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Specialized Processor Structure for Signal Processing in Piecewise–polynomial Bases

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ABSTRACT: *In this work the results of research of algorithms of fast spectral transformations in various bases are shown. The algorithm of fast transformation in the piecewise-quadratic bases, using good differential properties of parabolic basic splines is proposed. The structure of the specialized processor of signals processing in piecewise-polynomial bases is developed. By modeling in MATLAB environment with application standard Simulink - components possibilities of the offered algorithms and structure of the specialized processor are shown.*

KEYWORDS: *parallel calculations, piecewise–polynomial bases, specialized processor structure, signal processing, spectral transformations*

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I. INTRODUCTION

In the various systems intended for automation of scientific researches, the increase in productivity of the decision of problems in many respects occurs due to search of new algorithms of parallel calculations, including by revealing so-called " internal parallelism " problems and creation of architecture of the computing systems providing distribution of resources on the big number of processors. The most simple and effectively sold hardware are piecewise–polynomial and piecewise-rational methods. Piecewise-constant bases have received the progress from the spectral theory of discrete orthogonal functions with the limited number of values. For such algorithms as fast spectral transformations the greatest possible speed can be reached, and in the systems containing quite a limited number of processors.

This work analyses deficiencies of known systems orthogonal piecewise–polynomial basic functions applied in processors of fast spectral transformations: weak convergence approximation, discontinuity, etc. principles of creation of systems of piecewise–polynomial basic functions of higher degrees by a method of integration with a variable top limit is being developed:

$$\psi(x) = \int_0^x \varphi(x)dx. \qquad (1)$$

Features of integrated systems of (Haar, Shauder and Harmut) functions, convergence speed of interpolated numbers on their basis and an opportunity of applying of fast spectral processing (FSP) algorithms to these systems are examined. It has been proven through mathematical transformations and modeling, that it is necessary to apply algorithms of (FSP), developed for orthogonal piecewise of bases to fast calculation of factors, to the derivatives of interpolated splines constructed according to readout of signals [1,3,4,5,7,9,10].

II. SYSTEM OF FUNCTIONS HAAR

For construction models of the signals received from real objects, traditional harmonious functions are widely applied. It tells that many signals received from real objects can be easily presented by set sine and cosine waves fluctuations, which is using for the device of Fourier analysis. Result of it is transition from time to frequency functions. However, representation of time function sine and cosine wave functions is only one from many representations.

Any full system of orthogonal functions can be applied for expansion in a series, which correspond with Fourier series.

Wide distributions to technical appendices have received orthogonal systems continual the explosive basic function, given on the valid axis, which also have algorithms of fast transformations. They can be split in two classes:

- global basal functions - which value are not equal to zero neither one subinterval. Walsh's functions has a concern to this class [10] [17], numerical [17] [20] [22], sawtooth;
- localizable basic functions, which nonzero values are set on the enclosed pieces. Examples are Haar [17][20] [26] and Harmut's [20] [22] [24] functions.

Comparison of bases by quantity of demanded arithmetic operations

Table 1

Bases of Haar involve attention of experts for two reasons:

No.	Basis's name	Quantity of demanded arithmetic operations	N=1024	Operation over complex numbers
1	Basis of harmonics of functions (FFT)	m·N	10240	Is present
2	Walsh's basis (FWT)	m·N	10240	Is absent
3	Harmut's basis (Harmut FT)	3·N-4	3068	Is absent
4	Haar's basis (Haar FT)	2·N-2	2046	Is absent

1. Reduction of number of the factors necessary for approximation (with the set accuracy) in relation to the general number of binary pieces.
2. Absence of "long" operations. Operations of addition, subtraction and displacement are used only.

The limitations of rectangular orthogonal Haar's bases is weak convergence of numbers on sectionally - to constant functions, i.e. necessity of storing of several hundreds factors for many functions with the purpose of maintenance of errors of the order of 0,1 %.

Searches on the methods of reduction of volume of tables of factors, improvements of parameters of "smoothness" obviously lead to systems piecewise–polynomial basic functions of higher degree. Most simply results to piecewise- linear basic functions (function of Shauder) as a result of integration with a variable upper limit orthogonal piecewise-constant functions of Haar:

$$Shd_k(x) = 2^p \int_0^x har_k(r)\,dr$$

It is necessary to consider also, that $Shd_0^0(x) \equiv 1$ and $Shd_0(x) = x$

Often in practical appendices of numbers piecewise-linear functions with the objective of getting the amplitudes of all basic functions equal to unit, it is convenient to operate with the "normalized" systems:

$$S\tilde{h}d_k(x) = 2^p \int_0^x har_k(r)\,dr$$

$$P = 0,1,\ldots; \qquad k = 0,1,2,\ldots \qquad (2)$$

Fig. 1. a. shows piecewise-constant functions of Haar, and Fig.1. b. - functions of Shauder and on Fig.1. c. piecewise-parabolic functions of Haar.

