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Laminar Forced Convection with Viscous Dissipation in Couette-Poiseuille Flow of Pseudo-Plastic Fluids

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Abstract
Semi-analytical solutions are derived to investigate the effect of viscous dissipation on the temperature distribution and heat transfer characteristics of Couette-Poiseuille flow for pseudo-plastic fluids. The fluid flow is steady, laminar and both hydro-dynamically and thermally fully developed, while the thermal boundary conditions considered are both plates being kept at asymmetric heat fluxes. For Couette- Poiseuille flow for pseudo-plastic fluids, the temperature distribution and the Nusselt number obtained are greatly affected by constant heat flux ratio together with velocity of the moving plate, power-law index, modified Brinkman number and a dimensionless parameter which is the constant of integration in solving the momentum equation.
Keywords: *Pseudo-plastic fluids, Viscous dissipation, Couette-Poiseuille flow, Brinkman number, Nusselt number, Asymmetric heat-flux*

Nomenclature

A_c cross-sectional area of channel $\left(m^2\right)$
Br_{q1} modified Brinkman number defined in Eq. (22)
c_p specific heat at constant pressure $\left(j/kgk\right)$
h convective heat transfer coefficient $\left(W/m^2k\right)$
k thermal conductivity $\left(W/mk\right)$
L width of plate $\left(m\right)$
n power-law index
Nu Nusselt number, defined in Eq. (37)
q_1 upper wall heat-flux $\left(W/m^2\right)$
q_2 lower wall heat-flux $\left(W/m^2\right)$
T temperature $\left(K\right)$
T_1 upper wall temperature $\left(K\right)$
T_2 lower wall temperature $\left(K\right)$
ΔT general temperature difference $\left(K\right)$
u velocity of the fluid $\left(m/s\right)$
U axial velocity of the moving upper plate $\left(m/s\right)$
U^* dimensionless velocity of the moving plate, defined in Eq. (17)
P pressure $\left(Pa\right)$
w half-channel height $\left(m\right)$
W channel height $\left(m\right)$
x coordinate in the axial direction $\left(m\right)$
y coordinate in the vertical direction $\left(m\right)$
Y dimensionless vertical coordinate

Greek symbols

α thermal diffusivity $\left(m^2/s\right)$
β parameter defined in Eq. (31)
λ constant of integration appearing in Eq. (3)
μ dynamic viscosity $\left(Pa\ s\right)$
ρ density $\left(kg/m^3\right)$
θ dimensionless temperature
θ_m mean dimensionless temperature

Subscripts

m mean

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I. INTRODUCTION

Effect of viscous dissipation plays an important role in the study of heat transfer characteristics and this effect is frequently encountered in many applications such as viscous fluid flow, high speed flows and fluid flow through microscale channels. For Newtonian fluids, there have been abundant studies investigating, either theoretically or experimentally, the viscous dissipation effect on the forced convection in ducts and channels [1-7], As pointed out in several recent investigations [8-12], the importance of the effects of viscous dissipation in the analysis of forced convection in porous media is also indisputable. This effect is comparatively well-known in polymeric fluids which are non-Newtonian in nature. Understanding the viscous dissipation effect in the flow of polymeric fluid is of paramount importance in connection with plastics material processing, application of paints and lubricants, processing of food stuffs, as well as movement of biological fluids. As the motion of polymeric fluids cannot be described by the Navier-Stokes equations, the study of such fluids is becoming a big challenge for the past few decades. Many important practical problems of engineering interest involve steady-state shear flows and to obtain better physical insights into the elucidation of some of the interesting phenomena, relatively simple generalized Newtonian fluids models such as power-law fluids and Bingham plastics have been employed to obtain approximate solutions to the heat and fluid flow problems [7], For unsteady flow phenomena, the linear viscoelastic fluid model is used to describe the elastic response of the polymeric fluids while the retarded-motion expansion is valid for slowly varying flows [7],

Many important industrial problems involve non-Newtonian flows and numerous studies of viscous dissipation effect on convection heat transfer in ducts and channels can be found in the literature. Different Theological models have been employed for solving problems in different flow regimes, such as the generalized Newtonian fluid model, the linear viscoelastic fluid model and the retarded-motion expansion model [7]. Previous studies of viscous dissipation effect of forced convection involving non-Newtonian flows primarily focused on the generalized Newtonian fluids, such as power-law fluids [13-20], Bingham fluids [21] and Herschel- Bulkley fluids [22], The effect of viscous dissipation has been investigated for third grade fluids [23] as well as the viscoelastic fluids obeying the Oldroyd-B model [24], the Phan-Thien-Tanner model [25,26], and the FENE-P model [27]. In the case of fluid flow past an unsteady stretching sheet, the viscous dissipation effect has been considered for power-law liquid film [28]. In addition, it has been validated that in the second-law analysis, the effect of viscous dissipation imposes pronounced influence on the entropy generation rate which is strongly related to the flow and temperature fields of the non-Newtonian fluids [29,30].

The Couette-Poiseuille non-Newtonian flow describes a macro- molecular fluid confined between two horizontal plates with either one moving in the longitudinal direction with a constant speed. The motion of the moving plate induces complexity on the solution method of the momentum equation due to the rheological properties of non-Newtonian fluids. Theoretical studies with different hydrodynamic and thermal conditions have been conducted.

For Newtonian fluids, analytical solutions for heat transfer with effect of viscous dissipation were derived for the Couette-Poiseuille flow between parallel plates which are kept at constant heat flux and one of the plates is insulated [31]. Various problems of non-Newtonian Couette-Poiseuille flow such as investigations of elastic effect [32] and slip effect [33] on the hydrodynamic behavior as well as stability problem with pressure-dependent viscosity [34,35], have been conducted. On the other hand, limited studies on Couette-Poiseuille non-Newtonian flow taking into account the viscous dissipation effect are available. The plane Couette- Poiseuille flow of power-law fluids with viscous dissipation was exactly solved for the case of constant heat flux at one wall with the other insulated [36]. By considering Couette-Poiseuille flow, with the stationary plate subjected to constant heat flux and the other plate moving with constant velocity but insulated, analytical solutions with effect of viscous dissipation were obtained for viscoelastic fluids [37]. Numerical study was carried out investigating viscous dissipation effect for Poiseuille-Couette non-Newtonian fluid flow with the boundary condition of constant heat flux at one wall and the other plate insulated [38]. Heat transfer on Couette-Poiseuille flow for viscoelastic non-Newtonian fluid with simplified Phan-Thien-Tanner constitutive equation was analyzed, where the stationary plate was maintained at constant temperature and the moving plate is insulated [39].

