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Short Summary on History of Incline Algebra and its Association with distributive lattice

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***Abstract**: In This paper we make a review on history of incline algebra and its association with distributive lattice. Also our focus in this paper are when is an incline a lattice and a distributive lattice? and when does a lattice have a nontrivial incline structure?*

***Keywords:** Incline structure, Fuzzy algebra, Boolean Algebra, Probability theory, Lattice, Distributive lattice*

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I. Introduction

In our real life various problems related with medical sciences, engineering, political, financial, social deciplines and numerous different arenas involve provisional data which are not always necessarily in crisp, appropriate and conclusive forms due to uncertainty associated with these problems. Such problems are usually being handled with the help of the topics like probability theory , fuzzy set theory, intuitionistic fuzzy sets, interval mathematics and rough sets.

II. Brief History of Incline Algebra and its its Association with Foundational Algebra.

As we know that, propositional logic plays an important in the study of uncertainty of real life and many complex and have problems of human body as well as nature will be studied with approximately.

In continuation of this eagerness to reveal the parallel problems In the 1800's mathematicians discovered that propositional logic could be represented by a new structure called Boolean algebra in which the logic that $0+0=0, 1+0=0+1=1$ but $1+1=1$, where 1 denote true and 0 the false statement. This representation literally can be extended from propositions like p and $(q \text{ or } r)$ to the more complicated logic of binary relations considering matrices over the Boolean algebra. That is, let a relation "$x < y$" be represented on a set $\{x_1, x_2, x_3 \dots \dots, x_n\}$ by a matrix A where $a_{ij} = 1$ if $x_i < x_j$ and $a_{ij} = 0$ if $x_i \nless x_j$. With natural definitions. transitivity becomes the matrix inequality $A^2 \leq A$.

In the 1960's it was extended to a type of multi- valued logic known as fuzzy sets. Here we consider the work of domain by taking real number as elements between 0 and 1 and binary operation of addition and by multiplication are respectively by $\max(a, b)$, and by $\min(a, b)$from which we get a notion of degree to which the relationship holds and the degree of a composition is the minimum of its components.

For example,"x_1, is an associate of x_2" with degree 0.6 can be stated as a_{12}=0.6.

Boolean algebra and the theory of fuzzy sets are two examples of a structure called incline.

In a incline generally, the degree of relationship in a composition could be less than either factor.

One example we can obtained when we take as elements the interval $[0, 1]$ and operations $max\ (a, b)$ and xy. Which is combines arithmetic and ordered structures. In an incline the additive operation is $max\ (x, y)$ taken within an order structure and the multiplicative operation can be any associative distributive operation such that xy is less than or equal to either factor.

This gives the reason for the name: under repeated operations, quantities tend to decrease. Inclines can also be used to represent automata and other mathematical systems, in optimization theory, to study inequalities for nonnegative matrices and matrices of polynomials.

For semi-ring theory, see Clifford and Preston (1961) on congruence, generators, relations, which are the same as for semi-groups. For lattice theory, see the elementary material in Birkhoff (1967).

Special cases of incline theory have appeared in Cuninghame-Green's Minimax Algebra (1979) and in U. Zimmerman (1981).

R. Cuninghame-Green (1979) calls a commutative semi- ring satisfying $x \oplus x = x$, a belt. He introduces a dual addition $x \oplus' x = x$ (in practice $\oplus$ is often maximum and $\otimes'$ is minimum) and a dual multiplication $\oplus'$ which differs only in that $-\infty \otimes' \infty = -\infty$ and $-\infty \otimes' \infty = \infty$ (in practice $x \otimes y$ is $x + y$).

The combined structure is called a blog. He studies conjugacy, duality, residuation, special matrices all of whose row sums are equal, simultaneous linear equations, a definition of linear independence based on $\sum x_i v_i =$

c having unique solutions x_i for some c, rank based on this concept, regularity based on a strong linear independence, semi-norms, Eigen values, eigenvectors, spectral inequalities, sequences of powers, permanents, strict invertibility $AB = BA = I$. He then presents applications of these concepts.

U. Zimmerman (1981) deals with more general order structures: lattice ordered groups, idempotent semi-rings (the same as belts above), and ordered semi-modules. He proves characterization and representation theorems for these. He then applies these to a similar set of problems to those considered by Cuninghame-Green: shortest and other paths in graphs, linear programs, scheduling, duality for matroid flows, transportation and assignment problems. He also takes up eigenvalues in the finite dimensional case.

H. Hashimoto (1983) has studied what he calls a path algebra. A path algebra is a commutative semi-ring with $0, 1$ satisfying $x + x = x$. Thus it does not have the incline inequality, so that many properties of inclines are false.

Basically incline has an structure which is associative, commutative w.r.t addition and distributive over multiplication such that $x + x = x, x + x * y = x$; $\forall x, y$. It has a semi-ring structure and partial order structure. In this chapter we consider the standard concepts: sub-algebra, quotient algebra, ideal, congruence, direct product, generators, relations for inclines. These apply in a way similar to their use for groups and semi-groups. Every incline is a semi-lattice.

Our main focus in this paper are:

(1) When is an incline a lattice a distributive lattice?

(2) When does a lattice have a nontrivial incline structure?

