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Common Fixed Point Theorem of Semi Compatible Mapping in Intuitionisticfuzzy Metric Space

Rajesh Shrivastava[1], Arihant Jain[2] and Amit Kumar Gupta[1]
[1]*Department of Mathematics, Govt. Science and Commerce College,Benazir, Bhopal (M.P.) India*
[2]*Department of Mathematics, Shri Guru Sandipani Girls' Institute of Professional Studies, Ujjain (M.P.) 456 550 India*
[1]*Department of Mathematics,Govt. Science and Commerce College,Benazir, Bhopal(M.P.) India*
Corresponding Auther:Rajesh Shrivastava

Abstract: The *object of this paper is to use the concept of semi-compatible mapping and prove a common fixed point theorems for six Semi compatible self-maps in Intuitionistic Fuzzy Metric space using implicit relation. We have also cited an example in support of our result.*
Keywords and Phrases: *Intuitionistic Fuzzy Metric space, Common fixed points, Compatible maps andSemi compatibility. Compatible AMS Subject Classification (2000).Primary 47H10, Secondary 54H25.*

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I. Introduction

In 1965 the concept of fuzzy sets was defined by Zadeh [15]. Since then, to use this concept in topology and analysis, many authors have expansively developed the theory of fuzzy sets and applications. As a generalization of fuzzy sets, Atanassov [3] introduced and studied the concept of intuitionistic fuzzy sets, In 2004, Park [10] defined the concept of intuitionistic fuzzy metric space with the help of continuous t- norms and continuous t-conorms. Recently, in 2006, Alaca et. al. [1] using the notion of intuitionistic fuzzy sets, defined the concept of intuitionistic fuzzy metric space with the help of continuous t–norms and continuous t-conorms as a generalization of fuzzy metric space which is introduced by Kramosil and Michalek [8]. Turkoglu et. al. [13] gave generalization of Jungck's [6] common fixed point theorem in intuitionistic fuzzy metric spaces. They first created the concept of weakly commuting and R-weaklycommuting mappings in intuitionistic fuzzy metric spaces. The concept of weakly compatible mappings is most general as each pair of compatible mappings is weakly compatible but the converse is not true. After that, many authors proved common fixed point theorems using different mappings in such spaces. Pant[9] introduced the concept of weakly compatible mappings is most general as every commuting pair is R-weakly commuting ,each pair of R-weakly commuting mapping is compatible and each pair of compatible mappings is weakly compatible but reverse is not true. Cho, Sharma and Sahu[4] have introduced the notion of semi compatible mapping. The aim of this paper is to use the concept of semi compatible mapping and prove common fixed point theorem for six semi compatible mapping in intuitionistic fuzzy metric space using implicit relations.

II. Preliminaries

DEFINITION (2.1)[10]: A binary operation $*: [0, 1] \times [0, 1] \to [0, 1]$ is continuous t-norm if $*$ is satisfying the following conditions:
(i) $*$ is commutative and associative;
(ii) $*$ is continuous;
(iii) $a * 1 = a$ for all $a \in [0, 1]$;
(iv) $a * b \le c * d$ whenever $a \le c$ and $b \le d$ for all $a, b, c, d \in [0, 1]$.
DEFINITION (2.2)[10]: A binary operation $\Diamond: [0, 1] \times [0, 1] \to [0, 1]$ is continuous t-conorm if $\Diamond$ is satisfying the following conditions:
(i) $\Diamond$ is commutative and associative;
(ii) $\Diamond$ is continuous;
(iii) $a \Diamond 0 = a$ for all $a \in [0, 1]$;
(iv) $a \Diamond b \ge c \Diamond d$ whenever $a \le c$ and $b \le d$ for all $a, b, c, d \in [0, 1]$.