Different thermal boundary conditions need to be considered in the design of thermal equipment. The heat transfer characteristics of Couette-Poiseuille pseudo-plastic fluid flow through parallel- plate channel with asymmetric wall heat fluxes is a fundamental problem which has not been reported in the literature and is worth investigating. This type of problems is encountered in a diversity of processing operations, such as in various extruders and bearings associated with lubrication problems. The current study complements a prior work by Tso et al. [19] in which they analytically investigated the effect of viscous dissipation on heat convection of fully developed laminar power-law fluid flow through fixed parallel plates with asymmetric heat fluxes. On the other hand, the effect of viscous dissipation in non-Newtonian Couette-Poiseuille flow is the prime investigation of the present study, accentuating the effect induced by the moving plate in Couette-Poiseuille flow on the viscous dissipation and hence the heat transfer characteristics of the problem. The variation of the velocity field is governed by the moving plate velocity while the temperature field is affected by both the moving plate velocity and the asymmetrical heat fluxes at the plate walls. Nonetheless, complicated by the motion of the moving plate, a closed form analytical solution for the momentum equation of Couette-Poiseuille flow is not obtainable for the case of power-law fluid due to its inherent non-linearity nature. By applying a semi-analytical technique to solve the momentum equation and energy equation, the present study is motivated for elucidating the changes entailed in the convection heat transfer characteristics under thermal asymmetries for pseudo-plastic fluids due to the incorporation of the effect of viscous dissipation.

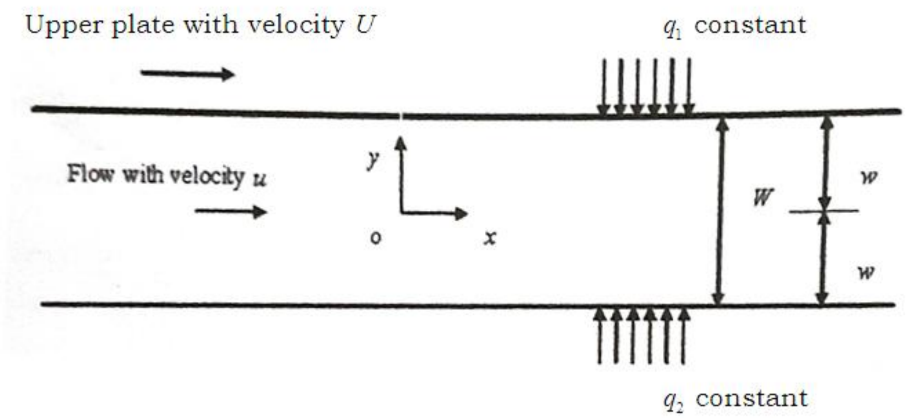


Fig. 1. Notation to the problem

II. STATEMENT OF PROBLEM AND MATHEMATICAL FORMULATION

Consider two flat parallel plates distanced W , where the upper plate is moving with constant velocity U and the lower plate is fixed. The coordinate system chosen is schematically shown in Fig. 1. The flow through the plates is considered at a sufficient distance from the entrance such that it is both hydro-dynamically and thermally fully developed. The axial heat conduction in the fluid is negligible. The polymeric fluid is considered to be pseudo-plastic and with constant properties. The thermal boundary conditions are the upper plate is kept at constant heat flux q_1 while the lower plate at different constant heat flux q_2 .

2.1 Momentum equation

Under steady-state operation, the equation of motion is given by:

$$\frac{d\tau_{yx}}{dy} = -\frac{dp}{dx} \tag{1}$$

where τ_{yx} is the shear-stress. By taking the pressure gradient $G = -{}^{dp}\!/_{dx}$ as constant along the axial direction, and integrating eq (1) with respect to X we obtain

$$\tau_{yx} = Gy + C_1 \tag{2}$$

and we define

$$C_1 = -GW\lambda \tag{3}$$

where λ is a constant of integration [7,40]. For non-Newtonian fluids, the Theological behavior of a power law fluid with constant fluid properties in between fixed parallel plates is described by the shear-stress relationship

$$\tau_{yx} = -\eta\left|\frac{du}{dy}\right|^{n-1}\frac{du}{dy} \tag{4}$$

where η is the consistency factor and du/dy is the velocity gradient perpendicular to the flow direction. The n is the power-law index, with $0 < n < 1$ for pseudo-plastic fluids and $n = 1$ for Newtonian fluids. Substituting Eq. (4) into Eq. (2) and after rearranging, we obtain

$$\left(\frac{du}{dy}\right)^n = \left(\frac{GW}{\eta}\right)\left[\lambda - \frac{y}{W}\right] \tag{5}$$

Integrating Eq. (5) with respect to y, we obtain

$$u = \left(\frac{GW}{\eta}\right)^{1/n}(-W)\frac{\left[\lambda - \left(y/w\right)\right]^{(1/n)+1}}{\left(1/n\right)+1} + C_2 \tag{6}$$

As the bottom plate is fixed, at $y = -W/2$ $u = 0$, we can evaluate C_2 as

$$C_2 = W\left(\frac{GW}{\eta}\right)^{1/n}\frac{\left[\lambda - \left(1/2\right)\right]^{(1/n)+1}}{\left(1/n\right)+1} \tag{7}$$

and by substituting C_2 into Eq. (6), we obtain

$$u = \frac{W}{\left(1/n\right)+1}\left(\frac{GW}{\eta}\right)^{1/n}\left[\left(\lambda + \frac{1}{2}\right)^{(1/n)+1} - \left(\lambda + \frac{1}{2}\right)^{(1/n)+1}\right] \tag{8}$$

Applying the boundary condition $u = U$ at $y = W/2$ gives

$$U = \frac{W}{\left(1/n\right)+1}\left(\frac{GW}{\eta}\right)^{1/n}\left[\left(\lambda + \frac{1}{2}\right)^{(1/n)+1} - \left(\lambda + \frac{1}{2}\right)^{(1/n)+1}\right] \tag{9}$$

By denoting $s = 1/n$, Eq. (9) becomes

$$U = \frac{W}{(s+1)}\left(\frac{GW}{\eta}\right)^{s}\left[\left(\lambda + \frac{1}{2}\right)^{s+1} - \left(\lambda + \frac{1}{2}\right)^{s+1}\right] \tag{10}$$

It can be observed that when U is set to zero, the condition $\left(\lambda + 1/2\right)^{s+1} = \left(\lambda - 1/2\right)^{s+1}$ has to be fulfilled and for $n < 1, \lambda$ is rendered as complex conjugates. For $n = 1$ (Newtonian fluid), $\lambda = 0$ and the velocity distribution is reduced to the classical results of plane Poiseuille flow for Newtonian fluids, as observed from Eq. (8). Following the steps performed in [7], and after rearranging and simplifying Eq (10) a dimensionless parameter Λ which relates λ with the plate distance, the pressure gradient, the moving plate velocity, the consistency factor and the power-law index, is given by

$$\Lambda = \frac{GW}{\eta}\left(\frac{W}{U}\right)^{\frac{1}{s}} = \left[\frac{s+1}{\left(\lambda + 1/2\right)^{s+1} - \left(\frac{1}{2} - \lambda\right)^{s+1}}\right]^{\frac{1}{s}}$$

Case I: $\Lambda \le (s+1)^{\frac{1}{s}}$ (11)

and

$$\Lambda = \frac{GW}{\eta}\left(\frac{W}{U}\right)^{\frac{1}{s}} = \left[\frac{s+1}{\left(\frac{1}{2}+\lambda\right)^{s+1} - \left(\frac{1}{2}-\lambda\right)^{s+1}}\right]^{\frac{1}{s}}$$

Case II: $\Lambda \le (s+1)^{\frac{1}{s}}$ (12)

Cases 1 and II are two distinguishable cases where there is no maximum velocity in the velocity profile for Case I and there is a maximum velocity in the velocity profile for Case II. Dividing Eq. (8) by Eq. (10), we obtain the velocity profile of Couette-Poiseuille flow in terms of λ as

$$\frac{u}{U} = \frac{\left(\lambda + \frac{1}{2}\right)^{s+1} - \left(\lambda + \frac{y}{W}\right)^{s+1}}{\left(\lambda + \frac{1}{2}\right)^{s+1} - \left(\lambda + \frac{1}{2}\right)^{s+1}} \tag{13}$$

Referring to Eqs. (11) and (12), Λ can be easily determined if the plate distance, the pressure gradient, the moving plate velocity, the consistency factor and the power-law index are specified. With a specific value of Λ, λ can then be evaluated numerically using Newton-Raphson method from Eq. (11) or Eq. (12). The iterations are stopped when the estimated relative error is smaller than 1×10^{-6}.

