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Monotone prime numbers with three prime digits

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Abstract
After defining, monotone prime numbers will be presented . How many monotone prime numbers with three prime digits are there in the interval (10^{p-1}, 10^p), (where p is a prime number)? On the one hand, it has been counted by computer among the prime numbers . On the other hand, the function (1), (2), (3), (4) gives the approximate number of monotone prime numbers with three prime digits in the interval (10^{p-1},10^p). Near-proof reasoning has emerged from the conformity of Mills' prime numbers with monotone prime numbers with three prime digits. The set of monotone prime numbers with three prime digits is probably infinite.

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I. Introduction

The sets of special prime numbers within the set of prime numbers are well-known. For instance, the Erdős-primes (the sum of the digits is prime) [8], Fibonacci-primes (F_0=0, F_1=1, $F_n=F_{n-1}+F_{n-2}$), Gauss-primes (in the form 4n+3), Leyland-primes (in the form x^y+y^x, where $1 \le x \le y$), Pell-primes (P_0=0, P_1=1, $P_n=2P_{n-1}+P_{n-2}$), Bölcsföldi-Birkás-Ferenczi primes (all digits are prime and the number of digits is prime), etc. Question: Which further sets of special prime numbers are there within the set of prime numbers? We have found a further set of special prime numbers within the set of prime numbers. It is the set of monotone prime numbers.

2. Monotone decreasing prime numbers with prime digits 2, 3, 5 [3], [9], [10], [11], [12],[13].
Definition:
a positive integer number is a monotone decreasing prime number with prime digits 2, 3, 5 if
a/ the positive integer number is prime, b/the digits of number are monotone decreasing primes,
c/ the number of digits is prime, d/ the sum of digits is prime, e/ all digits are 2 or 3 or 5.

The set of prime numbers meeting the conditions a/ and c/ is also well-known: it is the set of prime-long prime numbers [3], [9]. Positive integer numbers meeting all the five conditions (a/, b/, c/, d/,e/) at the same time are monotone decreasing prime numbers with prime digits 2, 3, 5 (Fig.1).

Monoton decreasing prime number p with three prime digits has the following sum form:

$$p=\sum_{j=0}^{k(p)} e_j(p).10^j \quad \text{where } e_j(p) \in \{2, 3, 5\} \text{ and } k(p)+1 \text{ is prime and } e_0(p) \in \{3\} \text{ and } \sum_{j=o}^{k(p)} e_j(p) \text{ is prime.}$$

The monotone decreasing prime numbers with three digits 2, 3, 5 are as follows (the last digit is : 3)
{{}, {}, {55333}, {}, {55555333333}, {}, {}, {}, \
{55555555555555555333333}, {55555333333333333333333333333,
55555555555555555555553333333}, {5555555555555555553333333333333}, \
{}, {55555555333333333333333333333333333333333}, {}, {}, \
{55555555553333333333333333333333333333333333333}, \
{55555555555555555555555555555555555555555555555555555333333}, \
{5555555555555555555555555555555333333333333333333333333333333}, {}, \
{}, {}, {}, {}, \
{555555555555555555555555555555555555555555555555555555555555555\
55555555555553333333}, {}, {}, {}, {}, etc.

M(p) is the factual frequency of monotone decreasing prime numbers with digits 2, 3, 5 in the interval (10^{p-1}, 10^p).
M(2)=0, M(3)=0, M(5)=1 , M(7)=0 , M(11)=1 , M(13)=0 , M(17)=0 , M(19)=0 , M(23)=1 , M(29)=2 , M(31)=1, M(37)=0, M(41)=1, M(43)=0, M(47)=0, M(53)=0, M(59)=1, M(61)=1, M(6=)=1, M(67)=1, M(71)=1,

M(73)=0, M(79)=0, etc. The factual frequency from p=2 to p=79 are only 0 or 1 or 2: the set of frequency values is approximately empty.

3. Monotone decreasing prime numbers with prime digits 2,, 3, 7 [3], [9], [10], [11], [12], [13].
Definition:
a positive integer number is a monotone decreasing prime number with prime digits 2, 3, 7 if a/ the positive integer number is prime, b/the digits of number are monotone decreasing primes, c/ the number of digits is prime, d/ the sum of digits is prime, e/ all digits are 2 or 3 or 7.

