

Comparative Analysis of Soft Locally P-connected and soft P-connected spaces

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ABSTRACT: This paper introduces soft locally pre-connectedness in soft topological spaces. A soft topological space (F_E, τ) is called soft locally pre-connected (soft locally P-connected) at $(x_{f_A}) \in F_E$ if and only if for every soft P-open set F_A containing (x_{f_A}) there exist a soft P-connected open set F_B such that $(x_{f_A}) \in F_B \subseteq F_A$. A detail study is carried out on the comparative analysis of soft locally pre-connected space with soft locally connected, soft pre -connected and soft connected spaces with examples.
AMS subject classification (2010): 54A10, 54A05 and 06D72.

KEYWORDS: soft Pre –closed set, soft pre-closure, soft pre-connected, soft pre-component, soft pre-locally connected.

I. INTRODUCTION.

Most of our real life problems in engineering, social and medical science, economies, environment etc. involve imprecise data and their solutions involve the use of mathematical principles based on uncertainty and imprecise. To handle such uncertainties, a number of theories have been proposed. Some of these are probability, fuzzy sets, intuitionistic fuzzy sets, interval mathematics and rough sets etc. Recently, Soft set plays an important role in this field. The origin of soft set theory could be traced to the work of Pawlak in 1993 titled Hard and Soft Set in proceeding of the International EWorkshop on rough sets and knowledge discovery at Banff. This motivated D.Molodstov's work in 1999 titled soft set theory[1]. Recently, in 2011 Muhammad Shabir and Munazza Naz[4] introduced the notation of soft topological spaces which are defined over an initial universe with a fixed set of parameters. In another corner, connectivity occupies very important place in topology. Many authors have presented different kinds of connectivity [2,7,8,9,10]. In this paper we wish to compare and analysis the related properties of softlocally pre-connected space and soft pre –connected space with examples.

II. PRELIMINARIES

For basic notations and definitions not given here, the reader can refer [1-9].

2.1. Definition. [3] A soft set F_A on the universe U is defined by the set of ordered pairs $F_A = \{(x, f_A(x)) : x \in E, f_A(x) \in P(U)\}$, where E is a set of parameters, $A \subseteq E$, $P(U)$ is the power set of U ,

and $f_A : A \rightarrow P(U)$ such that $f_A(x) = \emptyset$ if $x \notin A$. Here, f_A is called an approximate function of the soft set F_A .

The value of $f_A(x)$ may be arbitrary, some of them may be empty, some may have non-empty intersection.

Note that the set of all soft set over U is denoted by $S(U)$.

2.2. Example. Suppose that there are six houses in the universe. $U = \{h_1, h_2, h_3, h_4, h_5\}$ under consideration, and that $E = \{x_1, x_2, x_3, x_4, x_5\}$ is a set of decision parameters. The x_i ($i = 1, 2, 3, 4, 5$) stand for the parameters

“expensive”, “beautiful”, “wooden”, “cheap” and “in green surroundings” respectively. Suppose that $A = \{x_1, x_3, x_4\} \subseteq E$ and $f_A(x_1) = \{h_2, h_4\}$, $f_A(x_3) = U$, $f_A(x_4) = \{h_1, h_3, h_5\}$. Then, we can view the soft set F_A as

consisting of the following collection of

approximations: $F_A = \{(x_1, \{h_2, h_4\}), (x_2, \{\emptyset\}), (x_3, \{U\}), (x_4, \{h_1, h_3, h_5\}), (x_5, \{\emptyset\})\}$. Otherwise we write the soft set as follows $F_A = \{(x_1, \{h_2, h_4\}), (x_2, U), (x_4, \{h_1, h_3, h_5\})\}$.

2.3. Definition. [11] The soft set $F_A \in S(U)$ is called a soft point in F_E , denoted by (x_{f_A}) , if for the element $x \in A$ and $f_A(x) \neq \emptyset$ and $f_A(x') = \emptyset$ for all $x' \in A - \{x\}$.

The soft point (x_{f_A}) is said to be in the soft set F_B , denoted by $(x_{f_A}) \in F_B$ if for the element $x \in A$ and $f_A(x) \subseteq f_B(x)$.

2.4. Definition. [5] Let $F_A \in S(U)$. The soft power set of F_A is defined by $\tilde{P}(F_A) = \{F_{A_i} : F_{A_i} \subseteq F_A, i \in I \subseteq N\}$ and its cardinality is defined by $|\tilde{P}(F_A)| = 2^{\sum_{x \in E} |f_A(x)|}$, where $|f_A(x)|$ is the cardinality of $f_A(x)$.

