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Temperature-Dependent Viscosity and Prandtl Number Effects on Free Convection Ethanol Boundary Layers about a Vertical Plate with Injection

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ABSTRACT : *This study focuses on the effect of temperature-dependent viscosity and Prandtl number on the steady, laminar flow of ethanol, about a vertical porous plate along with injection. The coupled non-linear partial differential equations governing the non-similar flow have been solved numerically using an implicit finite difference scheme in combination with the quasilinearization technique. Numerical results indicate that temperature-dependent viscosity and Prandtl number, both have a major role on skin friction and heat transfer parameters as well as velocity and temperature fields. Also, it is observed that the effect of variable fluid properties along with injection plays a significant role in the control of laminar boundary layer.*

KEYWORDS -*Heat transfer, Injection, Skin friction, Temperature, Temperature-dependent Viscosity, Velocity.*
MSC 2010 Codes*– 76M20, 76N20, 76R10*

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I. INTRODUCTION

Applications of heat transfer are generally based on the constant physical properties of the ambient fluid in fluid dynamics research. However, it is known that these properties may change with temperature, especially the fluid viscosity and the Prandtl number. Numerous researchers have studied the effect of temperature-dependent viscosity (or variable viscosity) on different geometries under various situations [1-7]. Abundant literature is available on the topic of the laminar boundary layer free convection flow over a porous vertical plate with suction and injection, having wide range of engineering applications. In fact, the case of uniform suction and blowing (injection) through an isothermal vertical wall was treated first by Sparrow and Cess [8]; they obtained a series solution which is valid near the leading edge. This problem was considered in more detail by Merkin [9], who obtained asymptotic solutions, valid at large distances from the leading edge, for both suction and blowing (injection). Free convection boundary layer flows are frequently encountered in environmental and engineering devices. Motivated by this, the present study is undertaken to investigate the effect of temperature-dependent viscosity and Prandtl number on the free convection boundary layer flow (of ethanol) over a vertical porous plate with injection. It may be remarked here that ethanol is a liquid in room temperature, used extensively in day-to-day products, including paints, cosmetics, fuel and drug industries.

II. GOVERNING EQUATIONS

We consider a semi-infinite porous plate, which is played vertical in a quiescent fluid (ethanol) of infinite extent maintained at an uniform temperature. The plate is fixed in a vertical position with leading edge horizontal. The physical co-ordinates (x,y) are chosen such that x is measured from the leading edge (origin) in the stream wise direction and y is measured normal to the surface of the plate. Indeed, the flow is assumed to be in the x-direction i.e., along the vertical plate in the upward direction and the y-axis is taken to be normal to the plate.The fluid properties are assumed to be isotropic and constant except for the fluid viscosity. The temperature difference between the surface of the plate (T_w) and the ambient fluid(T_∞) is taken to be small. In the range of temperature (T) considered (i.e. 0-50^{0}C),the variation of both density (ρ) and specific heat (c_p) of ethanol with temperature is small and hence they are taken as constants.*[See Table I]* However, the viscosity (μ) and the Prandtl number (Pr) are assumed to vary as an inverse linear function of temperature:

$$\mu = 1/(b_1 + b_2 T) \qquad (1)$$

$$Pr = 1/(c_1 + c_2 T) \qquad (2)$$

Where $b_1 = 53.804$, $b_2 = 1.584$, $c_1 = 0.0428$ and $c_2 = 0.0006$ (3)

The numerical values required for these correlations, are taken from Table-I [10].

Table-I
***Values of thermo-physical properties of ethanol at different temperatures* [10]**

Temperature (T^0C)	Density(ρ) ($gr./m^3$)	Specific heat(c_p) ($(J \times 10^7/kg\ ^0K)$)	Thermal conductivity (k) ($erg \times 10^5/cm.s\text{-}^0K$)	Viscosity(μ) ($gr. \times 10^{-2}/cm\text{-}s$)	Prandtl number (Pr)
0	0.8037	2.2416	0.1721	1.7730	23.0906
10	0.7981	2.2419	0.1703	1.4662	20.1064
20	0.7901	2.4283	0.1670	1.2003	17.4532
30	0.7821	2.5296	0.1611	1.0035	13.7531
40	0.7741	2.3242	0.1585	0.8384	12.6316
50	0.7626	2.1226	0.1479	0.7062	11.8755

