# Bölcsföldi prime numbers

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**Abstract:** After defining, Bölcsföldi prime numbers will be presented from 23 to 33333333223. How many Bölcsföldi prime numbers are there in the interval  $(10^{p-1}, 10^p)$  (where p is a prime number)? On the one hand, it has been counted by computer among the prime numbers with up to 19-digits. On the other hand, the function (1) gives the approximate number of Bölcsföldi prime numbers in the interval  $(10^{p-1}, 10^p)$ . Near-proof reasonig has emerged from the conformity of Mills' prime numbers with Bölcsföldi prime numbers. The set of Bölcsföld prime numbers is probably infinite.

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#### I. Introduction

The sets of special prime numbers within the set of prime numbers are well-known. For instance, the Erdős-primes (the sum of the digits is prime) [8], Fibonacci-primes ( $F_0=0$ ,  $F_1=1$ ,  $F_n=F_{n-1}+F_{n-2}$ ), Gauss-primes (in the form 4n+3), Leyland-primes (in the form  $x^y+y^x$ , where  $1 \le x \le y$ ), Pell-primes ( $P_0=0$ ,  $P_1=1$ ,  $P_n=2P_{n-1}+P_{n-2}$ ), Bölcsföldi-Birkás primes (all digits are prime, the number of digits is prime, the sum of digits is prime), etc. Question: Which further sets of special prime numbers are there within the set of prime numbers? We have found a further set of special prime numbers within the set of prime numbers. It is the set of Bölcsföldi prime numbers.

### 2. Bölcsföldi prime numbers [3], [9], [10], [11].

Definition:a positive integer number is a Bölcsföldi prime number, if

a/ the positive integer number is prime, b/ all digits are 2 or 3, c/ the number of digits is prime, d/ the sum of digits is prime.

The set of prime numbers meeting the conditions a/ and c/ is also well-known: it is the set of prime-long prime numbers [3], [9]. Positive integer numbers meeting all the four conditions (a/, b/, c/, d/) at the same time are Bölcsföldi prime numbers (Fig.1, Fig.2).

Bölcsföldi prime number p has the following sum form:

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\begin{array}{ll} k(p) \\ p = \sum e_j(p).10^j & \text{where } e_j(p) \in \{2,3\} \text{ and } k(p) + 1 \text{ is prime and } e_0(p) \in \{3\} \\ i = 0 & \text{i} = 0 \end{array} \quad \begin{array}{ll} k(p) \\ \text{and } \sum e_j(p) \text{ is prime.} \\ i = 0 & \text{i} = 0 \end{array}
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The Bölcsföldi prime numbers are as follows (the last digit can only be 3):

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23,
223,
32233,
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32223,

2222333, 2223233, 2232323, 2233223, 2332333, 2333323, 3223223, 3223333, 3233323, 3332233,

32232333233, 32233322333, 32323223333, 32323333223, 3233232333, 3233233333, 32332323233,

3233322323, 32333332333, 33222323333, 332233332323, 33223333233, 33233322233, 33322233323, 3333222233, 33333222233, 33333322233, etc.

T(p) is the factual frequency of Bölcsföldi prime numbers in the interval  $(10^{p-1}, 10^p)$ .

T(2)=1, T(3)=1, T(5)=3, T(7)=10, T(11)=39, T(13)=104, T(17)=2526, T(19)=4915, etc. S(p) function gives the number of Bölcsföldi prime numbers in the interval  $(10^{p-1}, 10^p)$ . We think that  $S(p)=1,0404x1,6^{p-1}$ , where p is prime.

The factual number of Bölcsföldi primes and the number of Bölcsföldi primes calculated according to function (1) are as follows:

