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Completely Semi Prime, Fuzzy Semiprime Ideals Of A Po Ternary Semigroup

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Abstract:*In this paper we introduced the termsof Completely fuzzy semiprimepo ideals and fuzzy semiprime ideals of poternarysemigroups and also introduced the concepts of fuzzy d-system and fuzzy n-system of poternarysemigroups.It is proved that every completely fuzzy prime ideal of poternarysemigroup T is a completely fuzzy semiprime ideal of T also proved that every fuzzy m-system of po ternary semigroup is fuzzy n-system.****Mathematical subject classification (2010): 20M07; 20M11, 20M12***

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I. Introduction:

The algebraic theory of semigroups was widely studied by Clifford[2,3]. The ideal theory in general semigroups was developed by Anjaneyulu[1]. Since then a series of researchers have been extending the concepts and results of abstract algebra. Padmalatha, A. Gangadhara Rao and A.Anjaneyulu[10] introduced posubsemigroup, posubsemigroup generated by a subset, two sided identity of a posemigroup, zero of a posemigroup, po ideal ,po ideal generated by a subset. On the other hand, P.M.Padmalatha , A.Gangadahara Rao, P.RamyaLatha [12] introduced completely prime, prime ideal of a posemigroupV.Sivaramireddy studied on ideals in partial ordered ternary semi groups [16].

The concept of a fuzzy set was introduced by Zadeh in 1965[6]. This idea opened up new thoughts and applications in a wide range of scientific fields. A. Rosenfeld applied the notion of fuzzy subset to several areas of mathematics, among other disciplines. N. Kuroki, J N Mordeson developed the fuzzy semigroups concept. N.Kehayopulu, M.Tsingelis introduced the notion of fuzzy subset of a posemigroups[7-9]. Motivated by the study of N.Kehayopulu, M.Tsingelis work in posemigroups we attempt in the paper to study the completely semiprimepo ideals and fuzzy semiprimepo ideals of partialordered ternary semigroups.

II. Preliminaries:

Definition 2.1: [5] A semigroup T with an ordered relation ≤ is said to be po Ternarysemigroupif T is a partially ordered set such that $a \le b \Rightarrow aa_1a_2 \le ba_1a_2$, $a_1a\,a_2 \le a_1ba_2$, $a_1a_2\,a \le a_1a_2b$ for all $a, b, a_1a_2 \in T$.

Definition 2.2:A function f from T to the closed interval [0,1] is called a fuzzy subset of T. The po ternary semigroup T itself is a fuzzy subset of T such that T(x) =1, ∀x ∈ T. It is denoted by T or 1.

Definition 2.3: Let A be a non-empty subset of T. We denote f_A, the characteristic mapping of A. i.e., The mapping of T into [0,1] defined by

$f_A(x)=\begin{cases}1 & \text{if } x \in A\\ 0 & \text{if } x \notin A\end{cases}$ Then f_A is a fuzzy subset of T

Definition 2.4:[5]**:** A fuzzy subset f of a po ternary semigroup T is called fuzzy Ternarysub semigroupof T if $f(xyz) \ge f(x) \wedge f(y) \wedge f(z) \forall\, x, y, z \in T$.

Definition 2.5: Let T be a po ternary semigroup. For H⊆ T
we define **(H]**={t∈T / t≤ h for some h∈H}. For H={a} we write (a]= ({a}] = { t∈T / t ≤a}

Definition 2.5: Let T be a po ternary semigroup. For H⊆ T
we define [H)={t∈T / h ≤ t for some h∈H}. For H={a} we write (a]= ({a}] = { t∈T / t ≤a}

Definition 2.6: Let f be a fuzzy subset of a po ternary semigroup T. We define (f]by
$(f](x) = \underset{x\le y}{\vee} f(y), \forall x \in T$.

Note 2.7: Clearlyf ⊆ (f].