The relation (1) and (2) are reasonably good approximations for liquids such as ethanol, particularly for small wall and ambient temperature differences. Further, the fluid added (injection) or removed (suction) is the same as that involved in flow. The Boussinesq's approximation employed for the fluid properties to relate density changes in the flow field. Under the above-mentioned assumptions, the boundary layer equations governing the steady, two-dimensional flow are [9]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (4)$$
$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = g\beta(T - T_\infty) + \frac{1}{\rho_\infty}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) \quad (5)$$
$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{1}{\rho_\infty}\frac{\partial}{\partial y}\left[\left(Pr^{-1}\mu\frac{\partial T}{\partial y}\right)\right] \quad (6)$$

The initial and boundary conditions are

$$x = 0, y > 0; u = v = 0, T = T_w$$
$$x > 0; y = 0; u = 0, v = -v_0(for\ suction),$$
$$x > 0; y = 0; u = 0, v = +v_0(for\ injection),$$
$$y \to \infty; x > 0; u = 0, T = T_\infty \quad (7)$$

Introducing the following transformations

$$u = \frac{\partial\psi}{\partial y}; \quad v = \frac{-\partial\psi}{\partial x};$$
$$\psi = \frac{v^2 g\beta(T_w - T_\infty)\xi^3}{V_0{}^3}\left[f(\eta,\xi) \pm \frac{\xi}{4}\right]; T = T_\infty + (T_w - T_\infty)G(\eta,\xi);$$
$$\eta = \frac{V_0 y}{\nu\xi}; \xi = V_0\left[\frac{4x}{\nu^2 g\beta(T_{wo} - T_\infty)}\right]^{1/4}; \quad (8)$$

to Eqns. (4) – (6), we see that the continuity Eq.(4) is identically satisfied and Eqns.(5)-(6) reduce respectively, to

$$(NF')' + 3fF' - 2F^2 \pm \xi F' + G - F\xi^2 = \xi\left(FF_\xi - F'f_\xi\right) \quad (9)$$

$$(NPr^{-1}G')' + 3fG' \pm \xi G' = \xi\left(FG_\xi - G'f_\xi\right) \quad (10)$$

where,

$$u = \frac{V_0{}^2 4x}{\nu\xi^2}F; \quad v = \frac{-V_0}{\xi}\left(3f + \xi f_\xi - \eta F \pm \xi\right);$$

$$f = \int_0^\eta F d\eta\,; N = \left(\frac{\mu}{\mu_\infty}\right) = \frac{b_1 + b_2 T_\infty}{b_1 + b_2 T} = \frac{1}{1 + a_1 G},$$
$$Pr = \frac{1}{c_1 + c_2 T} = \frac{1}{a_2 + a_3 G}, \quad a_1 = \left(\frac{b_2}{b_1 + b_2 T_\infty}\right)\Delta T_w,$$
$$a_2 = c_1 + c_2 T_\infty\,,\ a_3 = c_2 \Delta T_w, \quad \Delta T_w = (T_w - T_\infty)\,; \qquad (11)$$

It is noted here that the upper and lower signs in Eqns. (9) and (10) is taken thought for suction and injection, respectively. The present study, however, restricted to the case of injection only.
The transformed boundary conditions are:

$$F = 0; G = 1\ at\ \eta = 0,\ F = 0; G = 0\ as\ \eta \to \infty \qquad (12)$$

The local skin friction and heat transfer parameter can be expressed as

$$\tau_w = \frac{V_0}{g\beta(T_{w0} - T_\infty)}\left(\frac{\partial u}{\partial y}\right)_{y=0} = \xi (F')_{\eta=0} \qquad (13)$$
$$Q = \frac{\nu}{V_0(T_{w0} - T_\infty)}\left(\frac{\partial T}{\partial y}\right)_{y=0} = \frac{-1}{\xi}(G')_{\eta=0} (\xi \neq 0\ ) \qquad (14)$$

Here, u and v are velocity components in x and y-directions respectively; F is dimensionless velocity; T and G are dimensional and dimensionless temperatures, respectively; ξ and η are transformed co-ordinates; ψ and f are the dimensional and dimension-less stream functions respectively; Pr is the Prandtl number; $a_1, a_2, a_3, b_1, b_2, c_1\ and\ c_2$ are constants; g is the acceleration due to gravity; β is the coefficient of thermal expansion; w and ∞ denote conditions at the edge of the boundary layer on the wall and in the free stream respectively, the subscript ξ and prime ($'$) denote, respectively partial derivatives with respect ξ and η .