The set of prime numbers meeting the conditions a/ and c/ is also well-known: it is the set of prime-long prime numbers [3], [9]. Positive integer numbers meeting all the five conditions (a/, b/, c/, d/,e/) at the same time are monotone decreasing prime numbers with prime digits 2, 3, 7. .
Monoton decreasing prime number p with three prime digits 2,3,7 has the following sum form:

$$p=\sum_{j=0}^{k(p)} e_j(p).10^j \quad \text{where } e_j(p) € \{2, 3, 7\} \text{ and } k(p)+1 \text{ is prime and } e_0(p) € \{3\} \text{ and } \sum_{j=o}^{k(p)} e_j(p) \text{ is prime.}$$

The monotone decreasing prime numbers with three prime digits 2, 3, 7 are as follows (last digit is 3) :
{{}, {733,
773}, {77773}, {}, {}, {7777777777333}, {}, {}, \
{77777777773333333333333}, {77777777777777773333333333333}, {}, {}, \
{77777777777777777777777777333333333333333}, {}, {}, {}, {}, \
{77777777773333333333333333333333333333333333333333333333333333}, {}, \
{777777777777777733333333333333333333333333333333333333333333333333333\
33}, {}, {}, {}, {}, {}, \
{777777777777777777777777777777777777777777777777777777777777777777777\
77777333333333333333333333333333}, {}, {}, {}, {}, {}, {}, {}, {}, \
{777777773333333333333333333333333333333333333333333333333333333333333\
3333333333333333333333333333333333333333333333333333333333333333333333\
3333333333,
7777777777333333333333333333333333333333333333333333333333333333333\
3333333333333333333333333333333333333333333333333333333333333333333333\
33333333333},

etc.
C(p) is the factual frequency of monotone decreasing prime numbers with prime digits 2, 3, 7 in the interval (10^{p-1}, 10^p).
C(2)=0, C(3)=2, C(5)=1, C(7)=0, C(11)=2, C(13)=1, C(17)= 0, C(19)=1 , C(23)=0 , C(29)=0, C(31)=1, C(37)=1, C(41)=1,
C(43)=0, C(47)=0, C(53)=0, C(59)=0, C(61)=0, C(67)=1, C(71)=0, etc.
The factual frequency from p=2 to p=71 are only 0 or 1 or 2: the set of frequency values is approximately empty.

4. Monotone increasing prime numbers with prime digits 2, 5, 7 [3], [9], [10], [11], [12],[13].
Definition:
a positive integer number is a monotone increasing prime number with prime digits 2, 5, 7 if a/ the positive integer number is prime, b/the digits of number are monotone increasing primes, c/ the number of digits is prime, d/ the sum of digits is prime, e/ all digits are 2 or 5 or 7.

The set of prime numbers meeting the conditions a/ and c/ is also well-known: it is the set of prime-long prime numbers [3], [9]. Positive integer numbers meeting all the five conditions (a/, b/, c/, d/,e/) at the same time are monotone increasing prime numbers with prime digits 2, 5, 7 (Fig.1).
Monoton increasing prime number p with three prime digits has the following sum form:

$$p=\sum_{j=0}^{k(p)} e_j(p).10^j \quad \text{where } e_j(p) € \{2, 5, 7\} \text{ and } k(p)+1 \text{ is prime and } e_0(p) € \{7\} \text{ and } \sum_{j=o}^{k(p)} e_j(p) \text{ is prime.}$$

The monotone increasing prime numbers with three prime digits 2, 5, 7 are as follows (the last digit is 7):

{{}, {227, 557, 577}, {}, {5555777}, {22555577777, 55555555777}, {2222225777777, 2222777777777}, {22222555555555777, 22555555555555577, 22555555555577777, 22555577777777777, 55555555555777777}, {5557777777777777777, 5777777777777777777}, {22222222222222225555777, 22222222222222555555777, 22222255777777777777777}, {22222222222222222222222222577, 22222222222222222222222255777, 22222222222222222255555555777, 22222222222222222255555777777, 22222222222222222255555777777, 22222255555555555557777777777,etc.

D(p) is the factual frequency of monotone increasing prime numbers with prime digits 2, 5, 7 in the interval (10^{p-1}, 10^{p}).
D(2)0, D(3)=3, D(5)=0 , D(7)=1, D(11)=2, D(13)=2 , D(17)=5, D(19)=2 , D(23)=3, D(29)=7 , D(31)=9, D(37)=4, D(41)=6, D(43)=7, D(47)=10, D(53)=8, D(61)=8, D(67)=10, D(71)=13, D(73)=11, D(79)=8, D(83)=10, D(89)=12, D(97)=18, D(101)=20, D(103)=17, D(107)=12, D(109)=15, D(113)=19, D(127)=20, D(131)=17, D(137)=13, D(139)=17, D(149)=16, D(151)=20, D(157)=15, D(163)=22, D(167)=20, D(173)=22, D(179)=19, D(181)=27, D(191)=22, D(193)=22, D(197)=14, D(199)=25, D(223)=33, D(227)=30, D(229)=34, etc.