Note 2.8:The set of all fuzzy subsets of T is denoted by F(T).
Definition 2.9: Let (T, ≤) be a po ternary semigroup and f,g,h be fuzzy subsets of T. For $x \in T$ the product fogoh is defined by (fogoh)(x) = $\begin{cases} \vee_{x\leq pqr} f(p) \wedge g(q) \wedge h(r) \text{ if } x \leq pqr \text{ exists} \\ 0 \quad \text{otherwise} \end{cases}$
Definition 2.10:[11] A nonempty subset A of a po ternary semigroup T is said to be po left ternary ideal or po left ideal of T if i) b, c ∈ T, a ∈ A ⇒bca∈ A ii) a ∈ A and t ∈ T such that t ≤ a ⇒ t ∈ A.
NOTE :A nonempty subset A of a po ternary semigroup T is a po left ternary ideal of T if and only if i) TTA ⊆ A ii) (A] ⊆ A.
Definition 2.11:A nonempty subset A of a po ternary semigroup T is said to be po lateral ternary ideal or po lateral ideal of T if i) b, c ∈ T, a ∈ A ⇒bac∈ A ii) a ∈ A and t ∈ T such that t ≤ a ⇒ t ∈ A.
NOTE **2.12:** A nonempty subset A of a po ternary semigroup T is a po lateral ternary ideal of T if and only if i) TAT UTTATT ⊆ A ii) (A] ⊆ A.
Definition 2.13: A nonempty subset A of a po ternary semigroup T is said to be po right ternary ideal or po right ideal of T if i) b, c ∈ T, a ∈ A ⇒abc∈ A ii) a ∈ A and t ∈ T such that t ≤ a ⇒ t ∈ A
NOTE **2.14**: A nonempty subset A of a po ternary semigroup T is a po right ternary ideal of T if and only if i) ATT ⊆ A ii) (A] ⊆ A.
Definition 2.15:A nonempty subset A of a po ternary semigroup T is said to be po ternary ideal or po ideal of T if i) b, c ∈ T, a ∈ A ⇒bca∈A, bac∈A, abc∈ A ii) a ∈ A and t ∈ T such that t ≤ a ⇒ t ∈ A.
NOTE **2.16**: A nonempty subset A of a po ternary semigroup T is a po ternary ideal of T if and only if i) TTA ⊆ A, TAT ⊆ A, ATT ⊆ A ii) (A] ⊆ A.
Definition 2.17:[11]LetT be a po ternary semigroup. A fuzzy subset f of T is called a fuzzy poleft idealof T if (i) x ≤ y then f(x) ≥ f(y) (ii) f(xyz) ≥ f(z), ∀ x,y,z ∈ T
Lemma 2.18: [10] Let T be a po ternary semigroup and f be a fuzzy subset of T. Then f is a fuzzy po left ideal of T if and only if f satisfies that (i) x ≤ y then f(x) ≥ f(y) ∀x,y, z ∈T (ii) Tofof⊆ f.
Definition 2.19: [11]Let T be a po ternary semigroup. A fuzzy subset f of T is called a fuzzy po right idealof T if (i) x ≤ y then f(x) ≥ f(y) (ii) f(xyz) ≥ f(x), ∀ x,y,z ∈T.
Lemma 2.20 [10] Let T be a po ternary Semigroup and f be a fuzzy subset of T. Then f is a fuzzy right ideal of T if and only if f satisfies that (i) x ≤ y then f(x) ≥ f(y) ∀ x,y, z ∈T (ii)fofoT⊆ f.
