Mathematical Modeling for Reliability Assessment of Uninterruptible Power Supply System in Sokoto State, Nigeria

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ABSTRACT: Uninterruptible Power Supply (UPS) is a system which is designed to provide a short-term power supply for our constant power demanding electronic equipment. This research paper presents a mathematical modeling for the reliability assessment of UPS system in Sokoto State, Nigeria, using part stress method. Data on failures of APC BG650MI in Sokoto were used as a case study, with special consideration given to factors like environmental impact, quality of power supply, service personnel, human factors (over and under usage). A comparative assessment was made on the reliability and reliability indices of the UPS when operated within Sokoto environment and when operated within the environment for which it was planned (USA). The result shows that lower reliability level is associated with the use of the UPS in Sokoto State of Nigeria, as compared with the country for which it was designed.

Keywords: Availability, failure rate, Maintainability, Modeling, MTBF, MTTF, Reliability.

I. INTRODUCTION

1.1 Background to the study

To help us better understand our world, we often describe a particular phenomenon mathematically by means of a function or an equation called model. Such mathematical model is an idealization of the real-world phenomenon and never a completely accurate representation [1]. Hence model can only approximate real-world behavior. However most models simplify reality, a good one can provide valuable result and conclusions. Also mathematical model can help us understand a behavior better or aid up in planning for the future. The model allows us to reach mathematical conclusions about the behavior of system as illustrated in fig. 1.1 [13]

![Fig 1.1: A flow of the modeling process beginning with an examination of the real-world data.](image)

Reliability is the ability of an apparatus, machine, or system to consistently Perform its intended or required function or mission, on demand and without degradation or failure. It can also defined as the probability of failure-free performance over an item's life, or a specified timeframe, under specified environmental and duty-cycle conditions; often expressed as mean time between failures (MTBF) or reliability coefficient [18][19], also called quality over time [16]
II. FAILURE RATE PREDICTION

Prediction model
The failure rate of the system is calculated by summing up the failure rates of each component, each category (based on probability theory). This applies under the assumption that a failure of any component is assumed to lead to a system failure. [4][9][17].

Failure rate prediction at reference conditions (part count)
The failure rate for equipment under reference conditions is calculated as follows:
\[
\lambda_{s,i} = \sum_{i=1}^{n} (\lambda_{\text{ref}})_i
\]
Where
\(\lambda_{s,i}\) is the failure rate of equipment made up of several components
\(\lambda_{\text{ref}}\) is the failure rate under reference conditions
\(n\) is the number of components

The reference conditions adopted are typical for the majority of applications of components in equipment. Reference conditions include statements about [5]
- Operating phase
- Failure criterion
- Operation mode (e.g. continuous, intermittent)
- Climatic and mechanical stresses
- Electrical stresses.

It is assumed that the failure rate used under reference conditions is specific to the component [7][8]

Failure rate prediction at operating conditions (part stress)
Components in equipment may not always operate under the reference conditions. In such cases, the real operational conditions will result in failure rates different from those given for reference conditions.

Therefore, models for stress factors, by which failure rates under reference conditions can be converted to values applying for operating conditions (actual ambient temperature and actual electrical stress on the components) are used.

\[
\lambda_{s,i} = \sum_{i=1}^{n} (\lambda_{\text{ref}} \pi_u \pi_I \pi_T)_i
\]
Where
\(\lambda_{\text{ref}}\) is the failure rate under reference conditions
\(\pi_u\) is the voltage dependence factor
\(\pi_I\) is the current dependence factor
\(\pi_T\) is the temperature dependence factor
\(n\) is the number of components [6][8][12].

The factors should be used for converting reference failure rates to field failure rates. The stress models are empirical and allow fittings of observed data [17].

The failure rate prediction process
The failure rate prediction process consists of the following steps.
- Define the equipment to be analysed
- Understand system by analyzing equipment structure
- Determine operational conditions: operating temperature, related stress.
- Determine the actual electrical stresses for each component.
- Select the reference failure rate for each component from the database.
- In the case of a failure rate prediction at operating conditions calculate the failure rate under operating conditions for each component using the relevant stress level.
- Sum up the component failure rates
- Document the results and the assumptions.

