

Using the Algorithmisation in Mathematics Teaching

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ABSTRACT:

The development of algorithmic and problem thinking is very important not only for the school environment, but also for a large number of activities in real life. In this paper, we try to point out the importance of using the algorithmization in teaching mathematics underlying many of its benefits. Contextually, mathematics and informatics intersect mostly in the term of algorithm, which can be defined as a kind of instructions designed to solve a particular problem..

KEYWORDS – Algorithm, mathematics, teaching, algorithmisation, informatics.

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I. INTRODUCTION

Mathematics is known to occupy a special place in science, culture and public life being one of the vital constituents of the world scientific and technical advance. It is the science of concepts with extreme generality, an excellent training school of thought in stages, which arranges things according to their complexity. As Pythagoras has stated: "...Mathematics is the way of understanding the universe."

Exploring mathematics by students can be supported by algorithms. Many objectives of mathematical education can be achieved at every level of education with multiple teaching benefits by applying the elements of algorithmisation. A teacher who is aware of these benefits can, for example, use algorithms to develop students' logical and mathematical thinking and to revise the material or to develop students' habit of validating the results.

Through analysing various methods of solving tasks, proving theorems and ways of presenting definitions and theorems, we can see that school mathematics has a two-fold nature: conceptual and algorithmic. We cannot, however, separate the conceptual and algorithmic mathematics because these two aspects are very closed to each other and they are equally important. In order to analyse conceptual elements, we need computational methods. On the other hand, algorithms treated separately from concepts are only automatic patterns for calculations.

The development in mathematics shows that these two aspects are closely related to each other.

The formulas for solving second degree equations were known in ancient times. Later on, mathematicians discovered algorithms for solving equations of the third and fourth degree. Throughout the years scientists were trying to discover general methods of solving algebraic equations of the n^{th} degree. But this was impossible until 1799 when Paolo Ruffini showed that for the equations of the fifth degree such methods do not exist, although his proof was not considered valid at that time. Few years later Niels Henrik Abel showed precisely that equations of more than the fourth degree cannot be solved by means of four basic operations (addition, subtraction, multiplication and division) and roots. However, further research showed that roots of some certain specific polynomials can be found. The French mathematician, Evariste Galois, who lived in the first half of the nineteenth century, specified which equations can be solved that way. In order to solve the problem, he created a new theory, which constitutes the basis for contemporary modern algebra. Galois introduced such terms as: group, normal subgroup and field, and his works became a driving force for the development of semigroup or quasigroup theory and ring theory, as well. Therefore, the search for general methods for solving much more complicated equations was one of the engines of development. The above brief history illustrates perfectly the way that mathematicians have worked. So, for them, discovering new concepts and relations is usually connected with making efforts to discover more general ways of solving different problems. Algorithms are created in such a way that they can resolve a very large class of tasks using just one and the same system of operations. Thus, they provide technical means to deepen the conceptual knowledge of mathematics.

II. MATHEMATICS AND ALGORITHMS

Education is a teaching method which employs a chain of exercises directed, integrated action at the level of teaching standardized scheme. Success depends on the ability of algorithms chosen to intervene as operational models that makes more efficient the learning experience.

The "algorithm" is a tool which enables the clearing of obstacles and the solving of didactic conflicts in the sense that it momentarily allows a clear apportionment of responsibilities. The teacher shows the algorithm, the pupil learns it and "applies" it correctly: if not, he must exert himself, but his uncertainty is nearly null. He is firmly told that there is a whole class of different situations to which the algorithm gives a solution (the conflict will resume when a choice of algorithm must be made for a given problem). The algorithm is practically the only "official"

method of release; this means that it has been the subject of making the teaching methods relating to it explicit. And it is used as a unique or nearly unique model for all the sub-cultural approaches in teaching .. [Brousseau, 1986, p 30].(see [1]).