Definition 2.21: [11]Let T be a po ternary semigroup. A fuzzy subset f of T is called apo lateral idealfuzzyof T if (i) x ≤ y then f(x) ≥ f(y) (ii) f(xyz) ≥ f(y), ∀ x,y, z ∈T
Lemma 2.22: [10] Let T be a po ternary Semigroup and f be a fuzzy subset of T. Then f is a fuzzy lateral ideal of T if and only if f satisfies that (i) x ≤ y then f(x) ≥ f(y) ∀ x,y, z ∈T
(ii)foTof⊆ f.
Definition 2.23: [11]Let T be a po ternary semigroup. A fuzzy subset f of Tis called a fuzzy idealof T if (i) x ≤ y then f(x) ≥ f(y) (ii) f(xyz) ≥ f(z), f(xyz) ≥ f(x) , f(xyz)≥ f(y)∀ x,,y, z ∈T.
Lemma 2.24 :[10] Let T be a po ternary semigroup and f be a fuzzy subset of T. Then f is a fuzzy ideal of T if and only if f satisfies that (i) x ≤ y then f(x) ≥ f(y) ∀ x,y, z ∈T (ii) fo foT⊆ f and Tofof⊆ f and foTof⊆ f.
Lemma 2.25:[7]Let T be a po ternary semigroup and ∅ ≠ A ⊆ T. Then A is a left ideal of T if and only if the characteristic mapping f_A of A is a fuzzy left ideal of T.
Lemma 2.26:[7]LetT be a po ternary semigroup and ∅ ≠ A ⊆ T. Then A is a right ideal of Tif and only if the characteristic mapping f_A of A is a fuzzy right ideal of T
Lemma 2.27:[7]Let T be a poternarysemigroup and ∅ ≠ A ⊆ T Then A is an ideal of T if and only if the characteristic mapping f_A of A is a fuzzy ideal of T.
Proposition 2.28:[13]Let f,g,h be fuzzy subsets of T. Then the following statements are true.
a. f ⊆ (f], ∀f ∈ F(T) b. If f ⊆ g then (f] ⊆ (g]
c .(f]o(g] ⊆ (fog], ∀f, g ∈ F(T)d. (f] = ((f]], ∀ f ∈ F(T)
e. For any fuzzy ideal f of T f = (f]
f. If f,g are fuzzy ideals of T, then fog ,f∪g are fuzzy ideals of T.
g . fo(g ∪ h] ⊆ (fog ∪ foh] h.(g ∪ h]of ⊆ (gof∪hof].
i. If a_λ is an ordered fuzzy point of T, then a_λ= (a_λ].
Definition 2.29:[13]Let T be a po ternary semigroup, a ∈ T and λ ∈ (0,1]. An ordered fuzzypoint **a_λ ,a_λ**: T → [0,1]definedby$a_\lambda(x) = \begin{cases} \lambda \text{ if } x \in (a] \\ 0 \text{ if } x \notin (a] \end{cases}$
clearlya_λ is a fuzzy subset of T. For every fuzzy subset f of T, we also denote $a_\lambda \subseteq f$ by $a_\lambda \in f$
Definition 2.30:[5] Let f be a fuzzy subset of X. Let t ∈ [0,1]. Define$f_t = \{x \in X/f(x) \geq t\}$. We call f_t a t-cut or a level set.
Definition 2.31:[12]A po (left/right/lateral) ideal of A of a po ternary semigroup T is said to be completely prime (left/right lateral) ideal of T provided x, y, z ∈ T and xyz ∈ A implies either x ∈ A or y ∈ A or z ∈ A..