2.2. Energy equation

The energy equation is given by

$$\rho c_p u \frac{\partial T}{\partial x} = k \frac{\partial^2 T}{\partial y^2} + \eta\left(\frac{du}{dy}\right)^{n+1} \tag{14}$$

where the second term on the right-hand side is the viscous-dissipative term. In accordance with the assumption of a thermally fully developed flow with uniformly heated boundary walls, the longitudinal conduction term is neglected in the energy equation [6], Following this, the temperature gradient along the axial direction is independent of the transverse direction and given as

$$\frac{\partial T}{\partial x} = \frac{dT_1}{dx} = \frac{dT_2}{dx} \tag{15}$$

where T_1 and T_2 are the upper and lower wall temperatures, respectively, as illustrated in

Fig 1 by taking $\alpha = k / \theta s$ and defining the mean velocity as

$$u_m = \frac{1}{W}\int_{w/2}^{w/2} u\,dy \tag{16}$$

utilizing the following non-dimensional parameters.

$$Y = \frac{y}{W}, \theta = \frac{T - T_1}{q_1 W / k}, U^* = \frac{U}{u_m} \tag{17}$$

Eq. (14) is non-dimensionalized as

$$\frac{d^2\theta}{dY^2} = b_1 - b_2(\lambda - Y)^{s+1} \tag{18}$$

where

$$b_1 = \left(\lambda + \frac{1}{2}\right)^{s+1} \beta U^*, \tag{19}$$

$$b_2 = \beta U^* + Br_{q_1} U^{*^{(s+1)/s}} \left[\frac{s+1}{\left(\lambda + 1/2\right)^{s+1} - \left(\lambda - 1/2\right)^{s+1}} \right]^{(s+1)/s}, \tag{20}$$

$$\beta = \frac{kW}{q_1} \frac{u_m}{\alpha \left[\left(\lambda + 1/2\right)^{s+1} - \left(\lambda - 1/2\right)^{s+1} \right]} \frac{\partial T_1}{\partial x} \tag{21}$$

In Eq. (20), the modified Brinkman number Br_{q_1}, in terms of q_1 denotes the ratio of the viscous dissipation term and the thermal diffusion term, and is given by

$$Br_{q_1} = \frac{\eta W^{-n} u_m^{n+1}}{q_1} \tag{22}$$

It should be noted that the definition of the Brinkman number is not unique for non-Newtonian fluid flows. Coelho and Pinho [41] developed a generalized Brinkman number for non-Newtonian duct flows independent of the rheological constitutive model. In the present study, the generalized Brinkman number can be expressed as

$$Br_{q_1} = \left[\frac{s+1}{\left(\lambda + 1/2\right)^{s+1} - \left(\lambda - 1/2\right)^{s+1}} \right]^{(s+1)/s} \frac{\eta W^{-n} u_m^{n+1}}{q_1} \tag{23}$$

As observed in Eq. (23), the constant of integration λ is absorbed into the generalized Brinkman number and its effect to the temperature profiles has been normalized. Since λ is one of the essential parameters under investigation in the present study, for defining the Brinkman number, instead of employing the generalized Brinkman number, we resort to the classical definition as depicted in Eq. (22), in which case the Brinkman number is independent of the constant of integration so that their effects to the heat transfer characteristics of viscous dissipative Couette-Poiseuille flow of pseudoplastic fluids can be studied individually. The thermal boundary conditions associated with Eq. (18) are

$$\theta\left(Y = \frac{1}{2}\right) = 0, \left.\frac{\partial \theta}{\partial Y}\right|_{Y=\frac{1}{2}} = 1 \tag{24}$$

and by utilizing Eq. (24), solution of Eq. (18) can be written as

$$\theta = d_1 (\lambda - Y)^{(s+3)} + d_2 Y^2 + d_3 Y + d_4 \tag{25}$$

where

$$d_1 = \frac{b_2}{(s+2)(s+3)}, \tag{26}$$

$$d_2 = \frac{b_1}{2}, \tag{27}$$

$$d_3 = \frac{4s\lambda - 2s + 8\lambda - 4 - 4b_2 \left(\lambda - 1/2\right)^{s+3} - 2b_1 s\lambda + b_1 s - 4b_1\lambda + 2b_1}{2(s+2)(2\lambda - 1)}, \tag{28}$$

and

$$\begin{aligned} d_4 = \Big[& 2b_1 s^2 \lambda - 8s^2\lambda + 4s^2 - b_1 s^2 + 20s + 8b_2 s \left(\lambda - 1/2\right)^{s+3} - 5b_1 s + 10 b_1 s\lambda \\ & - 40 s\lambda + 24 + 8b_2 \left(\lambda - 1/2\right)^{s+3} s - 48\lambda - 6b_1 + 12 b_1 \lambda \\ & + 16 b_2 \left(\lambda - 1/2\right)^{s+3} \Big] \Big/ 8\left(2s^2\lambda - s^2 5s + 10 s\lambda - 6 + 12\lambda\right) \end{aligned} \tag{29}$$

To evaluate β in Eq. (21), we employ a third thermal boundary condition, given by

$$\left.\frac{\partial \theta}{\partial Y}\right|_{Y=-\frac{1}{2}} = -\frac{q_2}{q_1} \tag{30}$$

and subsequently it can be expressed as

$$\beta = A_1\left(1+\frac{q_2}{q_1}\right) - A_2 Br_{q_1} \tag{31}$$

where

$$A_1 = \frac{(s+2)\left(1-4\lambda^2\right)}{\left\{\begin{array}{l}\left(\lambda-\frac{1}{2}\right)^{s+1}\left(s-8\lambda^2-4s\lambda^2+2\right) \\ +\left[(4\lambda-2)\left(\lambda-\frac{1}{2}\right)^{s+3}-(4\lambda+2)\left(\lambda-\frac{1}{2}\right)^{s+3}\right]\end{array}\right\}U^*} \tag{32}$$

$$A_2 = \frac{\left\{\begin{array}{l}U^*(s+1)\Big/\left[\left(\lambda-\frac{1}{2}\right)^{s+1}-\left(\lambda-\frac{1}{2}\right)^{s+1}\right]^{(s+1)/s} \\ \left[(4\lambda-2)\left(\lambda-\frac{1}{2}\right)^{s+3}-(4\lambda+2)\left(\lambda-\frac{1}{2}\right)^{s+3}\right]\end{array}\right\}}{\left\{\begin{array}{l}\left(\lambda-\frac{1}{2}\right)^{s+1}\left(s-8\lambda^2-4s\lambda^2+2\right) \\ +\left[(4\lambda-2)\left(\lambda-\frac{1}{2}\right)^{s+3}-(4\lambda+2)\left(\lambda-\frac{1}{2}\right)^{s+3}\right]\end{array}\right\}U^*} \tag{33}$$