DEFINITION (2.3)[1]: A 5-tuple $(X, M, N, *, \Diamond)$ is said to be an intuitionistic fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm, $\Diamond$ is a continuous t-conorm and M, N are fuzzy sets on $X^2 \times (0, \infty)$ satisfying the following conditions:-

(i) $M(x, y, t) + N(x, y, t) \leq 1$, for all $x, y \in X$ and $t > 0$;
(ii) $M(x, y, 0) = 0$, for all $x, y \in X$;
(iii) $M(x, y, t) = 1$, for all $x, y \in X$ and $t > 0$ if and only if $x = y$;
(iv) $M(x, y, t) = M(y, x, t)$, for all $x, y \in X$ and $t > 0$;
(v) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$, for all $x, y, z \in X$ and $s, t > 0$;
(vi) For all $x, y \in X$, $M(x, y, \cdot): [0, \infty) \to [0, 1]$ is continuous;
(vii) $\lim_{t\to\infty} M(x, y, t) = 1$, for all $x, y \in X$ and $t > 0$;
(viii) $N(x, y, 0) = 1$ for all $x, y \in X$;
(ix) $N(x, y, t) = 0$ for all $x, y \in X$ and $t > 0$ if and only if $x = y$;
(x) $N(x, y, t) = N(y, x, t)$ for all $x, y \in X$ and $t > 0$;
(xi) $N(x, y, t) \lozenge N(y, z, s) \geq N(x, z, t + s)$ for all $x, y, z \in X$ and $s, t > 0$;
(xii) For all $x, y \in X$, $N(x, y, \cdot) : [0, \infty) \to [0, 1]$ is continuous;
(xiii)$\lim_{t\to\infty} N(x, y, t) = 0$,for all x, y in X.
Then (M, N) is called an intuitionistic fuzzy metric on X. The functions M(x, y, t) and N(x, y, t) denote the degree of nearness and the degree of non-nearness between x and y with respect to t, respectively.

REMARK[2.1]: Every fuzzy metric space (X, M, *) is an intuitionistic fuzzy metric space of the form (X, M, 1-M, *, ◊) such that t-norm * and t-conorm ◊ are associated as:
$x \lozenge y = 1- ((1-x) * (1-y))$, for all $x, y \in X$

REMARK[2.2]: In intuitionistic fuzzy metric space (X, M, N, *, ◊), M(x, y, .) isnon-decreasing and N(x, y, ·) is non-increasing for all x, y ∈ X.

EXAMPLE[2.1]: Let (X, d) be a metric space, define t-norm a * b=Min {a, b} and t- conorma ◊ b = Max{a b} and for all x,y∈ X and t > 0

$$M_d(x,y,t) = \frac{t}{t+d(x,y)} \quad , N_d(x,y,t) = \frac{d(x,y)}{t+d(x,y)}$$

Then (X, M, N, *, ◊) is an intuitionistic fuzzy metric space. We call this intuitionistic fuzzy metric (M, N) induced by the metric d the standard intuitionistic fuzzy metric.

DEFINITION (2.4)[1]: Let (X, M, N, *, ◊) be an intuitionistic fuzzy metric space. Then
(a) A sequence $\{x_n\}$ in X is said to be Cauchy sequence if for all t > 0 and p > 0,

$$\lim_{n\to\infty} M(x_{n+p}, x_{n,\, t}) = 1, \lim_{n\to\infty} N(x_{n+p}, x_{n,\, t}) = 0$$

(b)) A Sequence $\{x_n\}$in X is said to be Convergent to a point x∈ X if , for all t >0,

$$\lim_{n\to\infty} M(x_{n+p}, x_{n,\, t}) = 1, \lim_{n\to\infty} N(x_{n+p}, x_{n,\, t}) = 0$$

Since * and ◊ are continuous, the limit is uniquely determined from (v) and (xi) of definition (2.3), respectively.

DEFINITION (2.5)[1]: An intuitionistic fuzzy metric space (X, M, N, *, ◊) is said to be complete if and only if every Cauchy sequence in X is convergent.

DEFINITION (2.6)[11]: Let A and B be mappings from an intuitionistic fuzzy metric space (X, M, N, *, ◊) into itself. Then the maps A and B are said to be compatible if, for all t > 0,

$\lim_{n\to\infty} M(ABx_n, BAx_n, t) = 1, \lim_{n\to\infty} N(ABx_n, BAx_n, t) = 0$
Whenever $\{x_n\}$ is a sequence in X such that- $\lim_{n\to\infty} Ax_n, = \lim_{n\, n\to\infty} Bx_n = x$, for some x∈ X.