III. Completely Fuzzy Semiprimepo Ideals And Fuzzy Semiprimepo Ideals

Definition 3.1: A fuzzy idealf of a po ternary semigroup T is said to be a completely fuzzysemiprimeidealif for any fuzzy point a_t of T such that $a_t{}^n \subseteq f$ for some odd natural number $n \in N$ then $a_t \subseteq f$.

Theorem 3.2: Let f be a fuzzy ideal of a po ternary semigroup T.f is completely fuzzy semiprime ideal iff for any ordered fuzzy point a_t of T such that $a_t{}^3 \subseteq f \Rightarrow a_t \subseteq f$.

Proof: Suppose f is completely fuzzy semiprime then clearly if $a_t{}^3 \subseteq f \Rightarrow a_t \subseteq f$.

Conversely suppose that $a_t{}^3 \subseteq f \Rightarrow a_t \subseteq f$.

We prove this by induction on n. This is true for $n = 3$.

Assume that this is true for $n = k$.

$\Rightarrow a_t{}^{k+2} o a_t{}^{k+2} o a_t{}^{k-4} \subseteq ToTof \subseteq f \Rightarrow a_t{}^{3k} \subseteq f \Rightarrow (a_t{}^k)^3 \subseteq f \Rightarrow a_t{}^k \subseteq f \Rightarrow a_t \subseteq f$ by inductive hypothesis.

Therefore f is completely fuzzy semiprime ideal.

Theorem 3.3: If f is completely fuzzy semiprime ideal of a po ternary semigroup T then for $x \in T$ for every $\lambda_1, \lambda_2, \lambda_3 \in (0,1] (i) x_{\lambda_1} o x_{\lambda_2} o x_{\lambda_3} \subseteq f \Rightarrow x_{\lambda_1} o x_{\lambda_2} oToTo x_{\lambda_3} \subseteq f$

(ii) $x_{\lambda_1} oToTo x_{\lambda_2} o x_{\lambda_3} \subseteq f$ (iii) $\subseteq f$.

Proof: Let f be completely fuzzy semiprime ideal of a po ternary semigroup T

Suppose $x_{\lambda_1} o x_{\lambda_2} o x_{\lambda_3} \subseteq f$.

Consider

$$\left(x_{\lambda_1} o x_{\lambda_2} oToTo x_{\lambda_3}\right)^3 = (x_{\lambda_1} o x_{\lambda_2} o x_{\lambda_3} o\, ToT) o (x_{\lambda_1} o x_{\lambda_2} o x_{\lambda_3} oToT)\, o (x_{\lambda_1} o x_{\lambda_2} o x_{\lambda_3} oToT)$$
$$\subseteq (x_{\lambda_1} o x_{\lambda_2} oToT) o (x_{\lambda_3} o x_{\lambda_1} o x_{\lambda_2}) oToT\, o (x_{\lambda_3} o x_{\lambda_1} o x_{\lambda_2}) oToT\, o x_{\lambda_2}$$

$\subseteq x_{\lambda_1} o x_{\lambda_2} oT$ o f $oToT\, o x_{\lambda_3}$

$\subseteq$f

$\Rightarrow \left(x_{\lambda_1} o x_{\lambda_2} o x_{\lambda_3} oToT\right)^3 \subseteq f \Rightarrow x_{\lambda_1} o x_{\lambda_2}\ o x_{\lambda_3} oToT \subseteq f$ since f is completely fuzzy semiprime ideal.

Consider

$\left(x_{\lambda_1} oToTo x_{\lambda_2} o x_3\right)^3 =$

$$\left(x_{\lambda_1} oToTo x_{\lambda_2} o x_3\right) o \left(x_{\lambda_1} oToTo x_{\lambda_2} o x_{\lambda_3}\right) o (\, x_{\lambda_1} oToTo x_{\lambda_2} o x_{\lambda_3})$$

$\Rightarrow= (x_{\lambda_1} oToT\ ) \text{ o } (x_{\lambda_2} o x_{\lambda_3} o x_{\lambda_1})\text{oToT o}(x_{\lambda_2} o x_{\lambda_3} o x_{\lambda_1}) oToTo x_{\lambda_2} o x_{\lambda_3}$

$$\subseteq (\, x_{\lambda_1} oToT\ )\text{o f oToTo f o}ToTo x_{\lambda_2} o x_{\lambda_3}$$
$$\subseteq f$$

therefore$x_{\lambda_1} oToTo x_{\lambda_2} o x_{\lambda_3} \subseteq f$ since f is completely fuzzy semiprime ideal.