Definition 2.1. An incline is an algebraic structure (ℑ,+,*) having a non-empty set ℑ and two binary operations + and * such that for all x,y,z in ℑ, if the following laws hold

[K1] *Associative laws*

(i)x + (y + z) = (x + y) +z,

*(ii)x * (y * z) =(x * y)* z.*

[K2] *Commutative laws*

(i)x + y = y + x,

*(ii)x * y = y * x.*

[K3] *Distributive laws*

*(i)x * (y + z) = (x * y) + (x * z),*

*(ii)(y + z) * x = (y * x) + (z * x).*

[K4] *Idempotent law: x + x = x.*

[K5] *Incline law*

*(i)x + (x * y) = x,*

*(ii)y + (x * y) = y.*

In brief an incline is an algebraic structure with two operations, addition and multiplication. It generalizes distributive lattices in that multiplication need not be idempotent.

We have Boolean algebras ⊂ fuzzy algebras ⊂ distributive lattices ⊂ inclines ⊂ semi-rings. Therefore, it may have applications to new models in various sciences.

Definition 2.2. Let $x,y \in ℑ$. The incline order relation denoted as "$\leq$" and is defined as $x \leq y \leftrightarrow x + y = y$.
From the incline axiom (K5) obiviously, we have

I. $x + y \geq x$ and $x + y \geq y$ for $x,y \in ℑ$,

II. $xy \leq x$ and $xy \leq y$ for $x,y \in ℑ$.

which are known as incline properties.

Here it is clear from the above structure that these operations make quantities to decrease "slide downhill." Hence, we decided to name it as incline and let ℑ denote an arbitrary incline where ℑ is the first letter of the Korean alphabet and is pronounced as "Gee-Uck" and it look like an incline.

Example 2.3: The Boolean algebra {0, 1} is trivial example of an incline under Boolean operations meet and join.

Example 2.4: The fuzzy algebra [0, 1] is also have an incline structure under the operations maximum and minimum.

Proposition 2.5: Every distributive lattice is an incline. An incline is a distributive lattice (semi-ring) if and only if $x^2 = x$; $\forall x$.

Proof. The first part of the statement is obvious.

Again, If $y \leq z$ then $y + z = z, \Longrightarrow x(y + z) = xz$ *i.e..* $xy \leq xz$. For any distributive lattice $x \wedge x = x$.

Conversely, we want to show that if $x^2 = x$ then $xy = x \wedge y$ true Here we have $xy \leq x$ from incline properties. Now from commutatively $xy \leq y$.

Suppose $u \leq x$ and $u \leq y$. Then $u = u^2 \leq xy$.
This proves that $xy = x \wedge y$.
Example 2.6: The set [0, 1] with the usual multiplication and partial order relation have an incline structure but not $x^2 = x$.
Explanation: It clearly follows from the definition that an incline has a semi-lattice structure with respect to addition and a semi-group structure under multiplication.

Any unspecified ordering in an incline it is assumed to be the semi-lattice ordering.

Proposition 2.7: Every incline is a semi-lattice ordered commutative semi-group in which $xy \leq x$, w.r.t the same operations.
Proof. Suppose $x \leq y$ in an incline. Then $zx + zy = z(x + y) = zy$. Therefore $zx \leq zy$. This proves that we have a semi-lattice ordered commutative semi-group.

The converse is false since a non-distributive lattice is a semi-lattice ordered commutative semi-group in which $xy \leq x$ but is not an incline.

III. Conclusion

In this paper, we written brief history on incline algebra and its given the answer when is an incline a lattice a distributive lattice? and when does a lattice have a nontrivial incline structure? Which established the relation of incline algebra with lattice and distributive lattice.

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References

[1]. Z.Q.Cao,K. H.Kim and F. W. Roush, *Incline Algebra and Applications,* John Wiley and Sons, New York,1984.
[2]. Jun, Y.B., Ahn, S.S., Kim, H.S., fuzzy sub-inclines (ideals) of incline algebra, fuzzy sets and system 123(2001), No. 2, 217-225.
[3]. Ahn, S.S., Jun, Y.B., Kim, H.S., On γ-ideals in incline algebra, Lommun, Korean Math Soc. 17(2002), No. 2, 229-235.
[4]. L. A. Zadeh, K.-S. Fu, E. Tanaka, and M. shimura, Eds. Fuzzy Sets and Their Applications to Cognitive and Decision Processes, Academic Press, New York, 1975.
[5]. D. Zimmermann, Linear and Combinatorial Optimization in Ordered Algebraic structures, Annals of Discrete Mathematics, No. 10, North-Holland, Amsterdam, 1981.
[6]. T. Qin, Jar metric principle (11) Science Exploratim, 1(1981), 101-108.
[7]. Y. Qin, Algorithms on optimal path problems, science Exploration, 1(1904), 63-94.
[8]. B. Tilson, Decomposition and complexity of finite semigroups, Semigroup Forum, 3(1971), 189-250,
[9]. R. F. Willians, Classfication of subahifts of finite type, Ann. Math., 2(1974), 380-381.
[10]. F. Ramsey. On a problem of formal logic, Proc. Lendos Math. Soc., 30(1930), 204-206
[11]. D. Allen. JE, A generalization of the fees theuren to a class of regular semigroups, Semigroup Forse 2 (1973), 321-331-
[12]. G. Birkhoff, Lattice Theory, Aner, Math, Sc. coling Pub. Vol. 25, Providence, R... 1967
[13]. Thi-qiang Can. An algebraic system generating the fuzzy subsets of a set, in Advances in Pussy at Theory and Applications, P. P. Mang. Ed., to appear
[14]. Thi-giang Cao, Modern control theory of psychological phenomena, Proceedings of CAA-JKER Bilateral Meeting on Control Systems, August 10-12, 1981, shanghai
[15]. Thi-qiang Can, Modere control theory of psychological phenomena, in Recent Developments in Control System and its Applications, Chinese Association of Automation. Ed., Gordon and Breach, New York, 1982, 192-190