The following data is needed
- Description of equipment including structural information
- All components categories and the number of components in each category.
- Failure rate at reference conditions for all components

Relevant stress factor for the components.[6][9]

**Types of power problems and effect on critical load (UPS)**

There are many types of power disturbances that cause electronic equipment like UPS to malfunction. These disturbances include [14] sag, noise, surge, brown-out etc. fig 2.1 below shows how the above mentioned disturbances are manifested [11].

![Disturbance on the ac line](image)

**Fig.2.1: disturbance on the ac line**

**III. ASSUMPTIONS**

As stated earlier, failure rate predictions and hence, the reliability assessment of a system demand that certain assumptions must be made. In this research, the following, assumption were made in the mathematical modeling for reliability assessment of the UPS under consideration

1. The components are assumed to be working at maximum rated power (maximum watts). It is generally assume therefore that weighting factor due to rating $W_R$ for capacitor, transistor diode and resistor are 6.0, 2.0 and 1.0 respectively [9][15][19][21]

2. The Operational environment is at normal room temperature of 28°C and there is no mechanical motion/vibration or shock experienced by the UPS.

3. No specific value is usually allotted to the derating factor of items such as crystal connector and heat sink, in the literature [19]. They are generally assumed to be unity in the computation of the overall failure rate. Thus is to be adopted in this thesis.

4. Electronic components may be connected either in series or parallel. While reliability prediction techniques such as similar complexity and prediction by function techniques take the method of connections into consideration, the parts stress method does not [9]. In this research, reliability is evaluated strictly and only on the basis of operational stress of the component parts of the system.
### 3.3 Reliability Indices

Two of such parameters that are commonly used according to Susan, (2010) are the Mean time before failure (MTBF) and Mean time to failure (MTTF).

### 3.3.1 Mean Time before Failure (MTBF)

For a repairable system, mean time before failure (MTBF) is the mean or average time before successful failures of the system [10]. The MTBF can be obtained by running a system for a predetermined length of time under specified conditions. Hence for the failure rate $\lambda$, which is the number of failure per unit time, MTBF is given by [9][15][19][21]

$$MTBF = \frac{1}{\lambda} \quad (3.1)$$

### 3.3.2 Mean Time to Failure (MTTF)

The mean time to failure (MTTF) is used for component or items that are not repairable example filament lamps, resistors, capacitors and so on, which are disposed off as soon as they fail.[19][21]. MTTF can be computed after testing a numbers of items, $N$ in a specified way (example by applying certain electrical, mechanical, heat or humidity conditions) until all failed.

If the times to failure are $(t_1, t_2, t_3, \ldots, t_n)$ then the observed MTTF is given [19] as

$$MTTF = \frac{\sum_{i=1}^{n} (t_i - t_0)}{N}$$

i.e.

$$MTTF = \frac{(t_1 - t_0) + (t_2 - t_0) + \cdots + (t_n - t_0)}{N}$$

where

- $t_0$ = starting (reference time)
- $(t_1 - t_0)$ = period to 1st failure
- $(t_2 - t_0)$ = period to 2nd failure
- $(t_n - t_0)$ = period to nth failure

$N$ = total number of failure components.

Consider the case in which a fixed number $N_0$ of identical components are tested.

Let

- $N_s$ = number surviving up to time t
- $N_f$ = number failed up to time t
- $N_0 = N_s + N_f$ = total number in operation at t=0

∴ Reliability at any time t becomes

$$R(t) = \frac{N_s}{N_0} \quad (3.2)$$

The failure rate $\lambda(t)$ is normally defined by the mathematical relation

$$\lambda(t) = \lim_{\Delta t \to 0} \frac{1}{N_s} \times \frac{\Delta N_f}{\Delta t}$$

$$\lambda = \frac{1}{N_s} \times \frac{dN_f}{dt} \quad (3.3)$$

where

- $N_s$ = number of serving items after a life test
- $\Delta N_f$ = number of failure item during the time interval, $\Delta t$.

Consider the case in which a fixed number $N_0$ of identical components are tested.