DEFINITION (2.7)[12]: Let A and B be mappings from an intuitionistic fuzzy metric space (X, M, N, *, ◊) into itself. Then the maps A and B are said to be Semi-compatible if,
$\lim_{n\to\infty} M(ABx_n, Bx, t) = 1, \lim_{n\to\infty} N(ABx_n, Bx, t) = 0$, for all t > 0,
Whenever $\{x_n\}$ is a sequence in X such that- $\lim_{n\to\infty} Ax_n, = \lim_{n\to\infty} Bx_n = x$, for some x∈ X.
DEFINITION (2.8)[7]: Two self-maps A and B in a intuitionistic fuzzy metric space (X, M, N, *, ◊) is said to be weak compatible if they commute at their coincidence points. i.e. Ax = Bx for some x in X, then ABx = BAx.
DEFINITION (2.9)[7]: Let (X, M, N, *,◊) be an intuitionistic fuzzy metric space, A and B be self-maps in X, Then a point x in X is called a coincidence point of A and B iff Ax = Bx. In this case y = Ax = Bx is called a point of coincidence of A and B.

It is easy to see that two compatible maps are weakly compatible but converse is not true.

Lemma (2.1)[1]: Let (X, M, N, *, ◊) be an intuitionistic fuzzy metric space and $\{y_n\}$ be a sequence in X , if there exist a number k∈(0, 1) such that

$M(y_{n+2}, y_{n+1}, kt) \geq M(y_{n+1}, y_n, t)$

$N(y_{n+2}, y_{n+1}, kt) \leq N(y_{n+1}, y_n, t)$

for all t > 0 and n=1, 2, 3, … , then $\{y_n\}$ is a Cauchy sequence in X.

Lemma (2.2)[14]: Let (X, M, N, *, ◊) be an intuitionistic fuzzy metric space and for all x, y in X, t > 0 and if there exists a number k ∈(0, 1)

$M(x, y, kt) \geq M(x, y, t)$ and $N(x, y, kt) \leq N(x, y, t)$, then x=y.

Proposition 2.1. Let $\{x_n\}$ be a Cauchy sequence in intuitionistic fuzzy metric space (X, M, N, *, ◊) with continuous t-norm t. If the subsequence $\{x_{2n}\}$ converges to x in X, then $\{x_n\}$ also converges to x.

A class of implicit relation. Let Φbe the set of all real continuous functions $\phi: (R^+)^4 \to R$, non-decreasing in the first argument with the property :

a. For u, v ≥ 0, $\phi(u, v, v, u) \geq 0$ or $\phi(u,v,u,v) \geq 0$ implies that u ≥ v.

b. $\phi(u, u, 1, 1) \geq 0$ implies that u ≥ 1.

Example 2.1. Define: $\phi(t_1,t_2,t_3,t_4) = 18t_1 - 16t_2 + 8t_3 - 10t_4$,

and $\Psi(t_1,t_2,t_3,t_4) = 18t_1 - 16t_2 + 8t_3 - 10t_4$. Then φ, Ψ∈Φ.

3. Main Result.

Theorem 3.1. Let A, B, L, M, S and T be self-mappings on anintuitionistic fuzzy metric space (X, M, N, *,◊) with continuous t-norm and continuous t-conorm satisfying:

(3.1.1) L(X)⊆ST(X), M(X)⊆(X);

(3.1.2) ST(X) and AB(X) are Complete subspace of X.

(3.1.3) the pairs (L, AB) is semi compatible and (M, ST) are weak-compatible;

(3.1.4) Either AB or L is continuous;

(3.1.5) for some φ, Ψ∈Φ, there exists k ∈(0, 1) such that for all x, y ∈X and t > 0,

$\phi[M(Lx,My,kt), M(ABx,STy,t), M(Lx,ABx,t), M(My,STy,kt)] \geq 0$

and $\Psi[N(Lx,My,kt), N(ABx,STy,t), N(Lx,ABx,t), N(My,STy,kt)] \leq 0$

then A, B,L,M, S and T have a unique common fixed point in X.