III.RESULTS AND DISCUSSION

The system of dimensionless nonlinear coupled partial differential equations (9)-(10) with boundary condition (12) has been solved numerically employing an implicit finite difference scheme with a quasilinearization technique [11,12].In order to assess the accuracy of the numerical method which we have used, the skin friction and heat transfer parameters (τ_w, Q) for injection have been obtained by solving Eqns.(9) and (10) for constant viscosity [N=1] case, taking Pr=1.0 and compared with those of Merkin [9] for injection. Our results are found to be in good agreement with those of [9], as shown in Fig.1, validating the accuracy of the numerical method used in the present study. The computed results for variable viscosity as well as Prandtl number have been presented in the graphical form and analyzed.

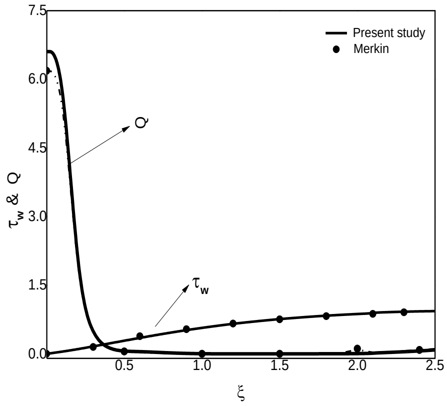

Fig. 1. Comparison of (a) skin friction and (b) heat transfer parameter with Merkin [9]for injection.

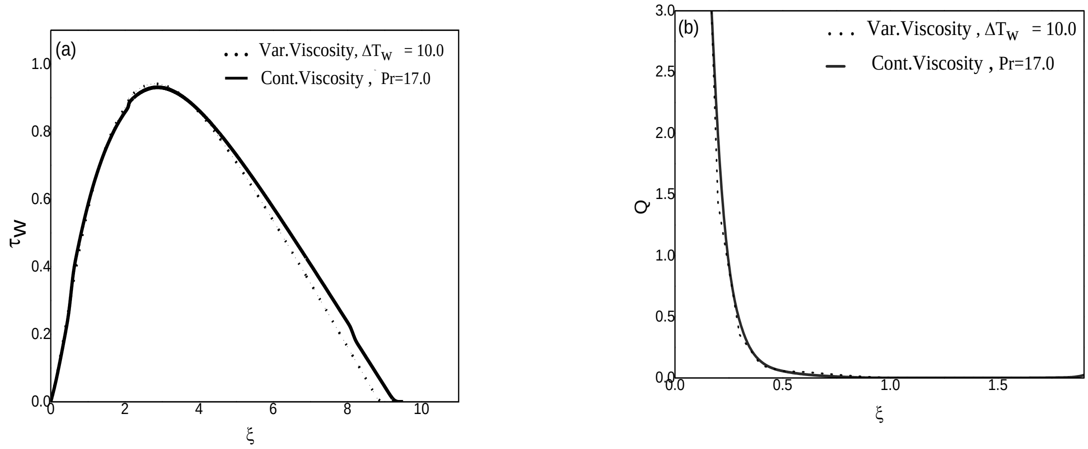

Fig.2. Variation of (a) skin friction and (b) heat transfer parameters along stream-wise directions.

Figure 2 describes the variation of skin friction $(\tau_w)$and heat transfer parameters (Q) with the stream wise coordinate ξ, in the presence of both variable fluid properties [T_∞ =28.7ºC, ΔT_w= 10.0] and constant fluid properties [N =1 and Pr = 17.0 for ethanol at room temperature] and injection. It is observed from Fig.2(a) that skin friction (τ_w)increases from zero to a maximum value in the neighborhood of ξ =2.6 for variable viscosity as well as constant viscosity, and then decreases as ξ further increases. It is also observed that the effect of variable fluid properties is to increase the skin friction and to decrease the heat transfer. In fact, $\tau_w$for variable fluid properties differs from that of constant fluid properties by about 10.35% [Fig.2(a)] while, the percentage of difference in the case of $(Q)$is around 0.005% [Fig.2(b)], at the stream-wise coordinate ξ =0.4. Further, it is observed that the zero-skin friction is moved downstream in the presence of variable fluid properties. Indeed, in the case of constant fluid properties zero skin friction occurs at the stream-wise location ξ = 9.3 whereas for variable fluid properties, the same occurs at ξ = 8.8. This justifies the delay in the boundary layer separation under the influence of variable viscosity, Prandtl number and injection.