Functions har_k (x) can be arranged a put in an orderly manner (by analogy ordering of functions (Walsh) in three ways:

1. To arrange binary fractions k in the natural order of increament – Adamar's ordering;
2. To arrange k in ascending order of inverted code - dyadic ordering;
3. To group k in group with number of categories after a comma 1,2,3, etc., and inside of each group to arrange fractions in ascending order - classical ordering;

Adamar's ordering of Haar functions leads to reception of the optimal algorithms FTH, but practical use of spectra in adamar is inconvenient ordering, as in such spectrum is not provided uniform convergence of the partial sums for continuous functions which are usually considered as signals.

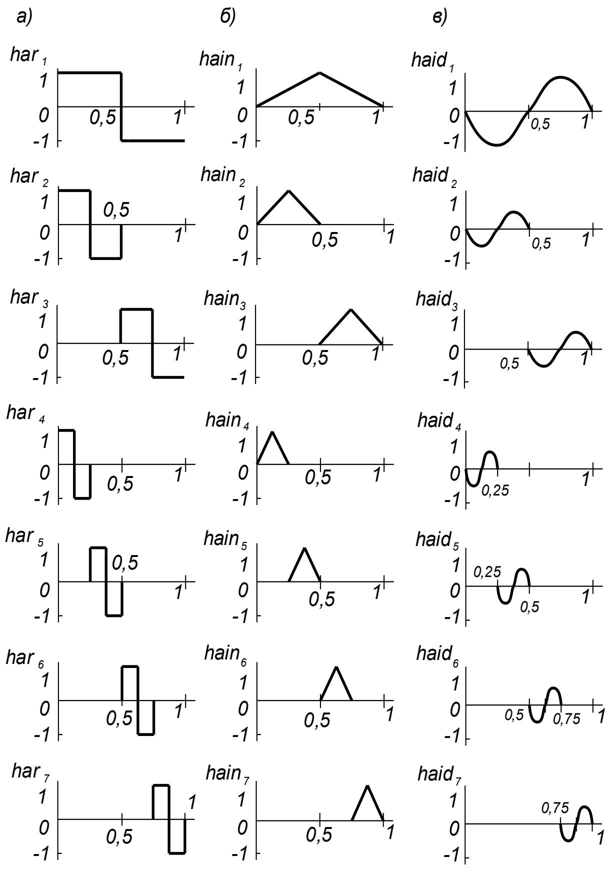

Fig.1. a) Piecewise-constant, b) Piecewise-linear and c) Piecewise-quadratic bases of Haar.

3. Algorithm of fast spectral transformations in piecewise-polynomial bases

One of the basic features of orthogonal bases is presence of fast algorithms for definition of spectral factors. Fast algorithms allow to reduce quantity of arithmetic operations and volume of necessary memory. As a result we obtained an increasing speed when using orthogonal bases for digital processing signals.

Having designated through any valid orthogonal piecewise-constant basis, we shall write down the formula of forward and backward fast spectral transformations for sequence of readout of a signal {xi}.

$$C_k = \frac{1}{2^p} \sum_{i=0}^{n-1} x(i) \cdot \varphi(k,i) \qquad (3)$$

$$X_i = \sum_{k=0}^{n-1} C_k \cdot \varphi(k,i) \qquad (4)$$

where k = Number of spectral coefficient, i = number of an element of sequence of the valid readout.
Algorithmically both of a kind of transformations differ from each other only by the presence of a constant multiplier.

Requirements for algorithms of fast spectral transformation consist first of all in a minimality of quantity of operations, simplicity of each operation and a minimality of the demanded volume of operative memory.