Proof. Let $x_0 \in X$. From condition (3.1.1),∃ $x_1, x_2 \in X$ such that

$Lx_0 = STx_1 = y_0$ and $Mx_1 = ABx_2 = y_1$

Inductively, we can construct sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$Lx_{2n} = STx_{2n+1} = y_{2n}$ and $Mx_{2n+1} = ABx_{2n+2} = y_{2n+1}$

for n = 0, 1, 2, ...

Step 1. Putting $x = x_{2n}$ and $y = x_{2n+1}$ in (3.1.5), we get

$\phi[M(Lx_{2n},Mx_{2n+1},kt),M(ABx_{2n},STx_{2n+1},t),M(Lx_{2n},ABx_{2n},t),M(Mx_{2n+1},STx_{2n+1},kt)] \geq 0$ and

$\Psi[N(Lx_{2n},Mx_{2n+1},kt),N(ABx_{2n}STx_{2n+1},t),N(Lx_{2n},ABx_{2n},t),N(Mx_{2n+1},STx_{2n+1},kt)] \leq 0$.

Letting n →∞, we get

$\phi[M(y_{2n},y_{2n+1},kt), M(y_{2n-1},y_{2n}, t), M(y_{2n},y_{2n-1},t), M((y_{2n+1},y_{2n},kt,)] \geq 0$

$\Psi[N(y_{2n},y_{2n+1}, kt), N(y_{2n-1},y_{2n}, t), N(y_{2n}, y_{2n-1}, t), N((y_{2n+1},y_{2n},kt,)] \leq 0$.

Using (a), we get

$M(y_{2n},y_{2n+1},kt) \geq M(y_{2n-1},y_{2n}, t)$ and $N(y_{2n},y_{2n+1},kt) \leq N(y_{2n-1},y_{2n}, t)$.

Therefore, for all n even or odd, we have

$M(y_n,y_{n+1},kt) \geq M(y_{n-1},y_n, t)$ and $N(y_n,y_{n+1},kt) \leq N(y_{n-1},y_n, t)$.

Therefore, by lemma 2.1, $\{y_n\}$ is a Cauchy sequence in X, Which is complete. Therefore $\{y_n\}$ converges to z ∈ X. it subsequence converges as follows:

$\{Mx_{2n+1}\} \to z$ and $\{STx_{2n+1}\} \to z$

$\{Lx_{2n}\} \to z$ and $\{ABx_{2n}\} \to z$.

Case I. AB is continuous. Since AB is continuous, $AB(AB)x_{2n} \to ABz$ and $(AB)Lx_{2n} \to ABz$

Since (L, AB) is semi compatible then $L(AB)x_{2n} \to ABz$.

Step 2. Putting $x = ABx_{2n}$ and $y = x_{2n+1}$ in (3.1.5) we get,

ϕ[M{L(AB)x_{2n},Mx_{2n+1} ,kt }, M{AB(AB)x_{2n}, ST x_{2n+1} ,t}, M{L(AB)x_{2n},AB(AB)x_{2n},t },
M{Mx_{2n+1}, STx_{2n+1} ,kt}]≥ 0
Ψ[{ N(L(AB)x_{2n},Mx_{2n+1} ,kt }, N{AB(AB)x_{2n}, ST x_{2n+1} ,t}, N{L(AB)x_{2n},AB(AB)x_{2n} ,t },
N{Mx_{2n+1}, STx_{2n+1} ,kt}] ≤ 0
Taking limit as n $\rightarrow\infty$ be get
ϕ[M{ABz,z,kt }, M{ABz, z ,t}, M{ABz, ABz ,t }, M{z, z ,kt}]≥ 0 and
Ψ[N{ABz,z,kt }, N{ABz, z ,t}, N{ABz, ABz ,t }, N{z, z ,kt}] ≤ 0
$\Rightarrow\phi$[M{ABz,z,kt }, M{ABz, z ,t}, 1, 1] ≥ 0 and
Ψ[N{ABz,z,kt }, N{ABz, z ,t},1, 1] ≤ 0
As $\phi$is non-decreasing and Ψ is non-increasing in the first argument, we have
$\Rightarrow\phi$[M{ABz,z,t }, M{ABz, z ,t}, 1, 1] ≥ 0 and
Ψ[N{ABz,z,t }, N{ABz, z ,t},1, 1] ≤ 0
Using (b) we have
M{ABz, z, t } =1 , for all t > 0
N{ABz, z, t } = 0, for all t > 0
Thus
ABz = z