Let

- $N_s$ = number surviving up to time t
- $N_f$ = number failed up to time t
- $N_0 = N_s + N_f$ = total number in operation at t=0

∴ Reliability at anytime t becomes

$$R(t) = \frac{N_s}{N_0}$$

and failure rate (for constant failure rate)

$$\hat{\lambda} = \frac{1}{N_f} \times \frac{dN_f}{dt} \quad \text{from Equation (3.3)}$$
\[ \lambda = \frac{1}{N_0 - N_s} \times \frac{dN_f}{dt} \]

or

\[ \int \lambda dt = \int \frac{1}{N_0 - N_s} dN_f \]

introducing the limits

\[ \int_0^\infty \lambda dt = \int_0^{N_f} \frac{1}{N_0 - N_s} dN_f \]

\[ \lambda t = \left[ \log_e \left( N_o - N_f \right) - \log_e \left( N_o - 0 \right) \right] \]

\[ - \lambda t = \log_e \left( \frac{N_0 - N_f}{N_0} \right) \]

Thus

\[ e^{-\lambda t} = 1 - \frac{N_f}{N_0} \] (3.4)

But from Equation (3.3)

\[ R(t) = \frac{N_s}{N_o} = \frac{N_0 - N_s}{N_0} = \left( 1 - \frac{N_s}{N_0} \right) \] (3.5)

Comparing equations (3) and (4), we have

\[ R(t) = e^{-\lambda t} \] (3.6)

The general expression for MTBF, \( m \) is [9][19][21]

\[ m = \int_0^\infty R(t) dt \] (3.7)

For the case when \( \lambda \) is constant from equation (3.6)

\[ R = e^{-\lambda t} \] so equation (3.7) becomes

\[ m = \int_0^\infty R(t) dt \]

\[ = \int \frac{1}{\lambda} e^{-\lambda t} dt \]

\[ = \frac{1}{\lambda} \left[ e^{\lambda t} - e^0 \right] \]

\[ m = \frac{1}{\lambda} \] (3.9)

If failure are due to chance and if the failure rate \( \lambda \) is constant, then

\[ \lambda = \frac{1}{MTTF} \] for non-repairable items

\[ \lambda = \frac{1}{MTBF} \] for repairable items

3.3.3 Equipment Availability

Equipment availability is the probability that an equipment will perform its required function at a stated instant of time or over a stated period of time [19]. Availability is a function of the utilization factor (\( \mu \)). The utilization factor of a unit or system is defined as the ratio of the operating time (\( t_{op} \)) to the sum of maintenance time (\( t_m \)), idle time (\( t_{id} \)) and operating time (\( t_{op} \)), which may occur between completion of maintenance and the use due to administrative reason [22]

Mathematically,

\[ A = \frac{MTBF}{MTBF + MTTF} \] (3.10)
3.4 Failure rates of Electronic components

The general expression for the failure rate of the parts stress method is given as;

\[ \lambda_i = \lambda_b \pi_E \pi_A \pi_Q \pi_N \ldots / 10^6 \text{hrs} \quad (3.11) \]

Where

- \( \lambda_i \): The failure of the \( i \)th part
- \( \lambda_b \): The basic failure rate obtained from derated data for each generic part against normalized stress and temperature factor.
- \( \pi_E \): Account for the environmental factors other than temperature
- \( \pi_Q \): Account for the quality factor
- \( \pi_N \): Account for the any additional factor that has not been taken care of above [17]

Reliability in UPS design

In each UPS design, it is possible to obtain the system reliability in Sokoto as compared to designed reliability for the target environment. Part stress method is used in this work to obtain the failure rate. Two failure rates therefore, will be estimated.

(a) Designed failure rate- as will be applicable to the system operating in the environment for which it was designed for.

(b) Relative failure rate- as will be applicable to the system operating in the Sokoto.

The conclusions arrived at can be used as guide lines for optimal design of system operating in the region of interest and hence assess the reliability of any UPS model operating in the region.

3.4.1 Program Organization

The main program are in six parts, namely the effective failure rate of UPS for designed country, effective failure rate of UPS in Sokoto State, reliability, Mean time to repair (MTTR), Availability of the UPS components and finally the redundancies of the parallel component in the UPS.

\[ \lambda_{Deff} = \sum nTK\lambda_{BD} \]

\[ \lambda_{Seff} = \sum nTK\lambda_{BS} \]

\[ R = \exp(-\lambda_{eff}t) \]

\[ MTTR = \frac{1}{\lambda_{eff}t} \]

\[ A = \frac{MTBF}{MTBF + MTTR} \]

\[ R_{PD} = 1-(1-R_D) \] and \[ R_{PS} = 1-(1-R_S) \]

Start

Read Data

\[ \lambda_{Deff} = \sum nTK\lambda_{BD} \]

\[ \lambda_{Seff} = \sum nTK\lambda_{BS} \]

R = \exp(-\lambda_{eff}t)

MTTR = \frac{1}{\lambda_{eff}t}

A = \frac{MTBF}{MTBF + MTTR}

R_{PD} = 1-(1-R_D) and \[ R_{PS} = 1-(1-R_S) \]

Stop

Fig3.1 flow chart
### IV. RESULTS AND DISCUSSIONS

4.1 Failure rate results

Tables 4.1 and 4.2 show failure rate results obtained for each component in the UPS under consideration.