When defining the Nusselt number in fully developed flow, it is usual to utilize the bulk mean fluid-temperature T_m, given by

$$T_m = \frac{\int_{A_c} \rho u T \, dA_c}{\int_{A_c} \rho u \, dA_c} \tag{34}$$

where A_c is the cross-sectional area of the channel. Using Eqs. (14) and (25), the numerator of Eq. (34) can be found. The dimensionless mean temperature is given by

$$\theta_m = \frac{k}{q_1 W}\left(T_m - T_1\right) \tag{35}$$

At this point, the convective heat transfer coefficient can be evaluated by the equation

$$q_1 = h\left(T_1 - T_m\right) \tag{36}$$

Defining Nusselt number to be

$$Nu = \frac{hW}{k} = \frac{q_1 W}{k\left(T_1 - T_m\right)} = -\frac{1}{0_m} \tag{37}$$

the explicit expression for Nusselt number can be evaluated. Since the explicit form of Eq. (37) is excessively huge, it is not presented here.

III. RESULTS AND DISCUSSION

As the general result is too complex, various particular cases will next be presented in order to reveal the heat transfer characteristics. The values of n selected for discussion are $\frac{1}{4}, \frac{1}{2}$ and 1.

3.1. Temperature profiles against the channel width for various parameters

Referring to Eq. (17), it is noted that the dimensionless temperature represents the temperature difference between the fluid and the upper impermeable wall, positive when the fluid temperature is higher, and vice versa. Fig. 2 shows the dimensionless temperature profiles of θ versus Y, where the constant heat flux ratio is kept as $2, U^* = 1$ and at $n = 0.5$ at five selected Br_{q_1} values such as $-5, -2, 0, 2$ and 5 with $\Lambda = 0.4$ and 0.8

for case I and 2.0 and 3.8 for case II as shown in Fig. 2a-d. The temperature distributions converge to zero θ at $X = 0.5$ by definition.

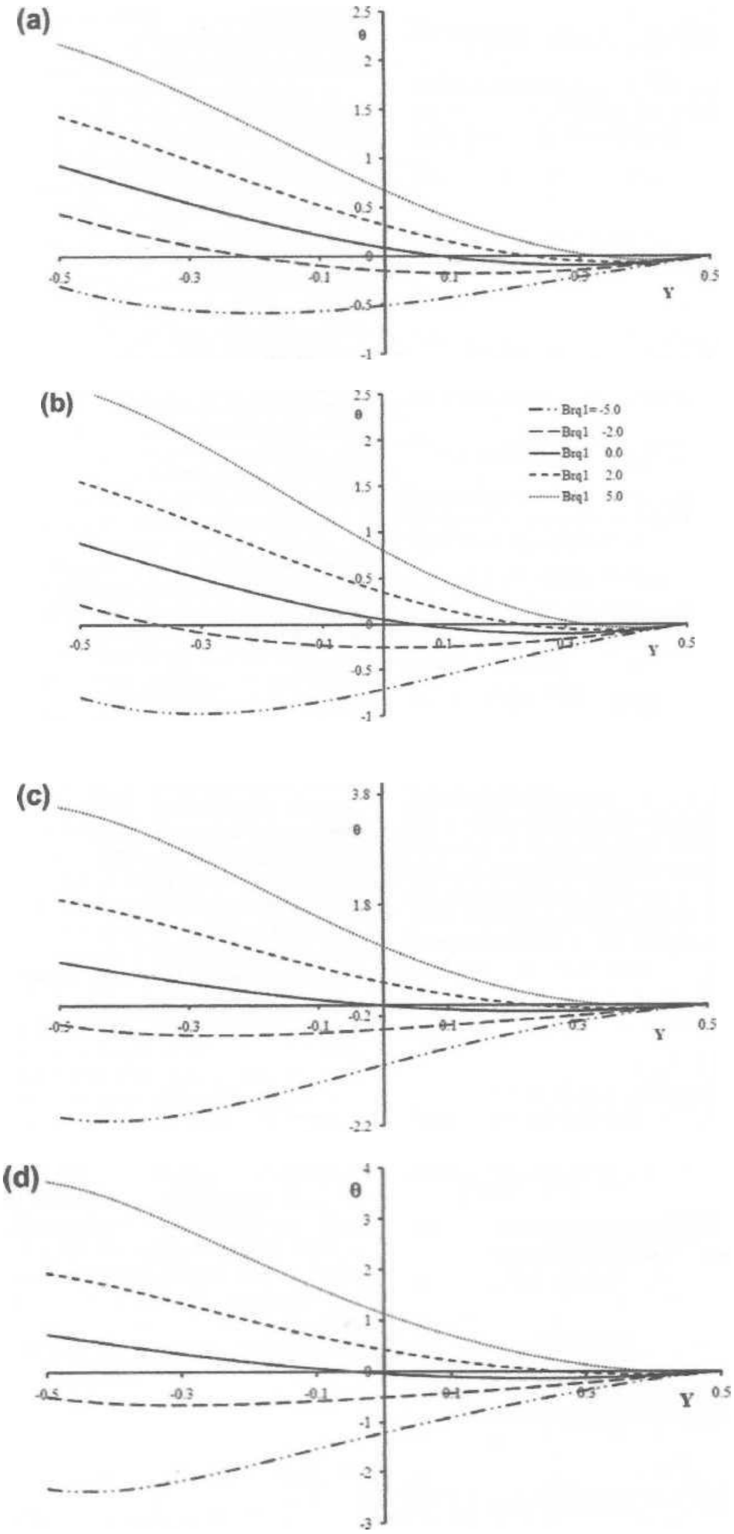
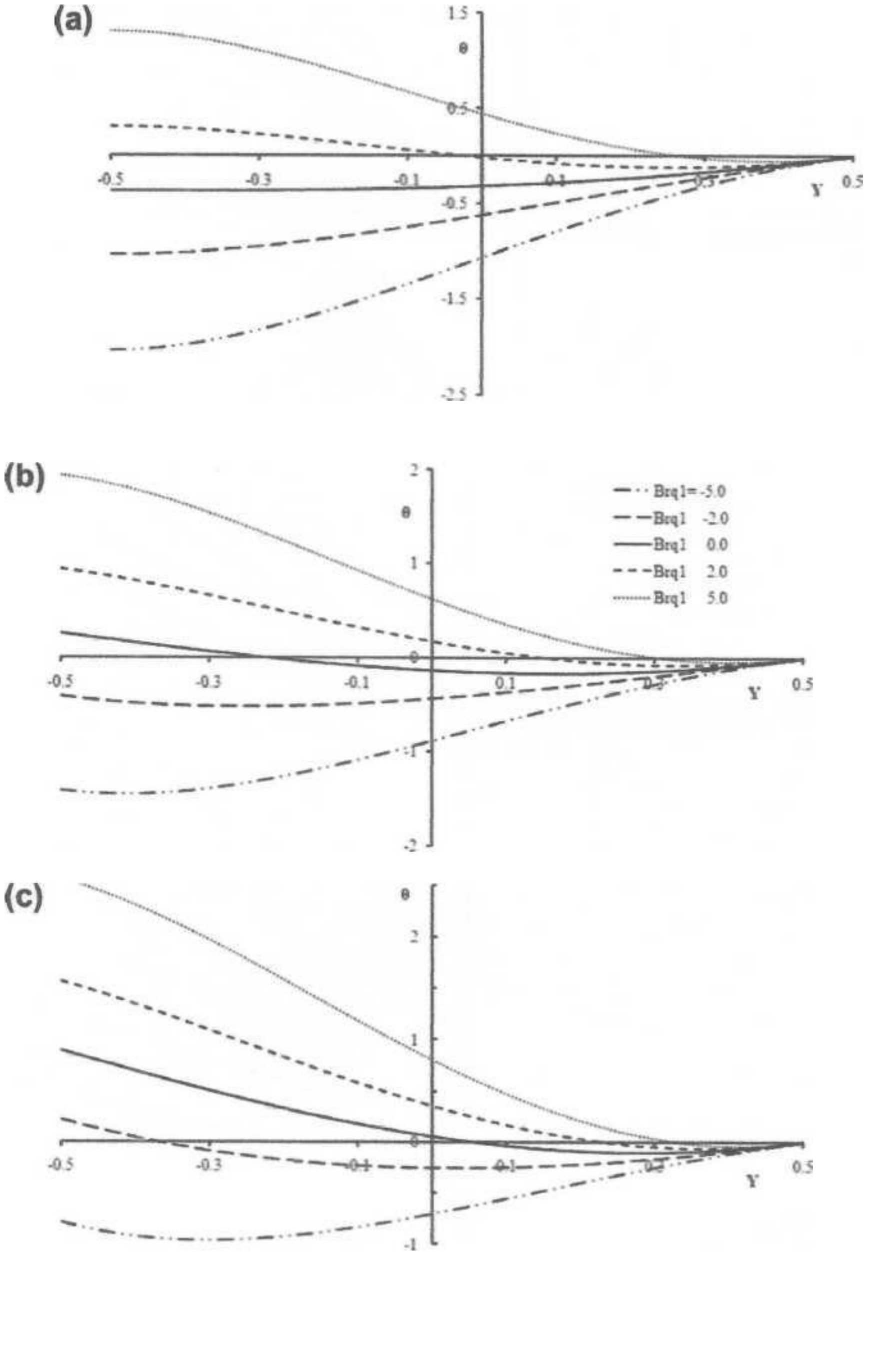