Q(p) function gives the number of monotone inecreasing prime numbers with prime digits 2, 5, 7 in the interval (10^{p-1},10^{p}). We think that the power funktion is
Q(p)=0,15p , where p is prime . (1)
The factual number of monotone increasing primes with digits 2,5,7 and the number of monotone increasing primes calculated according to functions (4) are as follows:

Number of digits p	The factual number of monotone increasing primes in the interval (10^{p-1},10^{p}) D(p)	The number of monotone increasing primes according to function (1) Q(p)= 0,15p	D(p)/Q(p)
2	0	0	-
3	3	0,45	6,66
5	0	0	-
7	1	1,05	0,95
11	2	1,65	1,21
13	2	1,95	1,03
17	5	2,56	1,95
19	2	2,85	0,70
23	3	3,45	0,87
29	7	4,35	1,61
31	9	4,65	1.94
37	4	5,55	0,72
41	6	6,15	0,96
43	7	6,45	1,09
47	10	7,05	1,42
53	10	7,95	1,26
59	8	8,85	0,90
61	8	8,85	0,90
67	10	10,05	0,96
71	13	10,65	1,22
73	11	10,95	1,00
79	8	11,85	0,68
83	10	12,45	0,80

89	12	13,35	0,90
97	18	14,55	1,24
101	20	15,15	1,32
103	17	15,45	1,12
107	12	16,05	0,75
109	15	16,35	0,92
113	19	16,95	1,12
127	20	19,05	1,05
131	17	19,65	0,87
137	13	20,55	0,63
139	17	20,55	0,83
149	16	22,35	0,72
151	20	22,65	0,88
157	15	23,55	0,64
163	22	24,45	0,90
167	20	25,05	0,80
173	22	25,95	0,85
179	19	25,95	0,73
181	27	27,15	0,99

5. Monotone increasing prime numbers with prime digits 3, 5, 7 [3], [9], [10], [11], [12], [13].
Definition:
a positive integer number is a monotone increasing prime number with prime digits 3, 5, 7 if a/ the positive integer number is prime, b/the digits of number are monotone increasing primes, c/ the number of digits is prime, d/ the sum of digits is prime, e/ all digits are 3 or 5 or 7.

The set of prime numbers meeting the conditions a/ and c/ is also well-known: it is the set of prime-long prime numbers [3], [9]. Positive integer numbers meeting all the five conditions (a/, b/, c/, d/,e/) at the same time are monotone ascendant prime numbers with prime digits 3, 5, 7 .
Monoton increasing prime number p with three prime digits 3, 5, 7 has the following sum form:

$$p=\sum_{j=0}^{k(p)} e_j(p).10^j \quad \text{where } e_j(p) € \{3, 5,7\} \text{ and } k(p)+1 \text{ is prime and } e_0(p) € \{7\} \text{ and } \sum_{j=o}^{k(p)} e_j(p) \text{ is prime.}$$

The monotone increasing prime numbers with three prime digits 3, 5, 7 are as follows (the last digit is 7):
{{}, {337, 557, 577}, {33377}, {3337777, 3355777, 5555777}, {33333333377, 33555555557, 55555555777}, {3333333335777, 3333355777777, 3333555555557, 3333555577777, 3333557777777, 3335557777777, 3355555555777}, {33333333333333577, 33333333377777777, 33355555577777777, 33355577777777777, 55555555555777777}, {3333333333335557777, 3333333333335777777, 3333333333355577777, 3333333333377777777, 3333333335555555557, 3333333355777777777, 3333335777777777777, 3333377777777777777, 3355555555555555777, 5557777777777777777, 5777777777777777777}, {33333333333333333333577, 33333333333333355555577, 33333333333333355555557, 33333555557777777777777}, {33333333333333333333335555777, 33333333333333333555555555577, 33333333335555555777777777777, 55555777777777777777777777777}, {3333333333333333333333333555577, 3333333333333333333333335555777, 3333333333555555555555557777777, 3333333335555555557777777777777, etc.

F(p) is the factual frequency of monotone increasing prime numbers with prime digits 3, 5, 7 in the interval $(10^{p-1}, 10^p)$.
F(2)=0, F(3)=3, F(5)=1, F(7)=3, F(11)=3, F(13)=7, F(17)=5, F(19)=11, F(23)=4 , F(29)=4, F(31)=11, F(37)=13 , F(41)=14, F(43)=11, F(47)=11, F(53)=13, F(59)=10, F(61)=14, F(67)=13, F(71)=10, F(73)=22, F(79)=17,

F(83)=8, F(89)=25, F(97)=16, F(101)=29, F(103)=17, F(107)=29, F(109)=31, F(113)=22, F(127)=25, F(131)=26, F(137)=19, F(139)=29, F(149)=31, F(151)=37, F(157)=45, F(163)=43, F(167)=27, F(173)=20, F(179)=, F(181)=62, F(191)=45, F(193)=38, F(197)=37, F(199)=56, F(223)=43, F(227)=47, etc.
R(p) function gives the number of monotone inecreasing prime numbers with prime digits 3, 5, 7 in the interval (10^{p-1},10^p). We think that the power funktion is
R(p)=0,21p , where p is prime . (2)