<table>
<thead>
<tr>
<th>Circuit Components</th>
<th>Description of components</th>
<th>No. of component (n)</th>
<th>Room temp in USA (TD)</th>
<th>Room temp in Sokoto (T(s))</th>
<th>Base failure rate for components in USA (λBD x10^-6)</th>
<th>Base failure rate for components in Sokoto (λBSx10^-6)</th>
<th>Volta ge stress ratio (K)</th>
<th>Effective failure rate in USA (λBDeff)</th>
<th>Effective failure rate in Sokoto (λBSeff)</th>
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<tr>
<td>F3</td>
<td>AC FUSE (1A,239V)</td>
<td>1</td>
<td>21</td>
<td>28</td>
<td>0.02</td>
<td>0.02</td>
<td>0.94</td>
<td>0.3948</td>
<td>0.5264</td>
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<tr>
<td>F2</td>
<td>RESISTOR 1A,125V</td>
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<td>28</td>
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<td>0.02</td>
<td>0.96</td>
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<td>28</td>
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<td>28</td>
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<td>0.02</td>
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<td>IC1</td>
<td>LM78DN (REGULATORS)</td>
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<td>1</td>
<td>21</td>
<td>28</td>
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<td>0.011</td>
<td>0.9435</td>
<td>0.376457</td>
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<td>0.078</td>
<td>0.41</td>
<td>0.073185</td>
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### Table 4.1 Failure for the BG650MI (series components)