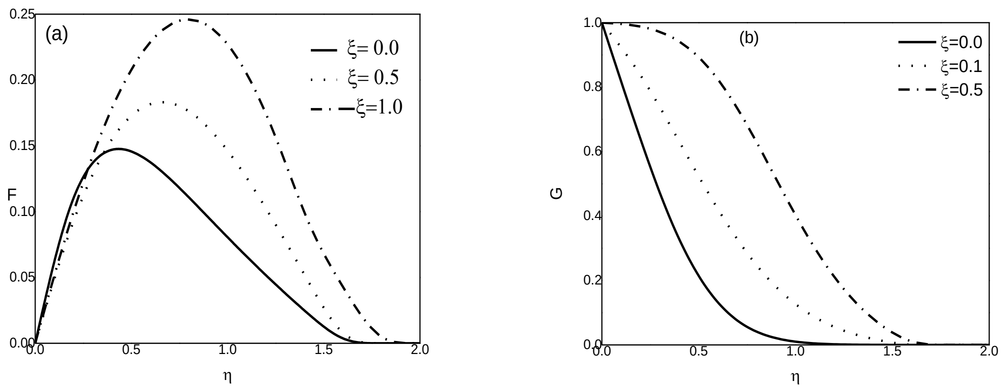

Fig. 3. Behavior of (a) velocity and (b) temperature profiles at different stream-wise locations.

The relevant velocity (F) and temperature(G) profiles are shown in Fig.3, for the case of variable fluid properties. It is observed that the thickness of momentum boundary layer increases with the increase of stream wise coordinate (ξ) [Fig.3(a)], which results in the increase of velocity of the fluid inside the boundary layer. On the other hand, as the thermal boundary layer thickness increases as ξ increases, enhancing the temperature inside the boundary layer [Fig.3(b)].

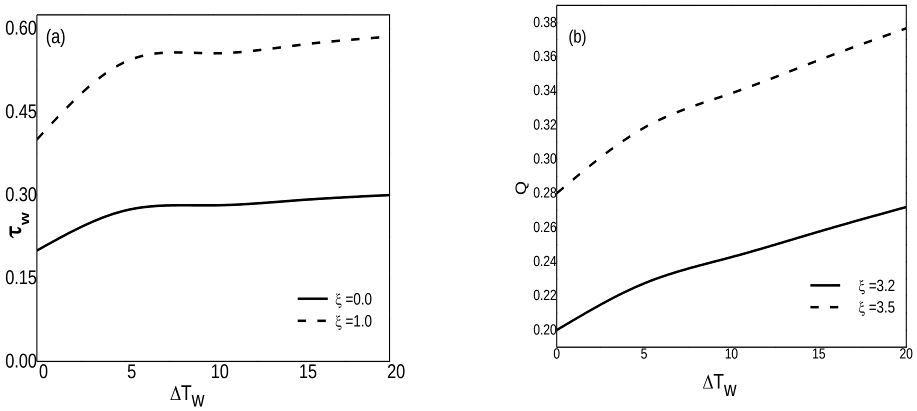

Fig. 4. Effect of ΔT_W on (a) skin friction and (b) heat transfer parameters at stream-wise locations.

The variation of viscosity and Prandtl number with temperature can be introduced in terms of the difference (ΔT_w) in the temperature of the wall and ambient fluid [Fig.4]. Since T_∞ =28.7^{0}C, the maximum value of ΔT_w is taken as 20^{0}C so that numerical computations are done with in the permissible temperature. In Fig 4(a)–(b) for different stream wise locations, it is observed that as ΔT_w increases both τ_w and Q increases. Further, it is also observed that as ξ increases, both skin friction and heat transfer increases. The rate of increase of skin friction is 49.34% for stream wise coordinate ξ = 0.0 and ξ = 1.0 at ΔT_w = 10^{0}C [Fig.4(a)] , while the rate of increase in heat transfer is 28.32% for ξ = 3.2 and ξ = 3.5 at the same value of ΔT_w [Fig.4(b)].

IV. CONCLUSIONS

The steady, laminar ethanol boundary layer flow (of ethanol) past a vertical porous flat plate is numerically investigated assuming both viscosity and Prandtl number as linear inverse functions of temperature. The computed results show that the flow/temperature field, skin friction and heat transfer characteristics are significantly affected by the temperature-dependent viscosity - Prandtl number and injection.

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