The factual number of monotone increasing primes and the number of monotone decreasing primes calculated according to functions (1) are as follows:

Number of digits p	The factual number of monotone increasing primes in the interval (10^{p-1},10^p) F(p)	The number of monotone increasing primes according to functions (2) R(p)= 0,21p	F(p)/R(p)
2	0	0	-
3	3	0,63	4,76
5	1	0,21	4,76
7	3	1,47	2,04
11	3	2,31	1,30
13	7	2,73	2,56
17	5	3,57	1,40
19	11	3,99	2,76
23	4	4,83	0,83
29	4	6,09	0,66
31	11	6,51	1,69
37	13	7,77	1,67
41	14	8,61	1,63
43	11	9,03	1,22
47	11	9,87	1,11
53	13	11,13	1,17
59	10	12,39	0,81
61	14	12,81	1,09
67	13	14,07	0,92
71	10	14,91	0,67
73	22	15,33	1.44
79	17	16,59	1,02
83	8	17,43	0,46
89	25	18,69	1,34
97	16	20,37	0,77
101	29	21,21	1,37
103	17	21,63	0,79
107	29	22,47	1,29
109	31	22,89	1.35
113	22	23,73	0,93
127	25	26,67	0,94
131	26	27,51	0,95
137	19	28,77	0,66
139	29	29,19	0,99

6. Number of the elements of the set of monotone increasing prime numbers with digits 3,5,7 [3], [9], [10], [11], [12], [13].

Let's take the set of Mills' prime numbers!
Definition: The number m=[M ad 3^n] is a prime number, where M=1,306377883863080690468614492602 is the Mills' constant, and n=1,2,3,… is an arbitrary positive integer number. It is already known that the number of the elements of the set of Mills' prime numbers is infinite. The Mills' prime numbers are the following: m=2, 11, 1361, 2521008887,…

The connection n→m is the following:1→2, 2→11, 3→1361, 4→2521008887,… The Mills' prime number m=[M ad 3^n] corresponds with the interval $(10^{m-1},10^m)$ and vice versa. For instance:2→$(10, 10^2)$, 11→$(10^{10},10^{11})$, 1361→$(10^{1360},10^{1361})$, etc. and vice versa. The number of the elements of the set of Mills' prime numbers is infinite. As a consequence, the number of the intervals $(10^{m-1}, 10^m)$ that contain at least one Mills' prime number is infinite. The number of monotone increasing primes with digits 3,5,7 in the interval $(10^{m-1},10^m)$ is R(m)=0,21m. The number of monotone increasing prime numbers with digits 3,5,7 is probably nfinite: lim R(p)=∞ is probably where p is prime, if p→∞.

II. CONCLUSION

Countless different sets of special prime numbers have been known. We have found the following set of special prime numbers within the set of prime numbers. There may be further sets of special prime numbers that we do not know yet. Finding them will be task of researchers of the future.

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REFERENCES:

[1]. http://oeis.org/A019546
[2]. Freud, Robert – Gyarmati, Edit: *Number theory* (in Hungarian), Budapest, 2000
[3]. http://ac.inf.elte.hu → VOLUMES → VOLUME 44 (2015)→ VOLLPRIMZAHLENMENGE→FULL TEXT
[4]. http://primes.utm.edu/largest.html
[5]. http://mathworld.wolfram.com/SmarandacheSequences.html
[6]. Dubner, H.:"Fw:(Prime Numbers) Rekord Primes All Prime digits" Februar 17. 2002
[7]. http://listserv.nodak.edu/scripts/wa.exe?A2=ind0202&L=nmbrthry&P=1697
[8]. Harman, Glyn: Counting Primes whose Sum of Digits is Prime. Journal of Integer Sequences (2012. , Vol. 15, 12.2.2.)
[9]. ANNALES Universitatis Scientiarum Budapestiensis de Rolando Eötvös Nominate Sectio Computatorica,2015, pp 221-226
[10]. International Journal of Mathematics and Statistics Invention, February 2017: http://www.ijmsi.org/Papers/Volume.5.Issue.2/B05020407.pdf
[11]. International Organisation of Scientific Research, April 2017 Bölcsföldi Birkás prime numbers: http://www.iosrjournals.org/iosr-jm/pages/v13(2)Version-4.html or http://dx.doi.org or www.doi.org Article DOI is: 10.9790/5728-1302043841

[12] International Refereed Journal of Engineering and Sciense 2018: Ács-Bölcsföldi-Birkás prime numbers: http://irjes.com/volume7issue6.html
[12]. International Organisation of Scientific . Research, 2021-08-26 Golden ratio prime numbers with three prime digits http://iiosrjen.org/pages/current-issue.html

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