Fig.2. shows the graph of forward FTH on Cooley-Tukey at N=16. Here and in the later graphs, continuous lines conform to operations of addition, and shaped lines - to operations of subtraction. Entrance readout are designated X_0, X_1..., X_N, and results are designated C_0, C_1, C_2..., C_N. Let's consider approximating number on piecewise-quadratic Harmut's functions:

$$f(x) \cong \sum_{k=0}^{n-1} C_k \, hid_k(x) \qquad (5)$$

The limitations of a given set is the absence of fast algorithm of calculation of factors. This limitation can be eliminated by the application of a parabolic spline. If we take the second derivative of a parabolic spline [2,4,6,7,8], interpolating on [0,1] function f (x), it will represent piecewise-constant function with changes of values of steps in units of a spline, and of some on piecewise-constant orthogonal basic functions. We shall write down, for example, decomposition of the derivative of a spline in a Harmut set:

$$S_2''(x) \cong \sum_{k=0}^{n-1} C_k \, hrm_k(x) \qquad (6)$$

Where hrm = Harmut's function.
According to theorems of the limited convergence and of integration of the closed systems as a result of integration of both parts we will get:

$$S_2'(x) = 2^p \int_0^x S_2''(u)\,du = \sum_{k=0}^{n-1} C_k \, hin_k(x) + S_2'(0) \qquad (7)$$

$$S_2(x) = \int_0^x S_2'(u)\,du + S_2(0) = \sum_{k=0}^{n-1} C_k \, hid_k(x) + S_2'(0) + S_2(0) \qquad (8)$$

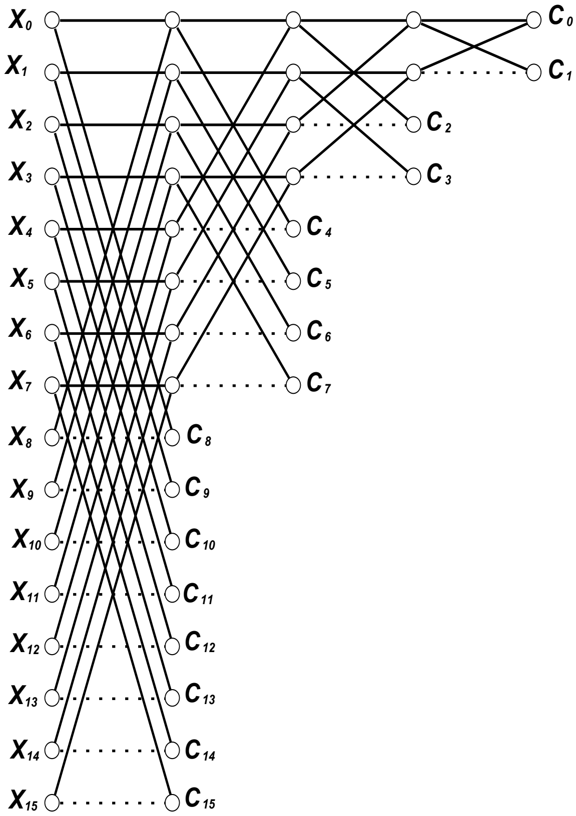

Fig.2. Graph of Haar fast transformation by Cooley-Tukey.

Whence follows, that factors of decomposition in a set by orthogonal functions of Harmut of the second derivative of the parabolic spline, interpolating the function in binary-rational units, are factors of decomposition of the first derivative of a spline on hin - to functions, and the spline on hid - to functions. The factor at a linear part of decomposition will be defined as value of the first derivative of spline S_2 (x) in a point x = 0, and constant component - as value of a spline in this point.

III. MODELLING STRUCTURE OF THE SPECIAL-PURPOSE PROCESSOR FOR PROCESSING OF SIGNALS IN PIECEWISE-POLYNOMIAL BASES

Research on graphs of fast spectral transformations in localized bases show, that their various modifications define the sets of variants of specialized computing structures. Researched graphs of fast spectral transformations in localized bases show, that their various modifications define sets of versions of special-purpose computing structures. A number of works were devoted to issues of development of structures and principles of their construction, in them opportunities of work in real time are marked, but the problem of achieving the maximal speed which is a major factor in problems DSP is not put.

The structure of the specialized processor in parallel-conveyor-based, carrying out fast transformation of Haar on algorithm Cooley-Tukey is shown on Fig.3. The structure consists of the block of buffer registers for storage N/2 of readout of data (where N-quantity of entrance readout), four processor blocks and a ultrafast operative memory (RAM) for storage both intermediate, and final factors.