Fig. 2. Temperature profiles at (a) $\Lambda = 0.4$, (b) $\Lambda = 0.8$, (c) $\Lambda = 2.0$ and (d) $\Lambda = 3.8$ at various Br_{q_1} for the case of $q_2/q_1 = 2U^* = 1$ and $n = 1/2$. The line legend is shown in (b).

Since the Brinkman number features the relative importance of viscous dissipation, it plays opposite roles for the heating and cooling processes. As depicted in Fig. 2, for the case of positive Brinkman number corresponding to the heating process where the fluid is being heated, the temperature gradient increases with the Brinkman number. The viscous dissipation generates internal heating inside the fluid leading to the increase of difference between the fluid and the wall temperatures. Higher Brinkman number which is attributed to higher viscosity and/or velocity of the fluid, contributes to higher internal heat generation. It is observed that the fluid temperature of a substantial portion of the channel exceeds the wall temperature, implying that the heat converted from the pumping power has overcome the applied heat energy to the walls. On the other hand, the negative Brinkman number corresponds to the cooling process where the fluid is being cooled. The value of the dimensionless temperature is negative due to the negative sign of the heat flux applied at the wall. The cooling process is opposed by the heating effect of the viscous dissipation. As expected, generally the motion of the upper plate tends to impart more heat into the fluid layers that are dragged along, unless off-set by the viscous dissipation effects. For different values of Λ , when Br_{q_1} is negative, the temperature profiles take only negative values, whereas, for positive Br_{q_1}, θ takes mostly positive values with minimum negative values.



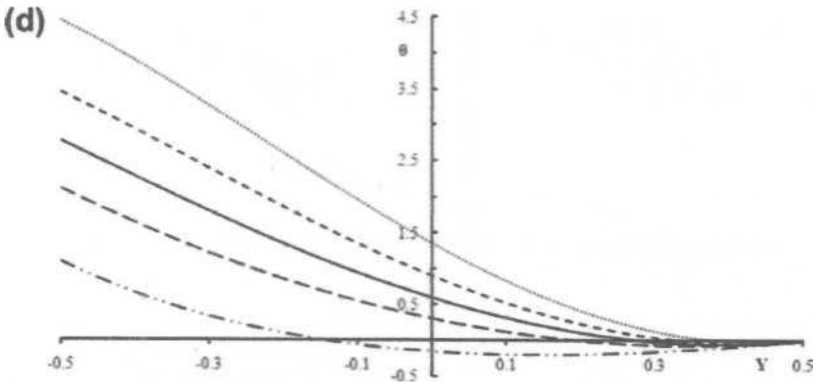
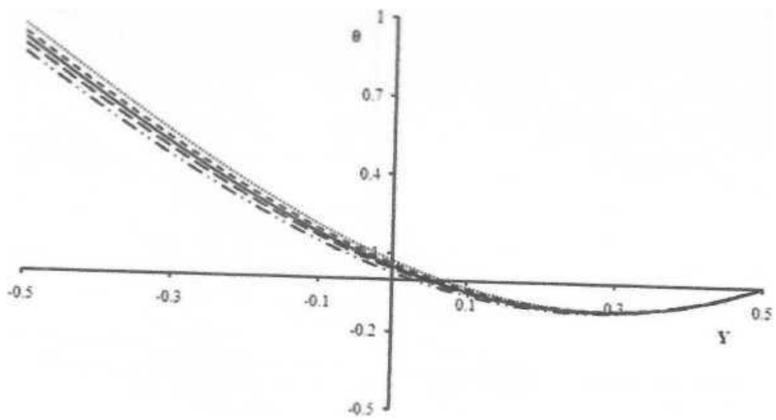


Fig. 3. Temperature profiles at (a) $q_2/q_1 = 0$ (b) $q_2/q_1 = 1$ (c) $q_2/q_1 = 2$ (d) $q_2/q_1 = 5$ at various Br_{q_1} for the case of $\Lambda = 0.8, U^* = 1$ and $n = 1/2$. The line legend is shown in (b).