Consider

$\left(x_{\lambda_1} oTo x_{\lambda_2} oTo x_{\lambda_3}\right)^3 =$

$$\left(x_{\lambda_1} oTo x_{\lambda_2} oTo x_{\lambda_3}\right) o \left(x_{\lambda_1} oTo x_{\lambda_2} oTo x_{\lambda_3}\right) o (x_{\lambda_1} oTo x_{\lambda_2} oTo x_{\lambda_3})$$
$$= x_{\lambda_1} oTo x_{\lambda_2} oTo\{\, x_{\lambda_3} \text{o} x_{\lambda_1} o (To x_{\lambda_2} oT) o\ (x_{\lambda_3}\ \text{o} x_{\lambda_1} \text{oT) oTo} x_{\lambda_3}\ \}$$

$\subseteq$f

therefore$x_{\lambda_1} oTo x_{\lambda_2} oTo x_{\lambda_3} \subseteq f$ since f is completely fuzzy semiprime ideal.

Corollary 3.4: Let f be a fuzzy ideal of a po ternary semigroup T. If f is completely semiprime then for every two ordered fuzzy points x_t, y_r, z_s of T such that $x_t o y_r o z_s \subseteq f$then $< x_t > o < y_r > o < z_s > \subseteq f$.

Theorem 3.5: Every completely fuzzy prime ideal of a po ternary semigroup T is a completely fuzzy semiprime ideal of T.

Proof: Let f be completely fuzzy prime ideal of a po ternary semigroup T and a_t be any ordered fuzzy point of T such that $a_t{}^3 \subseteq f \Rightarrow a_t o a_t o a_t \subseteq f \Rightarrow a_t \subseteq f$.

Therefore f is completely fuzzy semiprime.

Theorem 3.6: Let f be fuzzy prime po ideal of a po ternary semigroup T. If f is completely fuzzy semiprime po ideal of T then f is completely fuzzy prime.

Proof: Let f be completely fuzzy semiprime ideal of T.

Let$x_t o y_r o z_s \subseteq f \Rightarrow < x_t > o < y_r > o < z_s > \subseteq f$ *by corollory*3.4

$\Rightarrow x_t \subseteq f$ *or* $y_r \subseteq f$ *or* $z_s \subseteq f$ since f is fuzzy ideal.

Therefore f is completely fuzzy prime.

Theorem3.7: The nonempty intersection of any family of completely fuzzy prime po ideal of a po ternary semigroup T is a completely fuzzy semiprimepo ternary ideal of T.

Proof :

Let $\{f_\alpha\}$ be an arbitrary family of completely prime fuzzy ideals of T such that $\cap f_\alpha \neq \emptyset$.
Clearly $\cap f_\alpha$ is a fuzzy ideal.
Let ${x_\lambda}^3 \in \cap f_\alpha \Rightarrow {x_\lambda}^3 \in f_\alpha$ for each α.
$\Rightarrow x_\lambda \in f_\alpha$ for each α, since f_α is completely fuzzy prime.
Therefore $\cap f_\alpha$ is completely fuzzy semiprime ideal of T

Definition3.8: A fuzzy po subset f of T is said to be a fuzzy d-system if $x_t \subseteq f \Rightarrow {x_t}^n \subseteq f$ for all odd natural number $n \in N$.

Theorem 3.9: Let f be fuzzy po ideal of a po ternary semigroup T. f is completely fuzzy semiprime iff $1-f$ is a fuzzy d-system of T if $1 - \mathrm{f} \neq \emptyset$.

Proof: Suppose that f is a completely fuzzy semiprime po ideal of T.
Let $x_t \subseteq 1-f \Rightarrow x_t \nsubseteq f \Rightarrow f(x) < t$
If possible ${x_t}^n \nsubseteq 1-f \Rightarrow {x_t}^n \subseteq f$ for every odd natural number $n \in N \Rightarrow {x_t}^3 \subseteq f \Rightarrow x_t \subseteq f$ which is contradiction.
Therefore ${x_t}^n \subseteq 1-f \Rightarrow 1-f$ is a fuzzy d-system.
Conversely suppose $1-f$ is fuzzy d-system of T.
Let ${x_t}^3 \subseteq f$ .Suppose $x_t \nsubseteq f \Rightarrow x_t \subseteq 1-f \Rightarrow {x_t}^n \subseteq 1-f$ for every odd natural number $n \in N$
$\Rightarrow {x_t}^3 \subseteq 1-f \Rightarrow {x_t}^3 \nsubseteq f$, which is contradiction.
Therefore $x_t \subseteq f \Rightarrow f$ is completely fuzzy semiprime po ideal.