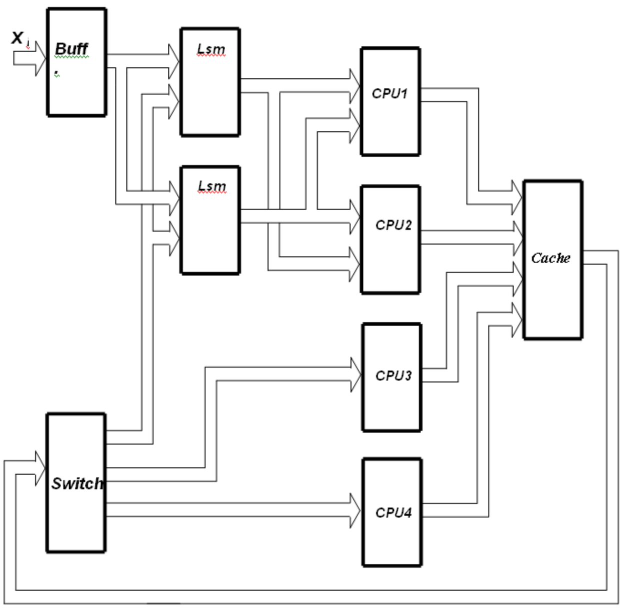

Fig.3. Structure of the specialized processor for digital processing a signal on algorithm Cooley-Tukey.

The degree of universality of the proposed computing structure is caused by the application of the piecewise-quadratic basic functions, allowing approximation with required accuracy of category of signals under research. High speed and relative simplicity of hardware realization are reached, in many respects, owing to use of principles of parallelization, conveyorization calculations and overlapping of operations of input with processing.

Transition to piecewise-quadratic functions Haar and Harmut and development of the computing structures which are carrying out transformations on piecewise-quadratic functions, allow to improve accuracy of approximation, to reduce quantity of the factors necessary for approximation, and by that to save a memory size.

Application of principles of parallelization and conveyorization, and also overlapping of operations of input with processing promote increase in the speed of specialized computing structure.

High-speed parallel – conveyor structure of the specialized processor was developed for performance of fast transformations in piecewise–polynomial the bases, differing by wide introduction of principles parallelization, conveyorization and overlapping of operations of input with processing.

IV. RESULT OF ALGORITHM MODELLING AND STRUCTURE OF THE SPECIALISED PROCESSOR.

In Fig. 4. The Simulink-model of the specialized processor is shown. For modeling of the proposed structure in the environment of MATLAB with application of resources Simulink following blocks are used:

- Two blocks of Sine Wave and Add block for feigning of an entry signal;
- Zero-Order Hold block for simulation of digitization and quantization of input analogue signal;

- Four blocks Integer Delay and summators, summators-subtracters for performing the operations of addition-subtraction under column (X.)
- Several blocks of XY Graph, Scope, To Workspace for the control of signals at various stages of processing and for link with MATLAB environment.

The input signal also can be entered in form of a data array from working environment MATLAB.

With the application developed Simulink models, modeling operation of structure has been done.

In table 1. Results of modeling of function $y=e^x$ in an interval [0, 1] for N=64 readout in piecewise-quadratic basis of Haar are shown. Results were achieved with the application developed Simulink models.