Step 3. Putting x = z and y =x_{2n+1} in (3.1.5) we get,
ϕ[M{Lz, Mx_{2n+1} ,kt }, M{ABz, ST x_{2n+1} ,t}, M{Lz, ABz ,t M{Mx_{2n+1}, STx_{2n+1} ,kt}] ≥ 0 and
Ψ[{ N(Lz, Mx_{2n+1} ,kt }, N{ABz, ST x_{2n+1} ,t}, N{Lz, ABz ,t }, N{Mx_{2n+1}, STx_{2n+1} ,kt}] ≤ 0
Taking limit as n $\rightarrow\infty$ be get
ϕ[M{Lz, z ,kt }, M{ABz, z ,t}, M{Lz, ABz ,t} M{z, z ,kt}] ≥ 0 and
Ψ[{ N(Lz, z ,kt }, N{ABz, z ,t}, N{Lz, ABz ,t }, N{z, z ,kt}] ≤ 0
$\Rightarrow\phi$[M{Lz, z ,kt }, 1, M{Lz, z ,t}, 1] ≥ 0 and
Ψ[{ N(Lz, z ,kt }, 1, N{Lz, z ,t },1] ≤ 0
As $\phi$is non-decreasing and Ψ is non-increasing in the first argument, we have
$\Rightarrow\phi$[M{Lz, z ,t }, 1, M{Lz, z ,t}, 1] ≥ 0 and
Ψ[N{(Lz, z ,t }, 1, N{Lz, z ,t },1] ≤ 0
Using (b) we have
M{Lz, z, t } =1 , for all t > 0
N{Lz, z, t } = 0, for all t > 0
Thus Lz = z
Combine the result from step II and step III , we have
ABz = Lz = z

Step 4. Putting x = Bz and y =x_{2n+1} in (3.1.5) we get,
ϕ[M{L(Bz), Mx_{2n+1} ,kt }, M{AB(Bz), ST x_{2n+1} ,t}, M{L(Bz), AB(Bz) ,t },
M{Mx_{2n+1}, STx_{2n+1} ,kt}] ≥ 0 and
Ψ[N{(L(Bz), Mx_{2n+1} ,kt }, N{AB(Bz), ST x_{2n+1} ,t}, N{L(Bz), AB(Bz) ,t },
N{Mx_{2n+1}, STx_{2n+1} ,kt}] ≤ 0
Taking limit as n $\rightarrow\infty$ we get
ϕ[M{Bz, z ,kt }, M{Bz, z, t}, M{Bz, Bz ,t }, M{z, z ,kt}] ≥ 0 and
Ψ[N{Bz, z ,kt }, N{Bz, z ,t}, N{Bz, Bz ,t } , N{z, z ,kt}] ≤ 0
$\Rightarrow\phi$[M{Bz, z ,kt }, M{Bz, z, t},1 , 1] ≥ 0 and
Ψ[N{Bz, z ,kt }, N{Bz, z ,t},1 , 1] ≤ 0
As $\phi$is non-decreasing and Ψ is non-increasing in the first argument, we have
$\Rightarrow\phi$[M{Bz, z ,t }, M{Bz, z, t},1 , 1] ≥ 0 and
Ψ[N{Bz, z ,t }, N{Bz, z ,t},1 , 1] $\leq$□ 0
Using (b) we have
M{Bz, z, t } =1 , for all t > 0
N{Bz, z, t } = 0, for all t > 0
Thus Bz = z
Combine the result from step 2, step 3 , and step 4we have
z = Az = Bz = Lz