The effect of viscous dissipation is seen in the value of modified Brinkman number. It is interesting to observe the behavior of the temperature profiles for various heat flux ratios and for various modified Brinkman number and hence to note the effect of viscous dissipation. Fig. 3 shows the temperature profiles for a fixed $\Lambda = 0.8, U^* = 1$ and $n = 0.5$ for various heat flux ratios such as $0, 1, 2$ and 5. From Fig. 3a, it is seen that when $q_2/q_1 = 0$ and $Br_{q_1} = 2$ and 5, the temperature profile are positive with minimum negative values, whereas, when $Br_{q_1} = 0, -2, -5$, the temperature profiles are all negative. Fig. 3b shows a similar pattern, but in this case, the absence of viscous dissipation leads the temperature profile to take both positive as well as negative values for the case of equal heat fluxes at both the plates. Fig 3c also follows similar pattern for the case of $q_2/q_1 = 2$ with temperature profile being purely negative only for when $Br_{q_1} = -5$. However, it is observed from Fig 3d that the temperature distributions manifests differently in the sense that all the values of θ are positive, except that when $Br_{q_1} = -5, \theta$ is near to zero.



(a)

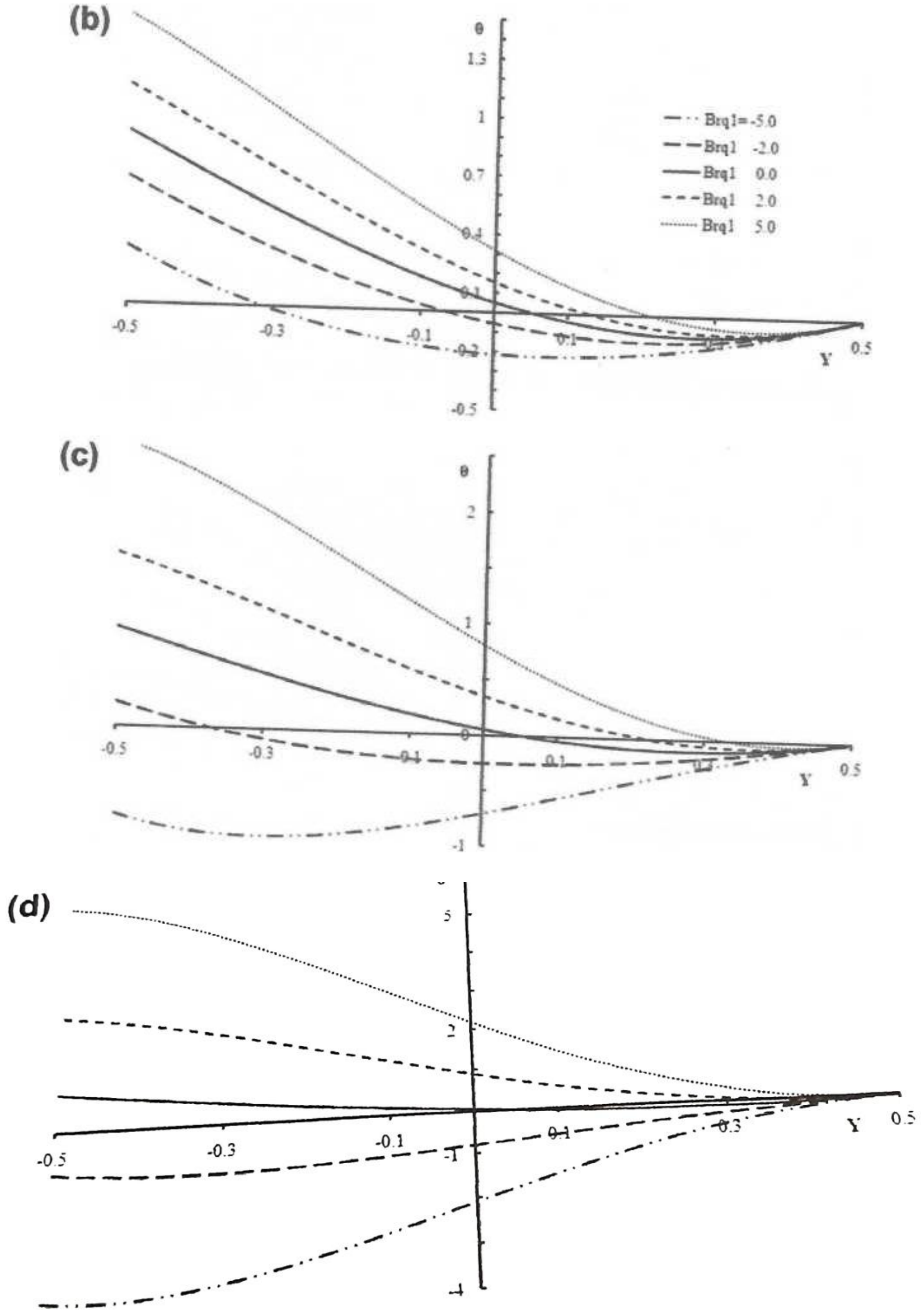


Fig. 4. Temperature profiles at (a) $U^* = 0.1$ (b) $U^* = 0.5$ (c) $U^* = 1$ and (d) $U^* = 2$ at various Br_{q_1} for the case of $\Lambda = 0.8$. $q_2/q_1 = 2$ and $n = 1/2$. The line legend is shown in (b).

Fig. 4 shows the temperature profiles for a fixed $\Lambda = 0.8$ and $q_2/q_1 = 2$ at $n = 0.5$ for various U^* values such as $U^* = 0.1, 0.5, 1$ and 2 and for various modified Brinkman values such as $Br_{q_1} = -5, -2, 0, 2$ and 5 . Fig. 4a shows there is a rapid decrease in the temperature, reaches negative minimum and then increases to zero θ at $Y = 0.5$ for $U^* = 0.1$. When $U^* = 0.5$, the values for the temperature are both positive and negative. However, when $U^* = 1$. $Br_{q_1} = -5$ and when $U^* = 2$. $Br_{q_1} = -5$, and -2 , the temperature are only negative.