<table>
<thead>
<tr>
<th>Circuit Components</th>
<th>Description of components</th>
<th>No. of component(s)</th>
<th>Room temp in USA (T_r)</th>
<th>Room temp in Sokoto (T_c)</th>
<th>Base failure rate for components in USA ($\lambda_{abs} X^{10^{-6}}$)</th>
<th>Base failure rate for component in Sokoto ($\lambda_{abs} X^{10^{-6}}$)</th>
<th>Voltage stress ratio (K)</th>
<th>Affective failure rate in USA ($\lambda_{affect}$)</th>
<th>Affective failure rate in Sokoto ($\lambda_{affect}$)</th>
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<tbody>
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<td>C3</td>
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<td>1</td>
<td>21</td>
<td>28</td>
<td>0.0047</td>
<td>0.025</td>
<td>0.5148</td>
<td>0.05081076</td>
<td>0.36036</td>
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<td>0.0047</td>
<td>0.025</td>
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<td>0.07514031</td>
<td>0.53291</td>
</tr>
<tr>
<td>C23</td>
<td>CERAMIC CAPACITOR</td>
<td>1</td>
<td>21</td>
<td>28</td>
<td>0.011</td>
<td>0.047</td>
<td>0.0002</td>
<td>0.00000462</td>
<td>0.0002632</td>
</tr>
<tr>
<td>C49</td>
<td>CERAMIC CAPACITOR</td>
<td>1</td>
<td>21</td>
<td>28</td>
<td>0.011</td>
<td>0.047</td>
<td>0.043</td>
<td>0.009933</td>
<td>0.0565588</td>
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<tr>
<td>C19</td>
<td>CERAMIC CAPACITOR</td>
<td>1</td>
<td>21</td>
<td>28</td>
<td>0.011</td>
<td>0.047</td>
<td>0.0466</td>
<td>0.0107646</td>
<td>0.0613256</td>
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<td>C4</td>
<td>FILM CAPACITOR</td>
<td>1</td>
<td>21</td>
<td>28</td>
<td>0.0008</td>
<td>0.011</td>
<td>0.1145</td>
<td>0.0019236</td>
<td>0.035266</td>
</tr>
<tr>
<td>C9</td>
<td>FILM CAPACITOR</td>
<td>1</td>
<td>21</td>
<td>28</td>
<td>0.0008</td>
<td>0.011</td>
<td>0.011</td>
<td>0.0001848</td>
<td>0.003388</td>
</tr>
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<td>C10</td>
<td>FILM CAPACITOR</td>
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<td>21</td>
<td>28</td>
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<td>0.011</td>
<td>0.034</td>
<td>0.0005712</td>
<td>0.010472</td>
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<td>21</td>
<td>28</td>
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<td>0.011</td>
<td>0.0348</td>
<td>0.00058468</td>
<td>0.0107184</td>
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<td>C6</td>
<td>FILM CAPACITOR</td>
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<td>21</td>
<td>28</td>
<td>0.0008</td>
<td>0.011</td>
<td>0.528</td>
<td>0.0088704</td>
<td>0.0162624</td>
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<tr>
<td>C17</td>
<td>FILM CAPACITOR</td>
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<td>21</td>
<td>28</td>
<td>0.0008</td>
<td>0.011</td>
<td>0.02262</td>
<td>0.000380016</td>
<td>0.006967</td>
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<td>C18</td>
<td>FILM CAPACITOR</td>
<td>1</td>
<td>21</td>
<td>28</td>
<td>0.0008</td>
<td>0.011</td>
<td>0.0267</td>
<td>0.0004856</td>
<td>0.008224</td>
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<tr>
<td>C22</td>
<td>FILM CAPACITOR</td>
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<td>28</td>
<td>0.0008</td>
<td>0.011</td>
<td>0.0002</td>
<td>0.00000336</td>
<td>0.0000616</td>
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<tr>
<td>C45</td>
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<td>1</td>
<td>21</td>
<td>28</td>
<td>0.0008</td>
<td>0.011</td>
<td>0.617</td>
<td>0.00103656</td>
<td>0.019004</td>
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<tr>
<td>C46</td>
<td>FILM CAPACITOR</td>
<td>1</td>
<td>21</td>
<td>28</td>
<td>0.0008</td>
<td>0.011</td>
<td>0.617</td>
<td>0.00106356</td>
<td>0.019004</td>
</tr>
<tr>
<td>C7</td>
<td>FILM CAPACITOR</td>
<td>1</td>
<td>21</td>
<td>28</td>
<td>0.0008</td>
<td>0.011</td>
<td>0.024</td>
<td>0.0004032</td>
<td>0.007392</td>
</tr>
<tr>
<td>J15,14,11,4,2</td>
<td>TOGGLE</td>
<td>6</td>
<td>21</td>
<td>28</td>
<td>0.035</td>
<td>0.61</td>
<td>1</td>
<td>4.41</td>
<td>102.48</td>
</tr>
</tbody>
</table>
The results in Tables 4.1 and 4.2 are obtained using equation (3.11)
4.2 Reliability assessment for BG650MI APC UPS

The effective failure rate of the BG650MI UPS in USA for which it is designed is

\[ \lambda_{BDeff} \text{(series)} + \lambda_{BDeff} \text{(parallel)} \]

\[ \lambda_{BDeff} = 1.9661 \times 10^{-4} \]

Hence reliability \( R = e^{-\lambda_{BDeff}} \)

Reliability of the BG650MI UPS for first year in USA is

For one year \( \Rightarrow 365 \text{days} \times 24 \text{hours} = 8760 \text{hrs} \)

\[ R = e^{-1.9661 \times 0.0001 \times 8760} = 0.1787 \approx 17.9\% \]

Similarly, effective failure rate of the BG650MI UPS in Sokoto, Nigeria, environment is

\[ \lambda_{BSeff} \text{(series)} + \lambda_{BSeff} \text{(parallel)} \]

\[ \lambda_{BSeff} = 9.8057 \times 10^{-4} \]

Reliability of the BG650MI for the first year in Sokoto is:

Hence reliability \( R = e^{-\lambda_{BSeff}} \)

Reliability of the BG650MI for the first year in Sokoto is;

For one year \( \Rightarrow 365 \text{days} \times 24 \text{hours} = 8760 \text{hrs} \)

\[ R = e^{-9.8057 \times 10^{-4} \times 8760} = 0.00185991 \approx 0.19\% \]

**TABLE 4.3 Reliability result for five years**

<table>
<thead>
<tr>
<th>YEAR 1</th>
<th>YEAR 2</th>
<th>YEAR 3</th>
<th>YEAR 4</th>
<th>YEAR 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>DESIGN</td>
<td>1.786548E-01</td>
<td>3.191753E-02</td>
<td>5.702218E-03</td>
<td>1.018728E-03</td>
</tr>
<tr>
<td>SOKOTO</td>
<td>1.85991E-04</td>
<td>3.4E-08</td>
<td>6.433966E-12</td>
<td>1.196710E-15</td>
</tr>
</tbody>
</table>