Mathematics, uses an algorithm in two directions: solve certain types of exercises and theoretical issues pertaining and to conduct practical activities. Didactic classification algorithms can be realized by reference to the classic criteria proposed by russian psychologist Landa. According to him there can be determined two categories of algorithms didactics: 1) identification algorithms - advancing a list of questions for referral ranked special class of problem, to develop a certain classifications value synthesis 2) solving algorithms – advancing a sequence of operations required for the accurate assessment of a situation of training in order to develop an effective decision. [4]

When studying in the area of applied mathematics, students have to master main numerical methods and algorithms of resolving mathematical problems, to have an idea about the applied programs packages available, to know how to use the numerical methods in resolving applied problems using the modern computer technologies, to develop numerical methods and algorithms and to implement the algorithms in a high-level programming language;... Based on the structure of academic activity of resolving applied problems, the following competencies have been singled out: significant skills of independent computer operation, programming, using the information processing methods and numerical methods of resolving basic problems; skills of independent algorithm construction and analysis; mastery of algorithmic modeling method during analysis of setting of mathematical problems; an ability to spot the applied aspect in solution of a scientific problem, to present and interpret the result correctly...[3]

Algorithms, invisible pieces of code forming the construction and mechanics of the modern era of machines - writes the English mathematician, Hannah Fry - gave everything to the world - from subscribing to information channels in social media, through search engines and satellite navigation, to the systems of recommendation of musical works - and are part of our modern infrastructure on a par with bridges, buildings and factories. We installed them in hospitals, courtrooms and cars. They are used by the police, supermarkets and film studios. They got to know our likes and dislikes; they tell us what to watch, what to read and who to date. At the same time, they have hidden possibilities, due to which they slowly change criteria of humanity” (2019, pp. 12-13).(see [7])

III. BENEFITS OF THE ALGORITHMIZATION

“Algorithm is an ordered set of unambiguous, executable steps that determine a finite process, which leads to the realization of a certain task.” (Brookshear, 2003, p. 181).

An algorithm is a step-by-step procedure designed to achieve a certain objective in a finite time, often with several steps that repeat or “loop” as many times as necessary. The most familiar algorithms are the elementary school procedures for adding, subtracting, multiplying, and dividing, but there are many other, more complex, algorithms in mathematics.

Mathematics advances in part through the development of efficient procedures that reduce difficult tasks to routine exercises that can be carried out without effort of thought. Alfred North Whitehead expressed this idea memorably in his book, *An Introduction to Mathematics* (1911):

“It is a profoundly erroneous truism, repeated by all copy books and by eminent people when they are making speeches, that we should cultivate the habit of thinking of what we are doing. The precise opposite is the case. Civilization advances by extending the number of important operations which we can perform without thinking about them.” (p. 61).

An effective algorithm can be used to efficiently solve an entire class of problems, without having to think through each problem from first principles. Knowing algorithms increases students’ mathematical power, which is a principal goal of school mathematics (NCTM, 1989).

Algorithmic and procedural thinking includes:

- understanding specific algorithms or procedures provided by other people,
- applying known algorithms to everyday problems,

- adapting known algorithms to fit new situations,
- developing new algorithms and procedures when necessary, and
- recognizing the limitations of algorithms and procedures so they are not used inappropriately.

By studying computational algorithms, students can learn things that will carry over to other areas of their lives. More and more, people need to apply algorithmic and procedural thinking in order to operate technologically advanced devices. Algorithms beyond arithmetic are increasingly important in theoretical mathematics, in applications of mathematics, in computer science, and in many areas outside of mathematics.

The use of algorithmization in mathematics teaching can be done in many ways (Krygowska, 1977; Rams, 1982; Wojcicka, 2005):

- teaching basic algorithms,
- execution of ready-made algorithms,
- comparing algorithms,
- algorithm analysis,
- algorithm supplementation.
- creating algorithms by students.

Introduction of algorithmization in teaching brings many educational advantages. So, as many authors have pointed out (see Krygowska, 1977; Rams, 1982; Wojcicka, 2005) the didactic benefits of teaching mathematics with the use of algorithmization are the following:

- knowledge of basic algorithms streamlines calculations, allows automatism,
- algorithm analysis allows you to see the precision and simplicity of a logical sequence of operations — method of operation,
- self-created algorithm:
 - requires logical thinking,
 - forces a very clear and unambiguous record solution plan,
 - develops reflective thinking- allows self-control;
- graphical writing allows:
 - non-verbal way of showing relationships and procedures,
 - comprehensive presentation of the method;
- algorithmization enables systematization of knowledge and it is an IT oriented teaching.

Algorithmization is one of the forms of mathematical modeling. Blum and Borromeo

Ferri (2009, p. 45) state that: “Mathematical modeling is the process of translating between the real world and mathematics in both directions.” Mathematical model can be presented in the form of: formula, equation, equation system, function and algorithm.