Definition 3.10: A fuzzy po ideal f of a po ternary semigroup T is said to be fuzzysemiprimeif g is a fuzzy po ideal of T and $g^n \subseteq f$ for some odd natural number n
then $g \subseteq f$.

Theorem 3.11: A fuzzy po ideal f of a po semigroup T is semiprime iff g is fuzzy po ideal of T such that $g^3 \subseteq f$ then $g \subseteq f$

Proof:Suppose f is fuzzy semiprime.
If $g^3 \subseteq f$ by definition $g \subseteq f$.
Conversely suppose that if $g^3 \subseteq f$ then $g \subseteq f$. We prove that if $g^n \subseteq f$ for some odd natural number n then $g \subseteq f$ by using induction on n.
Since if $g^3 \subseteq f$ then $g \subseteq f$, it is true for $n = 3$.
Assume that $g^k \subseteq f$ for some, $1 \leq k \leq n \Rightarrow g \subseteq f$.
Now assume $g^{k+1} \subseteq f \Rightarrow g^{k+1} o g^{k+1} o g^{k+1} \subseteq f$ since f is fuzzy po ideal
$\Rightarrow g^{3k+3} \subseteq f \Rightarrow (g^{k+1})^3 \subseteq f \Rightarrow g^{k+1} \subseteq f \Rightarrow g \subseteq f$.
By induction, f is fuzzy semiprime po ternary ideal.

Theorem 3.12: Every fuzzy prime po ideal of a po ternary semigroup is fuzzy semiprimepo ideal.
Proof: Let f be fuzzy prime po ideal of a po ternary semigroup T.
Let $g^3 \subseteq f$ where g is a fuzzy po ideal $\Rightarrow g \subseteq f$ since f is fuzzy po prime.
Therefore f is fuzzy semiprime po ideal.

Theorem 3.13: If f is a fuzzy po ideal of a po ternary semigroup T then the following are equivalent.
(a) f is a fuzzy semiprime po ideal.
(b) For an ordered fuzzy point $a_t, < a_t >^3 \subseteq f \Rightarrow a_t \subseteq f$.
(c) For any $a_t, Toa_toToa_toToa_toT \subseteq f \Rightarrow a_t \subseteq f$.

Proof: $(a) \Rightarrow (b)$ is obvious. $(\boldsymbol{b}) \Rightarrow (\boldsymbol{c})$:
Let a_t be a fuzzy point of T such $that\ Toa_toToa_toToa_toT \subseteq f$.

Here $< a_t > = (a_t \cup a_toToT \cup Toao_toT \cup ToToa_to \cup Toa_toToa_toT)$
$\Rightarrow < a_t >^3 = (a_t \cup a_toToT \cup Toao_toT \cup ToToa_to \cup Toa_toToa_toT)o$
$(a_t \cup a_toToT \cup Toao_toT \cup ToToa_to \cup Toa_toToa_toT)$o
$(a_t \cup a_toToT \cup Toao_toT \cup ToToa_to \cup Toa_toToa_toT)$
$\subseteq$ To$(a_t \cup a_toToT \cup Toao_toT \cup ToToa_to \cup Toa_toToa_toT)$
$\subseteq$ (Toa_t) $\cup$(Toa_to Toa_toToa_t)
$\subseteq$ Toa_to To$a_toToa_toToa_t$)
$\subseteq f$

$(\boldsymbol{c}) \Rightarrow (\boldsymbol{a})$:
For any a_t, $Toa_toToa_toToa_toT \subseteq f$ then $a_t \subseteq f$.
Let g be any fuzzy po ideal of T such that $g^3 \subseteq f$.
Suppose if possible $g \nsubseteq f \Rightarrow$ there exists a fuzzy point $a_t \subseteq g$ and $a_t \nsubseteq f$.
Since $a_t \subseteq g$. Now $Toa_toToa_toToa_toT \subseteq g^3 \subseteq f \Rightarrow a_t \subseteq f$, Which is a contradiction. $\Rightarrow g \subseteq f$. Therefore f is a fuzzy semiprime po ternary ideal of T.