Step 5. Since L(X) $\subseteq$ ST(X). there exist v ϵ X such that z = Lz = STv . Putting x = x_{2n} and y = v in (3.1.5) we get,

$\phi[\, M\{Lx_{2n,}Mv ,kt \}, M\{ABx_{2n,}\ STv ,t\}, M\{Lx_{2n,}\ ABx_{2n} ,t \},\ M\{Mv, STv ,kt\}] \geq 0$ and

$\Psi[\, N\{Lx_{2n,}Mv ,kt \}, N\{ABx_{2n,}\ STv ,t\}, N\{Lx_{2n,}ABx_{2n} ,t \},\ N\{Mv, STv ,kt\}] \leq 0$

Taking limit as $n \to\infty$ be get

$\phi[\, M\{z, Mv ,kt \}, M\{z, STv ,t\}, M\{z, z ,t \},\ M\{Mv, STv ,kt\}] \geq 0$ and

$\Psi[\, N\{z, Mv ,kt \}, N\{z, STv ,t\}, N\{z, z ,t \},\ N\{Mv, STv ,kt\}] \leq 0$

$\Rightarrow\phi[\, M\{z, Mv ,kt \}, 1, 1,\ M\{Mv, STv ,kt\}] \geq 0$ and

$\Psi[\, N\{z, Mv ,kt \}, 1, 1,\ N\{Mv, STv ,kt\}] \leq 0$

As $\phi$is non-decreasing and Ψ is non-increasing in the first argument, we have

$\Rightarrow\phi[\, M\{z, Mv ,t \}, 1, 1,\ M\{Mv, STv ,t\}] \geq 0$ and

$\Psi[\, N\{z, Mv ,t \}, 1, 1,\ N\{Mv, STv ,t\}] \leq 0$

Using (b) we have

$M\{z, Mv, t \} =1$, for all $t > 0$

$N\{z, Mv , t \} = 0$, for all $t > 0$

Thus $z = Mv$

Therefore z = Mv = STv . since (M, ST) is weakly compatible , we get that M(STv) = ST(Mv).

That is , Mz = STz.

Step 6 . Putting $x = x_{2n}$ and y = z in (3.1.5) and using step 5 , we have

$\phi[\, M\{Lx_{2n,}Mz ,kt \}, M\{ABx_{2n,}\ STz ,t\}, M\{Lx_{2n,}\ ABx_{2n} ,t \},\ M\{Mz, STz ,kt\}] \geq 0$ and

$\Psi[\, N\{Lx_{2n,}Mz ,kt \}, N\{ABx_{2n,}\ STz ,t\}, N\{Lx_{2n,}ABx_{2n} ,t \},\ N\{Mz, STz ,kt\}] \leq 0$

Taking limit as $n \to\infty$ be get

$\phi[\, M\{z, Mz ,kt \}, M\{z, Mz,t\}, M\{z, z ,t \},\ M\{Mz, Mz ,kt\}] \geq 0$ and

$\Psi[\, N\{z, Mz ,kt \}, N\{z, Mz ,t\}, N\{z, z ,t \},\ N\{Mz, Mz ,kt\}] \leq 0$

$\Rightarrow\phi[\, M\{z, Mz ,kt \}, M\{z, Mz,t\}, 1,\ 1] \geq 0$ and

$\Psi[\, N\{z, Mz ,kt \}, N\{z, Mz ,t\}, 1,\ 1] \leq 0$

As $\phi$is non-decreasing and Ψ is non-increasing in the first argument, we have

$\Rightarrow\phi[\, M\{z, Mz ,t \}, M\{z, Mz,t\}, 1,\ 1] \geq 0$ and

$\Psi[\, N\{z, Mz ,t \}, N\{z, Mz ,t\}, 1,\ 1] \leq 0$

Using (b) we have

$M\{z, Mz , t \} =1$, for all $t > 0$

$N\{z, Mz , t \} = 0$, for all $t > 0$

Thus z = Mz = STz.