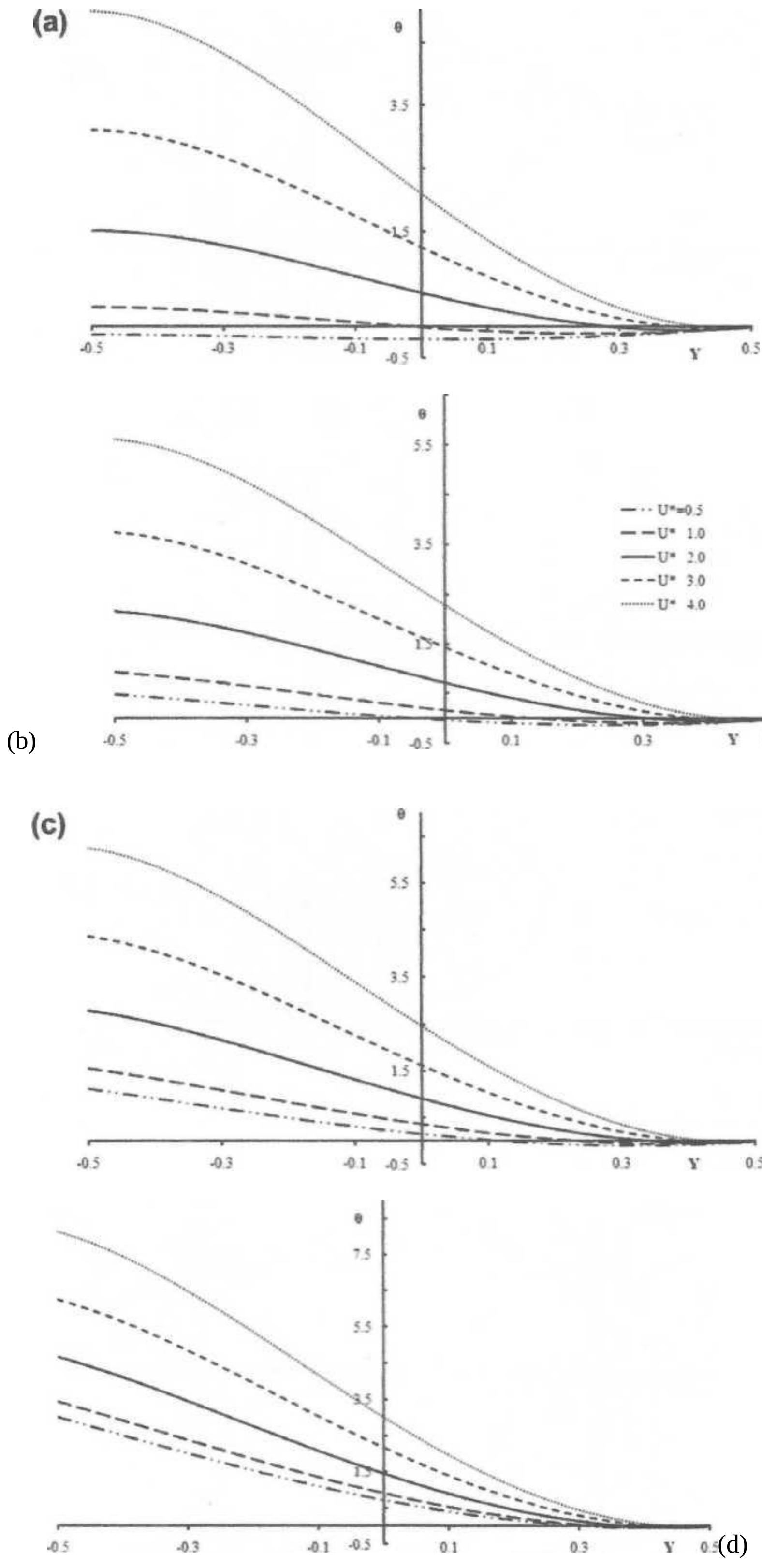


Fig. 5. Temperature profiles at (a) $q_2/q_1 = 0$ (b) $q_2/q_1 = 1$ (c) $q_2/q_1 = 2$ (d) $q_2/q_1 = 5$ at various U^* values for the case of $\Lambda = 0.8, Br_{q_1} = 2 n = 1/2$. The line legend is shown in (b).

Fig. 5 shows the temperature profiles for a fixed $\Lambda = 0.8, Br_{q_1} = 2$ and $n = 0.5$ for various U^* values such as $U^* = 0.5, 1, 2, 3$ and 4 and specified values of $q_2/q_1 = 0, 1, 2$ and 5. For the above specified values, it is observed that the temperature decreases and reaches to zero θ. For the insulated lower plate, when $U^* = 0.5$, the negative values taken by θ is minimum near to zero.

3.2. Heat transfer characteristics of cases of unequal heat fluxes

3.2.1. Newtonian fluids

Nusselt number expressed in Eq. (37) characterizes the heat transfer between the fluid and the upper wall, with the inclusion of the effect of viscous dissipation. For a Newtonian fluid $(n = 1)$ $\Lambda = 0.5$ and $U^* = 1$ we have the result as

$$Nu = \frac{1}{0.2528 - 0.2313\left(q_2/q_1\right) - 0.1136\,Br_{q_1}} \tag{38}$$

It can be deduced from Eq. (38), at a given ratio of q_2/q_1. the behavior of Nu at $n = 1$ versus Br_{q_1}, will form a rectangular hyperbola on both sides of an asymptote of

$$Br_{q_1} = 2.2256 - 2.0360\left(q_2/q_1\right) \tag{39}$$

3.2.2. Pseudo plastic fluids

For the pseudo plastic fluids $(n < 1)$, when $n = 1/4$, $\Lambda = 0.5$ and $U^* = 1$,

$$Nu = \frac{1}{0.2766 - 0.2224\left(q_2/q_1\right) - 0.1515\,Br_{q_1}} \tag{40}$$

and when $n = 1/2$, $\Lambda = 0.8$ and $U^* = 1$,

$$Nu = \frac{1}{0.2664 - 0.2159\left(q_2/q_1\right) - 0.1515\,Br_{q_1}} \tag{41}$$

It is observed form Eq. (40), at a given ratio of q_2/q_1, the behavior of Nu at $n = 1/4$ versus Br_{q_1} will from a rectangular hyperbola on both sides of an asymptote of

$$Br_{q_1} = 1.8265 - 1.4686\left(q_2/q_1\right) \tag{42}$$

and from Eq. (41), at a given ratio of q_2/q_1 the behavior of Nu at $n = 1/2$ versus Br_{q_1} will form a rectangular hyperbola on both sides of an asymptote of