Theorem 3.14: Every completely fuzzy semi prime po ideal of a po ternary semigroupT is a fuzzy semiprimepo ideal of T.
Proof: Suppose that f is completely fuzzy semiprime po ideal of T.
Let a_t be any ordered fuzzy point of T such that $< a_t >^n \subseteq f$ for some odd natural number $n \in N$.Now $a_t o a_t o a_t \dots . . o a_t$(n times)$\subseteq < a_t{}^n > \subseteq < a_t >^n \subseteq f$
$\Rightarrow a_t{}^n \subseteq f \Rightarrow a_t \subseteq f \Rightarrow < a_t > \subseteq f$ by theorem 3.13.
Therefore f is a fuzzy semiprime po ideal of T.
Theorem 3.15: Let T be a commutative po ternary semigroup and f be a fuzzy po ideal of T. Then f is completely fuzzy po semiprime iff f is fuzzy po semiprime.
Proof: Suppose f is completely fuzzy posemiprime. By theorem 3.14,f is a fuzzy semiprime po ideal of T. Conversely, suppose that f is fuzzy semiprime po ideal of T.
Let a_t be any ordered fuzzy point of T, $a_t{}^n \subseteq f$ for some odd natural number $n \in N$.
Now $a_t{}^n \subseteq f \Rightarrow < a_t >^n \subseteq f \Rightarrow < a_t > \subseteq f$ since f is fuzzy semiprime $\Rightarrow a_t \subseteq f$
Therefore f is completely fuzzy semiprimepo ideal of T.
Theorem 3.16: The non-empty intersection of arbitrary family of fuzzy prime po ideals of a po ternary semigroup T is a fuzzy semi prime po ideal.
Proof: Let $\{f_\alpha\}$ be an arbitrary family of fuzzy prime po ideals of T such that$\cap f_\alpha \neq \emptyset$.
Clearly $\cap f_\alpha$ is a fuzzy po ideal
Let a_t be any ordered fuzzy point of T such that $< a_t{}^3 > \subseteq \cap f_\alpha \Rightarrow < a_t{}^3 > \subseteq f_\alpha$ for each α
$\Rightarrow < a_t > \subseteq f_\alpha$ for each $\alpha \Rightarrow < a_t > \subseteq \cap f_\alpha$
Therefore intersection of arbitrary family of fuzzy prime po ideals of a po ternary semigroup T is a fuzzy semi prime po ideal.
Definition 3.17: Let f be a fuzzy po subset of a po ternary semigroup T.f is said to be fuzzyn-systemof T provided if $f(x) > t \Rightarrow \exists\, c \in T, s \in T \ni f(c) > t$and $c \leq xsx$.
Theorem3.18: Every fuzzy m-system of a po ternary semigroup T is a fuzzy n-system.
Proof: Let f be a fuzzy m-system of a po semigroup T. Let $f(x) > t$ for some $x \in T$.
Since $f(x) > t$ and $f(x) > t, f$ is fuzzy m-system
$\Rightarrow \exists\, c \in T, s \in T \ni f(c) > t \vee t \vee t = t$ and $c \leq xsx$
$\Rightarrow f(c) > t$ and $c \leq xsx$ whenever $f(x) > t$
$\Rightarrow f$ is fuzzy n-system. Therefore every fuzzy m-system is a fuzzy n-system.
Corollary 3.19: Let f be a fuzzy semiprime po ideal of a po ternary semigroup T. If $x_r oTox_r \subseteq f$ for some ordered fuzzy point x_r of T then$x_r \subseteq f$
