}{ds} \approx 0, \tag{3}$$

and

$$\frac{d\xi}{ds} \approx 0. \tag{4}$$

Under above assumptions, we get

$$ds \approx dt \tag{5}$$

As $\frac{dx^0}{ds} \approx 1$, we get

$$\frac{d^2 x^A}{dt^2} = \bar{\Gamma}^A_{00}. \tag{6}$$

Let us consider flat coordinate system, so, one obtain the metric as

$$\bar{g}_{AB} = diag\,(1,-1,-1,-1,-1) + \bar{\gamma}_{AB} \tag{7}$$

Now

$$\bar{\Gamma}^A_{00} = \frac{1}{2} g^{AB} \left(2g_{B0,0} - \bar{g}_{00,B}\right) \tag{8}$$

Let us consider $\bar{g}_{B0,0} \approx 0$, so we obtain

Let us consider $\bar{g}_{B0,0} \approx 0$, so we obtain

$$\Gamma^A_{00} = \frac{1}{2}\gamma_{00,A} \tag{9}$$

Then the field equation reads

$$\frac{d^2 x^A}{dt^2} = \frac{1}{2}\frac{\partial \bar{\gamma}_{00}}{\partial x^A} \tag{10}$$

In view of Newton's law

$$\frac{d^2 x^A}{dt^2} = \frac{\partial \varphi}{\partial x^A} \tag{11}$$

Comparing eqs. (10) and (11) one obtains

$$\bar{\gamma}_{00} = 2\varphi. \tag{12}$$

III. Vacuum Solution:

In view of field equation, one obtains

$$\bar{R}_{AB} = \bar{T}_{AB} - \frac{1}{2} g_{AB} \bar{T} \tag{14}$$

The oo component of eq. (14) reduces to

$$\bar{R}_{oo} = \bar{T}_{oo} - \frac{1}{2} g_{oo} \bar{T} \tag{15}$$

which is the Newton's law. For the vacuum solution $\bar{T}_{oo} = 0$ and $\bar{T} = 0$, which provides

$$\bar{R}_{oo} = 0. \tag{16}$$

In view of static solution, one obtains

$$\partial_\xi^2 \bar{\gamma}_{oo} + \partial_x^2 \bar{\gamma}_{oo} + \partial_y^2 \bar{\gamma}_{oo} + \partial_z^2 \bar{\gamma}_{oo} = 0 \tag{17}$$

In spherical polar coordinates and the assumption of spherical symmetry, we get

$$\partial_\xi^2 (r\bar{\gamma}_{oo}) + \partial_r^2 (r\bar{\gamma}_{oo}) = 0 \tag{18}$$

In order to obtain the solution of eq. (18), let us put

$$r\bar{\gamma}_{oo} = R(r)\chi(\xi), \tag{19}$$

Hence, we obtain

$$\frac{1}{R}\frac{\partial^2 R}{\partial r^2} = -\frac{1}{\chi}\frac{\partial^2 \chi}{\partial \xi^2} = k^2 \quad \text{(say)}, \tag{20}$$

where k is a constant. The solution reads

$$R = D_1 e^{kr} + D_2 e^{-kr} \tag{21}$$

and

$$\chi = Q_1 Cos\,(k\xi) + Q_2 \sin\,(k\xi) \tag{22}$$

where D_1, D_2, Q_1, and Q_2 are constants.

Let us put $D_1 = 0$ due to weak field approximation, one obtains

$$\left(r\bar{\gamma}_{oo}\right) = P + D_2 \bar{e}^{kr}\left(Q_1 \cos(k\xi) + Q_2 \sin(k\xi)\right), \qquad (23)$$

where P is adding constant. However for $k\xi \square 0$, we get $Cos\,(k\xi) \to 1$ and $\sin(k\xi) \to 0$. Hence,

$$\bar{\gamma}_{oo} = \frac{P}{r} - KM\,\frac{\bar{e}^{kr}}{r} \qquad (24)$$

where K and M are constants. Let us replace P by *(K+1) M,* so one obtains

$$\bar{\gamma}_{oo} = \frac{M}{r}\left[1 + K\left(1 - \bar{e}^{kr}\right)\right]. \qquad (25)$$

Hence, inside a galaxy the Newtonian potential assumes the form

$$\Phi = \frac{GM}{r}\left[1 + K\left(1 - \bar{e}^{kr}\right)\right]. \qquad (26)$$

For r small, one obtains

$$\bar{e}^{kr} \square 1 \qquad (27)$$

Hence

$$\Phi = \frac{GM}{r} \qquad (28)$$

IV. Concluding Remarks:

We have obtained static spherically symmetric vacuum solution in 5-dimensional spacetime. This solution may be used to demonstrate the galactic velocity profile in absence of additional dark matters.

References

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