Fig.4.1: Reliability against time graph

From fig. 4.1: we can appreciate and compare the reliability of the UPS in country which the system was designed for USA (series1) and that of Sokoto State (series 2) and see that the UPS under consideration is relatively higher in reliability in the country which the system was designed for than in the applied State. The reliability exponential decaying function graph shows that the UPS has a higher failure rate in Sokoto State, due to factors which are associated with them like voltage fluctuations, very high dust, surge frequency, high relative humidity, high temperature etc.

The ratio of the failure rate of Sokoto State to designed country is \( 9.8057 \times 10^{-4} : 1.96613 \times 10^{-4} \approx 5:1 \).

Comparatively, the rate of failure of the UPS in the country which the system was designed for is very much less than Sokoto case.

4.3 Reliability Indices Results

**MTTF Results**

From the failure rate of the system obtained for the designed and applied country, we obtain the Mean Time To Failure (MTTF) of the system as follows.

\[ \text{MTTF} = \frac{1}{\lambda_{BDeff}} \quad (4.1) \]
MTTF\(_D=\frac{1}{\lambda_{BDeff}} \tag{4.2}\)

The mean time the system is expected to function before failure (MTTF) in Sokoto state is 0.12yrs as against for the designed country which is approximately 0.58 yrs. The rate is about five times higher than the Sokoto case. It is possible to compare the result obtained for the failure rates obtained in computer model and the MTTF\(_S\) of table 4.9 of the data collection from utility maintenance centre UDUSOK in Sokoto on BG650MI.

The MTTF and the failure rate can be obtained as follows

\[
\text{MTTF Data} = \frac{\sum_{i=1}^{N} \text{MTTF}}{N} = \frac{86}{60} = 1.43333 \pm 1.4\text{yrs}
\]

Failure rate Data

\[
\lambda_{Data} = \frac{1}{14\times365\times24} = 81.53\times10^{-6} \text{hrs which can be compared modeled failure rate of } 98.8057\times10^{-6}
\]

The table 4.4 below shows the MTTR of the country for which the system was designed (USA) for over a period of five (5) years. The same information is depicted in the fig4.2. It is observed that MTTR decreases as the number of years increases. This is to be expected, because there are adequate maintenance facilities and personnel.

<table>
<thead>
<tr>
<th>YEAR 1</th>
<th>YEAR 2</th>
<th>YEAR 3</th>
<th>YEAR 4</th>
<th>YEAR 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>DESIGN</td>
<td>0.580618</td>
<td>0.290309</td>
<td>0.193539</td>
<td>0.145154</td>
</tr>
</tbody>
</table>

Table 4.4: mttr for the designed country

![Figure 4.2: Mean Time to Repair against time graph](image)

The MTTR for Sokoto cannot be obtained because of poor maintenance facilities, personnel and record keeping scheme common Nigeria.

**Availability (A)**

From equation (3.10)

\[
A= \frac{\text{MTBF}}{\text{MTBF + MTTR}}
\]

The above expression for the MTTR only hold for the designed country, where maintenance is regular and spare parts are readily available to put the system in order. Then the availability expression above will be true. This is however, not true for the Sokoto whose level of maintenance culture is low and the level of the maintenance personnel is very poor due to low technological know-how.

Table 4.3 and fig 4.3 gives the calculated values and graph respectively of availability of the design country for the period under consideration (5 years). The result shows that the availability of the equipment increases with the number of year. This could have accounted for the fact that maintenance is regular and spare parts are readily available.

<table>
<thead>
<tr>
<th>YEAR 1</th>
<th>YEAR 2</th>
<th>YEAR 3</th>
<th>YEAR 4</th>
<th>YEAR 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>DESIGN</td>
<td>0.999885896</td>
<td>0.999942925</td>
<td>0.999961949</td>
<td>0.999971461</td>
</tr>
</tbody>
</table>

Table 4.5 availability for the designed country
4.4 Fault analysis of BG650MI UPS

Sixty UPS were obtained from utility maintenance centre UDUS and considered for failure rate analyses, the fault that are associated with them hence obtained the components that are constantly prone to failure.