Proof: Let f be fuzzy semiprime po ideal ofT. Let $x_r oTox_r \subseteq f$
Consider$(Tox_r oT)^3 = (Tox_r oT)o(Tox_r oT)o(Tox_r oT) \subseteq To(x_r oTox_r)oT \subseteq TofoT \subseteq f$
$\Rightarrow (Tox_r oT)^3 \subseteq f$ and f is a fuzzy semiprime po ideal of T.
$\Rightarrow (Tox_r oT) \subseteq f$. we know $(x_r)^3 \subseteq Tox_r oT \subseteq f \Rightarrow x_r \subseteq f$
Theorem 3.20: Let f be a fuzzy ideal of a po ternary semigroup T. If f is fuzzy semiprime po ideal iff $1 - f$ is a fuzzy n-system if $1 - f \neq \emptyset$
Proof: Let f be a fuzzy semiprime po ideal of T.
Let $(1 - f)(x) > t \Rightarrow f(x) < 1 - t \Rightarrow x_{1-t} \nsubseteq f$
From corollary 3.19, $x_{1-t} oT\, o\, x_{1-t} \nsubseteq f$ since f is fuzzy semiprime
$$\Rightarrow (xsx)_{1-t} \nsubseteq f \Rightarrow f(xsx) < 1 - t \Rightarrow (1 - f)(xsx) > t$$
$\Rightarrow 1 - f$ is a fuzzy n-system.
Conversely, suppose that $1 - f$ is fuzzy n-system and $1 - f \neq \emptyset$
Let g be fuzzy po ideal of T such that $g^3 \subseteq f$.
Suppose $g \nsubseteq f \Rightarrow$ there exist an ordered fuzzy points $x_\lambda \subseteq g$ and $x_\lambda \nsubseteq f$
$$\Rightarrow f(x) < \lambda \Rightarrow (1 - f)(x) > 1 - \lambda$$
$\Rightarrow$ there exists $c, s \in T$ such that $(1 - f)(c) > 1 - \lambda$ and $c \leq xsx \Rightarrow f(c) < \lambda$
Since $c \leq xsx \Rightarrow f(c) \geq f(xsx) \Rightarrow f(xsx) < \lambda$
But $x_\lambda \subseteq g$, By lemma 7.6.1(3) of E.Book, $x_\lambda o x_\lambda o x_\lambda \subseteq gogog = g^3 \subseteq f$
$\Rightarrow (x_\lambda o x_\lambda o x_\lambda)(t) \leq f(t) \Rightarrow f(t) \geq \lambda$ for every $t \in T$.
But $xsx \in T \Rightarrow f(xsx) \geq \lambda$ which is contradiction. Therefore $g \subseteq f$.
$\Rightarrow$ fis fuzzy semiprime po ideal of T.
Theorem 3.21: If f is a fuzzy n-system of a po ternary semigroupT and $f(x) > t$ for some $x \in T$ then there exists a subset M of T such that f is fuzzy m-system on M.
Proof: Define $c_1 = x$ since $f(c_1) > t$ then there exists $c_2 \in T, s_1 \in T$ such that $f(c_2) > t$ and $c_2 \leq c_1 s_1 c_1$ since f is fuzzy n-system.

since $f(c_2) > t$ then there exists $c_3 \in T, s_2 \in T$ such that $f(c_3) > t$ and $c_3 \leq c_2 s_2 c_2$ and so on In general, if c_i has been defined, choose c_{i+1} as $c_{i+1} \in T, s_i \in T$ such that $f(c_{i+1}) > t$ and $c_{i+1} \leq c_i s_i c_i$. Construct $M = \{c_1, c_2, \dots \dots c_i, c_{i+1}, \dots \dots .\}$ clearly M is a subset of T. Let $c_i, c_j \in M$ for $i \leq j \Rightarrow f(c_i) > t, f(c_j) > t$ and also clearly $c_{j+1} \in M \Rightarrow$ f is a fuzzy m-system on M.

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