Activities related to mathematical modeling are connected with the activities involved in creating an algorithm to solve the problem (Broda, Smolucha, 2011; Perrenet, Zwaneveld, 2012). They are (Blum, Borromeo Ferri, 2009):

- constructing,
- simplifying,
- mathematizing (recording mathematical relations between considered variables),
- working mathematically,
- interpreting,
- validating.

These activities are important not only in mathematics, but also in the real world.(see [5])

Teaching mathematics can be done through constructing algorithms by students. A number of mathematical issues can be expressed in the form of an algorithm. This is confirmed in works of many authors such as, for example, Z. Krygowska and T. Rams. Meanwhile, A. Engel has pointed out that “...One cannot wholly understand something until one cannot explain it to the computer, i.e. express it in the form of an algorithm”. Z Krygowska writes: “It is the most important is that students constructed algorithms as the solutions for the problems by themselves. It can be achieved at any level because every level of contact with mathematics manifests its operative character, and specific categories of mathematical thinking need to be trained at every level of education”.

By creating algorithms the student gets used to the formalization of some of mathematical issues. Creating algorithms also develops reflective thinking and enables self-control. Having proposed the scheme, the students execute it by selecting a variety of initial parameters. They try to choose the examples independently and often enthusiastically so as to be able to go through each possible “path” of an algorithm. Therefore, the efforts to make a wide selection of examples become the student’s goal.

Through working with algorithms one can achieve the objectives associated with intellectual and personality development. These include such objectives as:

- the development of the habit of logical and correct reasoning and practical application of the principles of logic,
- the development of critical and creative thinking and the ability to draw conclusions or to make and verify hypotheses,
- shaping the ability to use patterns and letter symbols in everyday situations,
- the development of the ability to use computer techniques to solve various mathematical problems,
- the development of the ability to lead a substantive discussion which aims at finding the optimal solution,
- the development of the habit of verifying the results and possible correction of errors,
- getting used to careful and solid work,
- teaching to present solutions to the problems and tasks clearly.

Algorithmisation cannot be treated merely as a tool for achieving specific demands but rather as one of many methods for improving knowledge, acquiring new skills and good habits. [6]

IV. CONCLUSION

We conclude citing Bruno Buchberger , who writes: "...The algorithmic development of mathematics will never be finished. Thus one need never fear that reduction to algorithms will be the end of mathematics. Higher and higher problem areas of mathematics will be algorithmically opened, making deeper and deeper mathematics necessary...The impossibility of finishing the algorithmization of mathematics is not just the practical experience of all the workers on that field-because every time, when another piece of mathematics is algorithmized, new horizons appear-it is also an intrinsic property of mathematics, which is provable (!) and constitutes a practical manifestation of Gödel's incompleteness theorem...This has the following consequence for the didactics of mathematics. On the one hand, it is very naive to exclude the computer from mathematics today; on the contrary, it is not just an auxiliary tool but the motive and driving force for the rigorous development of the basic aim of mathematics, which is to make difficult problems solvable-by extensive thinking-systematically and even automatically..." [8]

REFERENCES

- [1]. Heinz Steinbring, Routine and Meaning in the Mathematics Classroom, Systematic Cooperation between Theory and Practice in Mathematics Education, 1988, London, Great Britain.
- [2]. Martin Cápáy , Martin Magdín, Alternative Methods of Teaching Algorithms, Procedia - Social and Behavioral Sciences 2013, 83 , 431 – 436.
- [3]. Olga Gavrishina, Yuri Zaharov, Competency-Based Approach in Training Mathematicians: Challenges of Time, Procedia - Social and Behavioral Sciences 2015, 214, 212 – 221.,
- [4]. Stamatescu Tudor , Active and Participative Modern Methods Applied in Teaching Mathematics in Secondary School , Annals of the "Constantin Brancusi" University of Targu Jiu, Engineering Series , 2016, No. 3.
- [5]. Anna Pyzara, Algorithmization as mathematical activity and skill, Didactica Mathematicae, 2018, Vol. 40, p. 97–117.
- [6]. Anna Pyzara, Algorithmisation in teaching mathematics, Annals of the Polish Mathematical Society, 5th Series: Didactica Mathematicae 2012, 34.
- [7]. Tadeusz Miczka, Specifics of Algorithmization in Data Culture, European Journal of Engineering and Formal Sciences, 2019, Volume 3, Issue 3.
- [8]. Bruno Buchberger, Computer Algebra: The End of Mathematics?, ACM SIGSAM Bulletin, 2002, Vol 36, No. 1.

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