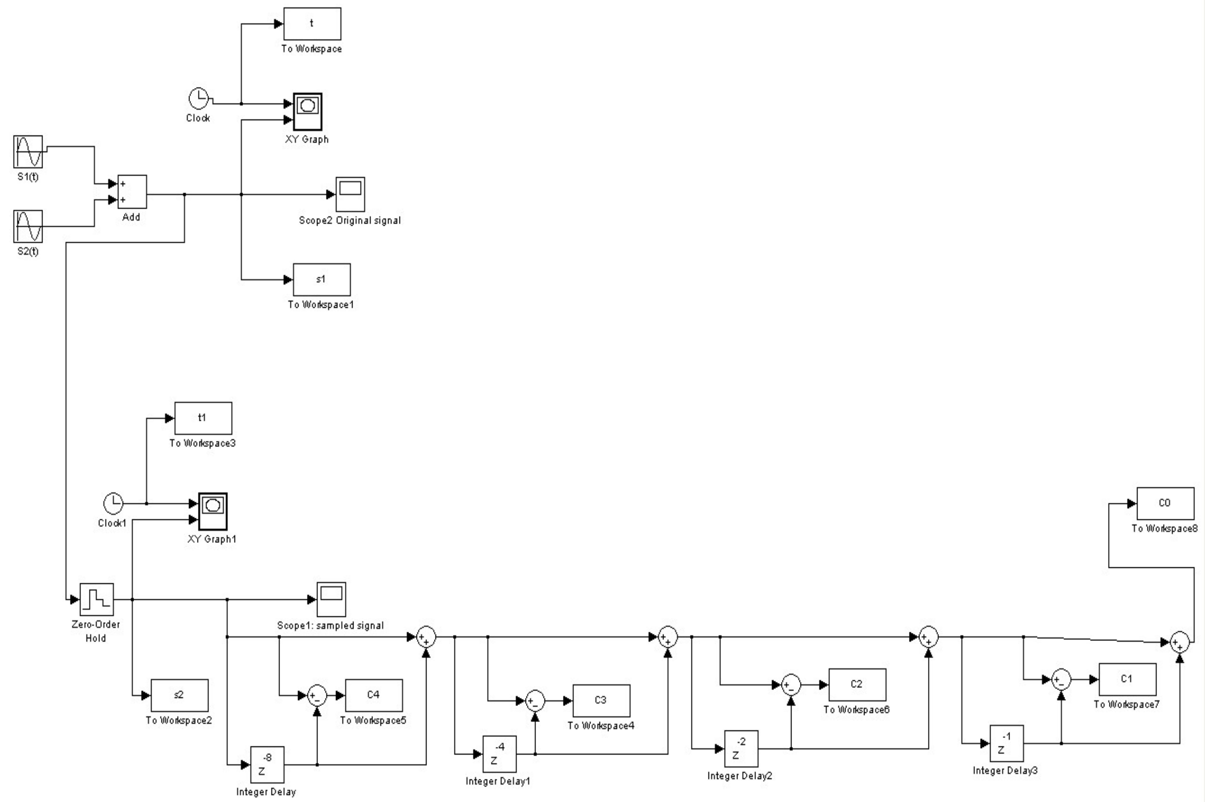

Fig. 4. The Simulink-model of the specialized processor.

Table 1.Executed: FT Haar Function: y=exp (x) N=64 Array of factors in Haar's piecewise-quadratic basis.

i	C_i	i	C_i	i	C_i	i	C_i
1	0.029808	17	0.045295	33	0.013813	49	-0.000006
2	0.080928	18	-0.000017	34	-0.000004	50	-0.000007
3	0.044062	19	-0.000017	35	-0.000004	51	-0.000007
4	0.40078	20	-0.000020	36	-0.000005	52	-0.000007
5	0.45037	21	-0.000019	37	-0.000004	53	-0.000007
6	-0.000353	22	-0.000021	38	-0.000005	54	-0.000007
7	-0.000453	23	-0.000023	39	-0.000005	55	-0.000008
8	0.041561	24	-0.000024	40	-0.000005	56	-0.000008
9	0.045246	25	-0.000025	41	-0.000006	57	-0.000008
10	-0.000073	26	-0.000028	42	-0.000005	58	-0.000008
11	-0.000084	27	-0.000030	43	-0.000005	59	-0.000009
12	-0.000093	28	-0.000032	44	-0.000006	60	-0.000010
13	-0.000105	29	-0.000034	45	-0.000006	61	-0.000009
14	-0.000120	30	-0.000035	46	-0.000005	62	-0.000010
15	0.000135	31	-0.000037	47	-0.000006	63	-0.000009
16	0.041988	32	0.042103	48	-0.000006	64	0.042132

V. CONCLUSION

Thus, as a result of research of systems of piecewise -constant, piecewise-linear and piecewise-quadratic bases their limitations are shown. For elimination piecewise of the limitation of these bases, and also on the improvement purpose approximation properties of investigated bases, the algorithm of fast transformation in the piecewise-quadratic bases, based on applications of a parabolic basic spline is proposed. The structure of the specialized processor for processing of signals of the bases under study were also proposed. The modeling of algorithms and structure of the specialized processor makes it possible to draw a conclusion that the proposed algorithm of fast transformation in piecewise-quadratic bases makes it possible to achieve the compression factor of signals from 1,05 to 3,70 at value of accuracy $10^{-4} \div 10^{-6}$.

This makes it possible to save a memory size necessary for the storage of spectral factors during the processing of real experimental files from 5 % to 24 % and during the processing of elementary functions (and functions consisting of their combinations) from 11 % to 70 %. The proposal structure of the specialized processor realizes an investigated class of signals with demanded accuracy, possesses the big factors of compression at the expense of application of piecewise-quadratic basis.

Application of results of researches in various systems of modeling, the control and management has shown that developed piecewise-polynomial methods of approximation of signals can be successfully realized algorithmically in the form of a system of program components, and also in hardware in the form of subsystems of digital signals processors.

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