$$Br_{q_1} = 1.7580 - 1.4247\left(q_2/q_1\right) \tag{43}$$

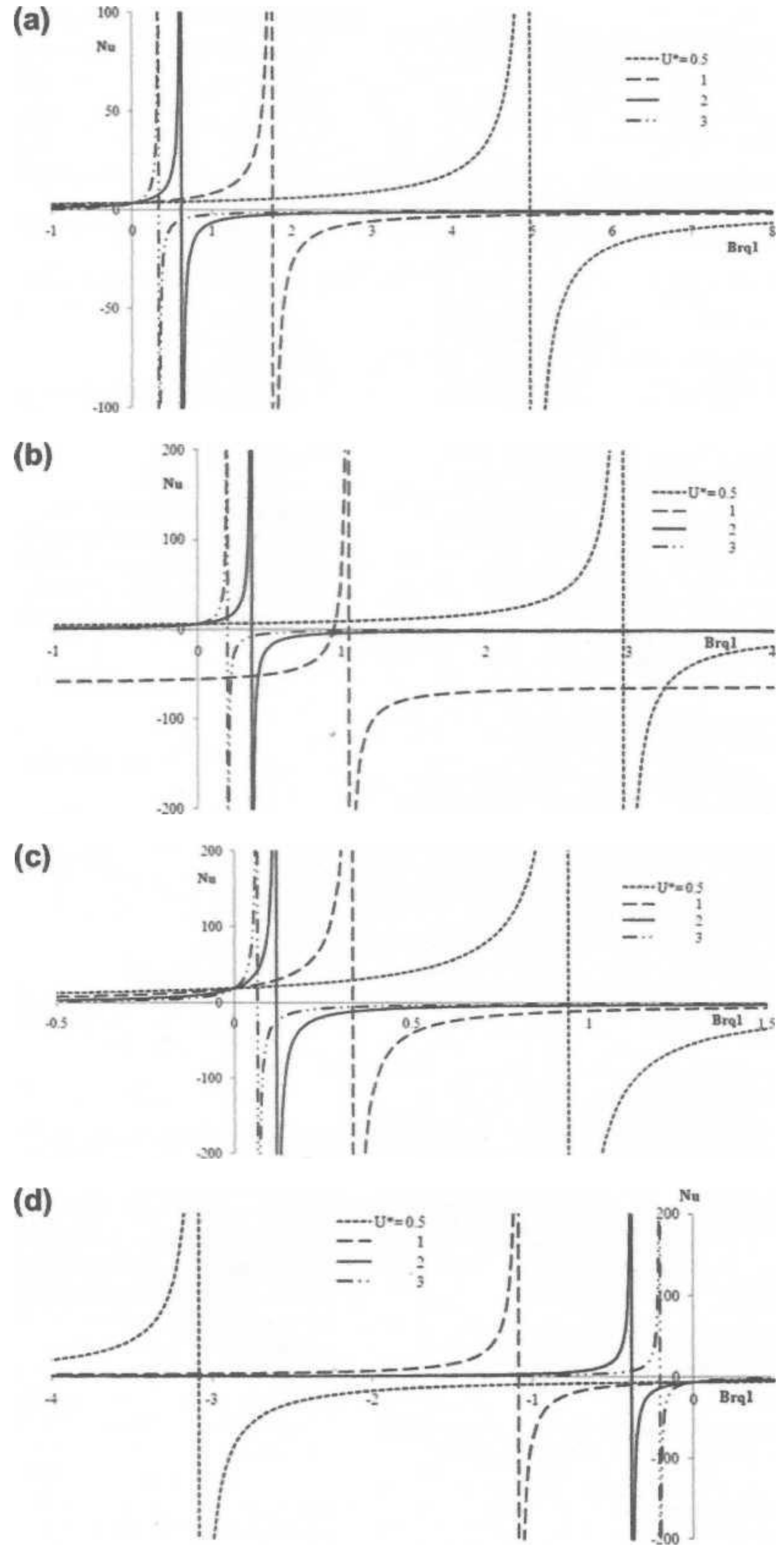


Fig. 6. Nusselt number versus Br_{q_1} at (a) $q_2/q_1 = 0$ (b) $q_2/q_1 = 0.5$ (c) $q_2/q_1 = 1$ and (d) $q_2/q_1 = 2$ at various U^* values for the case of $\Lambda = 0.8$ and $n = 1/2$.

Fig. 6 illustrates the variation of the Nusselt number against the modified Brinkman number at various U^* values such as $U^* = 0.5, 1, 2$ and 3 for the case of $\Lambda = 0.8$ and $n = 1/2$ and at various q_2/q_1 values such as $q_2/q_1 = 0, q_2/q_1 = 1/2, q_2/q_1 = 1$ and $q_2/q_1 = 2$. All the curves display rectangular hyperbolic profiles and the values of Br_{q_1} inducing singularities on the Nusselt number are shown in Table 1. For both pseudo-plastic fluids ($n = 1/4$ and $n = 1/2$) and Newtonian fluids $(n = 1)$, the Nusselt number profiles against Br_{q_1} are asymptotic, as can be deduced from Eqs. (38), (40) and (41).

3.3. Heat transfer characteristics of special case of lower plate insulated

For the case of lower plate insulated, $q_2 = 0$, and for Newtonian fluid, we obtain the result

$$Nu = \frac{1}{0.2528 - 0.1136 Br_{q_1}} \tag{44}$$

From Eq. (44), it is observed that when $q_2 = 0$ at $n = 1, Nu$ versus Br_{q_1} is asymptotic and the asymptote appears at $Br_{q_1} = 2.2256$. For pseudo plastic fluid when $n = 1/4$,

$$Nu = \frac{1}{0.2766 - 0.1515 Br_{q_1}} \tag{45}$$

From Eq. (45), it is observed that when $q_2 = 0$ at $n = 1/4, Nu$ versus Br_{q_1} is asymptotic and the asymptote appears at $Br_{q_1} = 1.8265$. $\theta(Y)$ For $n = 1/2$,

$$Nu = \frac{1}{0.2664 - 0.1515 Br_{q_1}} \tag{46}$$

From Eq. (46), it is observed that when $q_2 = 0$ at $n = 1/2, Nu$ versus Br_{q_1} is asymptotic and the asymptote appears at $Br_{q_1} = 1.7580$. For $n = 1/2$, the asymptotic values of Br_{q_1} and the graphical representations are depicted in table 1 and fig 6 respectively

Table 1 asymptotic values of Br_{q_1} at various q_2/q_1 and U^* for the case of $n = 1/2$ and $\Lambda = 0.8$

q_2/q_1	U^*			
	0.5	1.0	2.0	3.0
0	4.9724	1.7580	0.6216	0.3383
0.5	2.9576	1.0457	0.3697	0.2012
1	0.9427	03333	0.1178	0.0641
2.0	-3.0870	-1.0914	-0.3859	-0.2100

3.4. Heat transfer characteristics of case of equal heat fluxes

Of particular interest here is the case when both the upper and lower plates are of equal heat flux, i.e. $q_1 = q_2$.

3.4.1. Newtonian fluids

For the Newtonian fluid, the Nusselt number reduced to

$$Nu = \frac{1}{0.0215 - 0.1136 Br_{q_1}} \tag{47}$$

The expression of Nu Nu in Eq. (47) corresponds to the classical problem of Couette-Poiseuille viscous-dissipative problem in parallel- plate channel, for fully developed flow of Newtonian fluid with isoflux boundary condition when $U^* = 1$ and $\Lambda = 0.5$. For the case of no viscous dissipation $Br_{q_1} = 0$, the Nusselt number becomes

$$Nu = 46.4248 \tag{48}$$

3.4.2. Pseudo plastic fluids

For the pseudo plastic fluids, when $n = 1/4$,

$$Nu = \frac{1}{0.0542 - 0.1515 Br_{q_1}} \tag{49}$$

When $n = 1/4$ and $Br_{q_1} = 0$. $Nu = 18.4470$. When $n = 1/2$,

$$Nu = \frac{1}{0.0505 - 0.1515 Br_{q_1}} \tag{50}$$

When $n = 1/2$ and $Br_{q_1} = 0$. $Nu = 19.7999$.

IV. CONCLUSIONS

Heat transfer with the effect of viscous dissipation for steady, laminar, both hydro-dynamically and thermally fully developed pseudo-plastic fluid through a channel of Couette-Poiseuille flow, where both the plates are kept at specified but different constant heat flux ratios being considered as thermal boundary conditions is discussed. The momentum equation is solved semi-analytically where the upper plate is kept stationary and the lower plate is moving with constant velocity and in turn the energy equation can be solved analytically. Due to the mathematical nature of the model, the results developed are invalid for the case when the moving plate velocity approaching zero, i.e. the plane Poiseuille flow problem. The study reveals various characteristics for temperature distributions and Nusselt numbers which are influenced by the velocity of the moving plate, the heat flux ratio, power-law index, modified Brinkman number and the values of X which is in terms of the pressure gradient and channel dimension. The velocity and temperature profiles as well as the explicit Nusselt number correlations predicted in the present study would be a useful analytical tool for the design and performance analysis in a diversity of processing operations, such as in various extruders and bearings associated with lubrication problems.

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