Step 7. Putting $x = x_{2n}$ and y = Tz in (3.1.5) and using step 5 , we have

$\phi[\, M\{Lx_{2n,}M(Tz) ,kt \}, M\{ABx_{2n,}\ ST(Tz) ,t\}, M\{Lx_{2n,}\ ABx_{2n} ,t \},\ M\{M(Tz), ST(Tz) ,kt\}] \geq 0$ and

$\Psi[\, N\{Lx_{2n,}M(Tz) ,kt \}, N\{ABx_{2n,}\ ST(Tz) ,t\}, N\{Lx_{2n,}ABx_{2n} ,t \},\ N\{M(Tz), ST(Tz) ,kt\}] \leq 0$

Taking limit as $n \to\infty$, we get

$\phi[\, M\{z, Tz ,kt \}, M\{z, Tz ,t\}, M\{z, z ,t \},\ M\{ Tz, Tz ,kt\}] \geq 0$ and

$\Psi[\, N\{z, Tz ,kt \}, N\{z, Tz ,t\}, N\{z, z ,t \},\ N\{ Tz, Tz ,kt\}] \leq 0$

$\Rightarrow\phi[\, M\{z, Tz ,kt \}, M\{z, Tz ,t\},1, 1] \geq 0$ and

$\Psi[\, N\{z, Tz ,kt \}, N\{z, Tz ,t\},1, 1] \leq 0$

As $\phi$is non-decreasing and Ψ is non-increasing in the first argument, we have

$\Rightarrow\phi[\, M\{z, Tz ,t \}, M\{z, Tz ,t\},1, 1] \geq 0$ and

$\Psi[\, N\{z, Tz ,t \}, N\{z, Tz ,t\},1, 1] \leq 0$

Using (b) we have

$M\{z, Tz , t \} =1$, for all $t > 0$

$N\{z, Tz , t \} = 0$, for all $t > 0$

Thus z = Tz. since Tz = STz , we also have z = Sz

Therefore, z = Az = Bz = Lz = Mz = Sz = Tz

That is z is common fixed point of six maps.

Case II: L is continuous. Since L is continuous, $LLx_{2n} \rightarrow Lz$ and $L(AB)x_{2n} \rightarrow Lz$
Since (L, AB) is semicompatible then $L(AB)\, x_{2n} \rightarrow ABz$.
By uniqueness of limit in intuitionistic fuzzy metric space, we obtained that Lz = ABz .
Step 8: Putting x = zand y = x_{2n+1} in (3.1.5) we get,

$\phi[\, M\{Lz, Mx_{2n+1}, kt\}, M\{ABz, STx_{2n+1}, t\}, M\{Lz, ABz, t\}, M\{Mx_{2n+1}, STx_{2n+1}, kt\}] \geq 0$ and

$\Psi[\, N\{Lz, Mx_{2n+1}, kt\}, N\{ABz, STx_{2n+1}, t\}, N\{Lz, ABz, t\}, N\{Mx_{2n+1}, STx_{2n+1}, kt\}] \leq 0$

Taking limit as $n \rightarrow \infty$, we get

$\phi[\, M\{Lz, z, kt\}, M\{Lz, z, t\}, M\{Lz, Lz, t\}, M\{z, z, kt\}] \geq 0$ and

$\Psi[\, N\{Lz, z, kt\}, N\{Lz, z, t\}, N\{Lz, Lz, t\}, N\{z, z, kt\}] \leq 0$

$\Rightarrow \phi[\, M\{Lz, z, kt\}, M\{Lz, z, t\}, 1, 1] \geq 0$ and

$\Psi[\, N\{Lz, z, kt\}, N\{Lz, z, t\}, 1, 1] \leq 0$

As ϕ is non-decreasing and Ψ is non-increasing in the first argument, we have

$\Rightarrow \phi[\, M\{Lz, z, t\}, M\{Lz, z, t\}, 1, 1] \geq 0$ and

$\Psi[\, N\{Lz, z, t\}, N\{Lz, z, t\}, 1, 1] \leq 0$

Using (b) we have

$M\{z, Lz, t\} = 1$, for all $t > 0$
$N\{z, Lz, t\} = 0$, for all $t > 0$

Thus we have z = Lz therefore z = Lz = ABz and the rest of the proof follows from step 4.
Uniqueness : let w be another fixed point of A, B, L, M, S and T then w = Aw = Bw = Lw = Mw = Sw = Tw.
By taking x = z and y = w in (3.1.5) and we have