| Num         | ber of digits The factual number of     | The number of                       |                             |  |  |  |  |
|-------------|-----------------------------------------|-------------------------------------|-----------------------------|--|--|--|--|
|             | Bölcsföldi primes Bölcsf                | földi primes                        | p.1                         |  |  |  |  |
| p           | in the interval $(10^{p-1}, 10^p)$ T(p) | according to function $S(p)=1,0404$ | $x_{1,6}^{p-1}$ $T(p)/S(p)$ |  |  |  |  |
| 2           | 1                                       | 1,6                                 | 0,63                        |  |  |  |  |
| 3           | 1                                       |                                     | 0,38                        |  |  |  |  |
| 2<br>3<br>5 | 3                                       |                                     | 0,44                        |  |  |  |  |
| 7           | 10                                      |                                     | 0,60                        |  |  |  |  |
| 11          | 39                                      |                                     | 0,34                        |  |  |  |  |
| 13          | 104                                     |                                     | 0,36                        |  |  |  |  |
| 17          | 2526                                    |                                     | 1,32                        |  |  |  |  |
| 19          | 4915                                    |                                     | 1,00                        |  |  |  |  |
|             |                                         | .,,                                 | -,                          |  |  |  |  |
|             | Fig.1                                   |                                     |                             |  |  |  |  |
|             |                                         |                                     |                             |  |  |  |  |
|             |                                         |                                     |                             |  |  |  |  |
|             |                                         |                                     |                             |  |  |  |  |
|             |                                         | c                                   |                             |  |  |  |  |
|             | Duissa Jana u                           |                                     |                             |  |  |  |  |
|             | / Prime-long p                          | orimes                              |                             |  |  |  |  |
|             | /                                       |                                     | _ \                         |  |  |  |  |
|             |                                         |                                     |                             |  |  |  |  |
|             | Primes a Full primes                    | Eiölcsföldi primes f Erdős-         | )]                          |  |  |  |  |
|             |                                         | primes                              | d //                        |  |  |  |  |
|             | \                                       |                                     | //                          |  |  |  |  |
|             |                                         |                                     |                             |  |  |  |  |
|             | \ Primes consisting o                   | of prime-digits                     |                             |  |  |  |  |
|             |                                         |                                     |                             |  |  |  |  |
|             |                                         | b                                   | /                           |  |  |  |  |
|             |                                         |                                     |                             |  |  |  |  |
|             | Primes consisting of                    | of prime-digits                     |                             |  |  |  |  |

|                                     |                     | Fig.2 |   |  |
|-------------------------------------|---------------------|-------|---|--|
| Prime num                           | bers a              |       |   |  |
| Primes consisting of prime-digits b |                     |       |   |  |
| <u> </u>                            | Prime-long primes   | c     |   |  |
|                                     | Erdős-prime         | es (  | b |  |
|                                     | Full primes         | e     |   |  |
|                                     | Bölcsföldi primes f |       |   |  |

# 3. Number of the elements of the set of Bölcsföldi prime numbers [3], [9], [10], [11].

Let's take the set of Mills' prime numbers! Definition: The number  $m=[M \text{ ad } 3^n]$  is a prime number, where M=1,306377883863080690468614492602 is the Mills' constant, and n=1,2,3,... is an arbitrary positive integer number. It is already known that the number of the elements of the set of Mills' prime numbers is infinite. The Mills' prime numbers are the following: m=2,11,1361,2521008887,...

The connection n $\rightarrow$ m is the following:1 $\rightarrow$ 2, 2 $\rightarrow$ 11, 3 $\rightarrow$ 1361, 4 $\rightarrow$ 2521008887,... The Mills' prime number m=[M ad 3<sup>n</sup>] corresponds with the interval  $(10^{m-1},10^m)$  and vice versa. For instance:2 $\rightarrow$ (10,  $10^2$ ),  $11\rightarrow$ ( $10^{10},10^{11}$ ),  $1361\rightarrow$ ( $10^{1360},10^{1361}$ ), etc. and vice versa. The number of the elements of the set of Mills' prime numbers is infinite. As a consequence, the number of the intervals  $(10^{m-1},10^m)$  that contain at least one Mills' prime number is infinite. The number of Bölcsföldi primes in the interval  $(10^{m-1},10^m)$  is  $S(m)=1,0404x1,6^{m-1}$ . The number of Bölcsföldi prime numbers is probably infinite:  $S(m)=1,0404x1,6^{m-1}$ .

 $p \rightarrow \infty$ 

#### **II.** Conclusion

Countless different sets of special prime numbers have been known. We have found the following set of special prime numbers within the set of prime numbers. There may be further sets of special prime numbers that we do not know yet. Finding them will be task of researchers of the future.

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