Table 4.6: Failure rate analysis of 60 BG650MI UPS

<table>
<thead>
<tr>
<th>S/No</th>
<th>MODEL</th>
<th>YEAR OF MANUFACTURE (YM) YR</th>
<th>YEAR OF 1ST FAILURE (YFF) YR</th>
<th>MTTF YFF–YM YR</th>
<th>DIAGNOSIS</th>
<th>REMARKS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>BG650MI</td>
<td>2005</td>
<td>2007</td>
<td>2</td>
<td>Bad o/p Transfer</td>
<td>Repaired</td>
</tr>
<tr>
<td>2</td>
<td>BG650MI</td>
<td>2007</td>
<td>2009</td>
<td>2</td>
<td>Blown MOV</td>
<td>Repaired</td>
</tr>
<tr>
<td>3</td>
<td>BG650MI</td>
<td>2005</td>
<td>2006</td>
<td>1</td>
<td>Blown FET</td>
<td>Repaired</td>
</tr>
<tr>
<td>4</td>
<td>BG650MI</td>
<td>2005</td>
<td>2006</td>
<td>1</td>
<td>Blown FET</td>
<td>Repaired</td>
</tr>
<tr>
<td>5</td>
<td>BG650MI</td>
<td>2005</td>
<td>2007</td>
<td>2</td>
<td>Blown MOV</td>
<td>Repaired</td>
</tr>
<tr>
<td>6</td>
<td>BG650MI</td>
<td>2005</td>
<td>2006</td>
<td>1</td>
<td>Blown MOV</td>
<td>Repaired</td>
</tr>
<tr>
<td>7</td>
<td>BG650MI</td>
<td>2005</td>
<td>2006</td>
<td>1</td>
<td>Blown MOV</td>
<td>Repaired</td>
</tr>
<tr>
<td>8</td>
<td>BG650MI</td>
<td>2005</td>
<td>2006</td>
<td>1</td>
<td>Blown MOV</td>
<td>Repaired</td>
</tr>
<tr>
<td>9</td>
<td>BG650MI</td>
<td>2007</td>
<td>2008</td>
<td>2</td>
<td>Blown MOV</td>
<td>Repaired</td>
</tr>
<tr>
<td>10</td>
<td>BG650MI</td>
<td>2005</td>
<td>2006</td>
<td>1</td>
<td>Blown MOV</td>
<td>Repaired</td>
</tr>
<tr>
<td>11</td>
<td>BG650MI</td>
<td>2005</td>
<td>2006</td>
<td>1</td>
<td>Blown FET</td>
<td>Repaired</td>
</tr>
<tr>
<td>12</td>
<td>BG650MI</td>
<td>2007</td>
<td>2009</td>
<td>2</td>
<td>Blown MOV</td>
<td>Repaired</td>
</tr>
<tr>
<td>13</td>
<td>BG650MI</td>
<td>2007</td>
<td>2008</td>
<td>1</td>
<td>Blown FET</td>
<td>Repaired</td>
</tr>
<tr>
<td>14</td>
<td>BG650MI</td>
<td>2005</td>
<td>2006</td>
<td>1</td>
<td>Blown FET</td>
<td>Repaired</td>
</tr>
<tr>
<td>15</td>
<td>BG650MI</td>
<td>2007</td>
<td>2009</td>
<td>2</td>
<td>Blown FET</td>
<td>Repaired</td>
</tr>
<tr>
<td>16</td>
<td>BG650MI</td>
<td>2005</td>
<td>2006</td>
<td>1</td>
<td>Blown FET</td>
<td>Repaired</td>
</tr>
<tr>
<td>17</td>
<td>BG650MI</td>
<td>2006</td>
<td>2009</td>
<td>3</td>
<td>Bad o/p Transfer</td>
<td>Repaired</td>
</tr>
<tr>
<td>18</td>
<td>BG650MI</td>
<td>2005</td>
<td>2008</td>
<td>3</td>
<td>Bad battery</td>
<td>Repaired</td>
</tr>
<tr>
<td>19</td>
<td>BG650MI</td>
<td>2005</td>
<td>2008</td>
<td>3</td>
<td>Not charging</td>
<td>Repaired</td>
</tr>
<tr>
<td>20</td>
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<td>2005</td>
<td>2007</td>
<td>2</td>
<td>Bad battery</td>
<td>Repaired</td>
</tr>
<tr>
<td>21</td>
<td>BG650MI</td>
<td>2005</td>
<td>2006</td>
<td>1</td>
<td>Blown board</td>
<td>Repaired</td>
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<tr>
<td>22</td>
<td>BG650MI</td>
<td>2006</td>
<td>2007</td>
<td>1</td>
<td>Blown FET</td>
<td>Repaired</td>
</tr>
<tr>
<td>23</td>
<td>BG650MI</td>
<td>2007</td>
<td>2009</td>
<td>2</td>
<td>Not charging</td>
<td>Repaired</td>
</tr>
<tr>
<td>24</td>
<td>BG650MI</td>
<td>2008</td>
<td>2010</td>
<td>2</td>
<td>Blown FET</td>
<td>Repaired</td>
</tr>
<tr>
<td>25</td>
<td>BG650MI</td>
<td>2009</td>
<td>2010</td>
<td>1</td>
<td>Blown MOV</td>
<td>Repaired</td>
</tr>
<tr>
<td>26</td>
<td>BG650MI</td>
<td>2005</td>
<td>2006</td>
<td>1</td>
<td>Blown MOV</td>
<td>Repaired</td>
</tr>
<tr>
<td>27</td>
<td>BG650MI</td>
<td>2005</td>
<td>2006</td>
<td>1</td>
<td>Blown MOV</td>
<td>Repaired</td>
</tr>
</tbody>
</table>