$\phi[\, M\{Lz, Mw, kt\}, M\{ABz, STw, t\}, M\{Lz, ABz, t\}, M\{Mw, STw, kt\}] \geq 0$ and

$\Psi[\, N\{Lz, Mw, kt\}, N\{ABz, STw, t\}, N\{Lz, ABz, t\}, N\{Mw, STw, kt\}] \leq 0$

$\Rightarrow \phi[\, M\{z, w, kt\}, M\{z, w, t\}, M\{z, z, t\}, M\{w, w, kt\}] \geq 0$ and

$\Psi[\, N\{z, w, kt\}, N\{z, w, t\}, N\{z, z, t\}, N\{w, w, kt\}] \leq 0$

$\Rightarrow \phi[\, M\{z, w, kt\}, M\{z, w, t\}, 1, 1] \geq 0$ and

$\Psi[\, N\{z, w, kt\}, N\{z, w, t\}, 1, 1] \leq 0$

As ϕ is non-decreasing and Ψ is non-increasing in the first argument, we have

$\Rightarrow \phi[\, M\{z, w, t\}, M\{z, w, t\}, 1, 1] \geq 0$ and

$\Psi[\, N\{z, w, t\}, N\{z, w, t\}, 1, 1] \leq 0$

Using (b) we have

$M\{z, w, t\} = 1$, for all $t > 0$
$N\{z, w, t\} = 0$, for all $t > 0$

Thus z = w and z is the unique common fixed point of the self-maps A, B, L, M, S and T.
On taking B= T = I (the identity maps on X) in the main theorem, we have the following:
Corollary 3.2 : Let A, L, M and S be self-mappings on an intuitionistic fuzzy metric space (X, M, N, *,◊) with continuous t-norm and continuous t-conorm satisfying:

(3.1.1) L(X) ⊆S(X), M(X) ⊆(X);
(3.1.2) S(X) and A(X) are Complete subspace of X.
(3.1.3) the pairs (L, A) is semi compatible and (M, S) are weak-compatible;
(3.1.4) Either A or L is continuous;
(3.1.5) for some $\phi, \Psi \in \Phi$, there exists $k \in (0, 1)$ such that for all $x, y \in X$ and $t > 0$,

$\phi(\, M(Lx,My,kt), M(Ax,Sy,t), M(Lx,Ax,t), M(My,Sy,kt)) \geq 0$

and $\Psi(\, N(Lx,My,kt), N(Ax,Sy,t), N(Lx,Ax,t), N(My,Sy,kt)) \leq 0$

then A, L,M and S have a unique common fixed point in X.
Example3.1: Let(x, d) be a Metric space, where X=[0, 1] and(X, M, N, *, ◊) be an intuitionistic fuzzy metric space, Define self-maps L, M, A and S as follows:

$$L(X)=M(X)=\begin{cases} 0, & x \in [0, \frac{5}{6}] \\ 1-x, & \text{otherwise} \end{cases},$$

$$A(X)=\begin{cases} 0, & x \in [0,\frac{4}{5}] \\ 1-x, & \text{otherwise} \end{cases}$$

$$S(X)=\begin{cases} 0, & x \in \left[0,\frac{3}{4}\right] \\ 1-x, & \text{otherwise} \end{cases}.$$

Then L(X)=M(X)=$[0,\frac{1}{6})$,A(X)=$[0, \frac{1}{5}]$ and S(X)=$[0,\frac{1}{4}]$, Hence the pair (L, A) and (M, A) are weak compatible and A(X) is complete . Further, for k =$\frac{1}{3}$ the condition (3.1.3) is satisfied. Thus, 0 is the unique common fixed point of the mappings A, L, M, andS.

III. Conclusion

In view of theorem3.1 is generalization of the result. The concept of semi compatible mapping and prove common fixed point theorem for six semi compatible mapping in intuitionistic fuzzy metric space using implicit relations.

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