Figure 4.3 Availability against time
Mathematical Modeling For Reliability Assessment Of Uninterruptible...

Table 4.7: fault analysis for 60 BG650MI ups

<table>
<thead>
<tr>
<th>S/NO</th>
<th>PARTS</th>
<th>NO. OF OCCURRENCE</th>
<th>% OF OCCURRENCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Blown Metal Oxide varistor (MOV)</td>
<td>23</td>
<td>38.33</td>
</tr>
<tr>
<td>2</td>
<td>Blown FET (Field Effects Transistor)</td>
<td>18</td>
<td>30.00</td>
</tr>
<tr>
<td>3</td>
<td>Blown Board</td>
<td>4</td>
<td>6.67</td>
</tr>
<tr>
<td>4</td>
<td>Bad Output Transformer</td>
<td>3</td>
<td>5.00</td>
</tr>
<tr>
<td>5</td>
<td>Bad Battery</td>
<td>6</td>
<td>10.00</td>
</tr>
<tr>
<td>6</td>
<td>Charging Problem</td>
<td>6</td>
<td>10.00</td>
</tr>
<tr>
<td>TOTAL</td>
<td></td>
<td>60</td>
<td>100%</td>
</tr>
</tbody>
</table>

MOV faults forms 38.33% of the total faults, blown FET forms 30% of the total faults, blown Board forms 6.67% of the total faults, bad Output Transformer forms 5% of the total faults, bad Battery forms 10% of the total faults, Charging Problem forms 10% of the total faults.

The result shows that Blown MOV and Blown FET accounts for the major faults that are associated with UPS in Sokoto. Anti surge suppressor and stabilizer will serve as a measure against these problems [2].

V. RECOMMENDATIONS

The following recommendations are derived from the research, to have high degree of reliability in Sokoto State:

www.ijesi.org
a. Use of air conditioner as means of improving the environmental condition that are obtainable in the USA. This will provide a means of removing excess heat away from the system and appreciably reducing the dust deposited on it.

b. Use of stabilizers and voltage surge suppressor to reduce the operating stresses on the system.

c. Exposing maintenance staff to the importance of keeping data that will help to determine the reliability of design, MTTR and availability of the system.

d. Training in Reliability Physics and Reliability Engineering should be made part of science/engineering curriculum study so as to increase the number of expert in maintenance engineering in Sokoto State.

e. Availability of component- Components that have been discovered to be constantly failing should be made available to the maintenance engineers, so that the availability of the system can be increased.

f. From the result of this research paper, it is possible for the designer of UPS system to make a type that can work durably in Sokoto State and Nigeria at large.

5.1 Suggestion for further work

Given the rapid rate of power failure in Sokoto State, UPS will constantly be needed to provide a short-time back-up power supply. The following are recommended for further work on this paper.

1. Collection and keeping of relevant maintenance data which can be used for further reliability analysis of UPS in the State.

2. The design and construction of UPS system that will meet the demand of the State and Nigeria at large by derating all UPS components to address all the factors affecting reliability in the State.

REFERENCES