2.5. Example. [5] Let $U = \{u_1, u_2, u_3\}$, $E = \{e_1, e_2, e_3\}$, $A = \{e_2, e_3\}$ and $F_A = \{(e_2, \{u_2, u_3\}), (e_3, \{u_1, u_2\})\}$. Then

$$F_{A_1} = F_A, F_{A_2} = F_\emptyset, F_{A_3} = \{(e_2, \{u_2, u_3\})\},$$

$$F_{A_4} = \{(e_2, \{u_2\})\}, F_{A_5} = \{(e_2, \{u_3\})\}, F_{A_6} = \{(e_2, \{u_2\}), (e_3, \{u_1, u_2\})\}, F_{A_7} = \{(e_2, \{u_3\}), (e_3, \{u_1, u_2\})\},$$

$$F_{A_8} = \{(e_2, \{u_3\}), (e_3, \{u_2\})\}, F_{A_9} = \{(e_2, \{u_2\}), (e_3, \{u_1\})\}, F_{A_{10}} = \{(e_3, \{u_1, u_2\})\}, F_{A_{11}} = \{(e_3, \{u_1\})\},$$

$$F_{A_{12}} = \{(e_3, \{u_2\})\}, F_{A_{13}} = \{(e_2, \{u_2, u_3\}), (e_3, \{u_1\})\}, F_{A_{14}} = \{(e_2, \{u_2, u_3\}), (e_3, \{u_2\})\},$$

$$F_{A_{15}} = \{(e_2, \{u_3\}), (e_3, \{u_1\})\}, F_{A_{16}} = \{(e_2, \{u_2\}), (e_3, \{u_2\})\}$$
 are all soft subset of F_A . So $|\tilde{P}(F_A)| = 2^4 = 16$.

2.6. Definition. [5] Let $F_E \in S(U)$. A soft topology on F_E denoted by $\tilde{\tau}$ is a collection of soft subsets of F_E

having the following properties:

- (i). $F_\emptyset, F_E \in \tilde{\tau}$
- (ii). $\{F_{E_i} \subseteq F_E : i \in I \subseteq N\} \subseteq \tilde{\tau} \Rightarrow \bigcup_{i \in I} F_{E_i} \in \tilde{\tau}$
- (iii). $\{F_{E_i} \subseteq F_E : 1 \leq i \leq n, n \in N\} \subseteq \tilde{\tau} \Rightarrow \bigcap_{i=1}^n F_{E_i} \in \tilde{\tau}$.

The pair $(F_E, \tilde{\tau})$ is called a soft topological space.

2.7. Example. Let us consider the soft subsets of F_A that are given in Example 2.5. Then $\tilde{\tau}_1 = \{F_A, F_\emptyset, F_{A_6}, F_{A_9}, F_{A_{11}}\}$, $\tilde{\tau}_2 = \{F_\emptyset, F_A, F_{A_5}, F_{A_8}, F_{A_4}\}$, $\tilde{\tau}_3 = \{\tilde{P}(F_A)\}$ are soft topologies on F_A .

2.8. Definition. Let $(F_E, \tilde{\tau})$ be a soft topological space. Then, every element of $\tilde{\tau}$ is called a soft open set.

Clearly F_\emptyset and F_E are soft open sets. The collection of all soft open set is denoted by $G_s(F_E)$. Let $F_C \subseteq F_E$. Then F_C is said to be soft closed if the soft set F_C^c is soft open in F_E . The collection of all soft closed set is denoted by $F_s(F_E)$.

2.9. Definition. Let $(F_E, \tilde{\tau})$ be a soft topological space over U. A soft separation of F_E is a pair F_A, F_B of non-null soft open sets over F_E such that $F_E = F_A \cup F_B, F_A \tilde{\cap} F_B = F_\emptyset$. A soft topological space $(F_E, \tilde{\tau})$ is said to be soft connected if there does not exist a soft separation of F_E .

2.10. Definition. A soft topological space $(F_E, \tilde{\tau})$ is called soft locally connected at $(x_{f_A}) \in F_E$ if and only if for every soft open set F_A containing (x_{f_A}) there exist a soft connected open set F_B such that $(x_{f_A}) \in F_B \subseteq F_A$. That means $(F_E, \tilde{\tau})$ is called soft locally connected if and only if it is soft locally connected at every point of F_E .

III. SOFT PRE –CLOSED SET AND SOFT PRE-CONNECTED SPACE

In this section we introduce the concept of soft pre-closed and soft pre-closure and its related properties in a soft topological space $(F_E, \tilde{\tau})$.

3.1. Definition. Let $(F_E, \tilde{\tau})$ be a soft topological space. A soft set F_A is said to be soft pre-closed set (soft P-closed) if there exist an soft closed set F_C such that $(F_A)^\circ \subseteq F_C \subseteq F_A$. The set of all soft P –closed set of F_E is denoted by $F_{sp}(F_E, \tilde{\tau})$ or $F_{sp}(F_E)$.

3.2. Example. Let us consider the soft set F_A and its soft subset as in Example 2.5 , where $U=\{u_1,u_2,u_3\}$, $E=\{e_1,e_2,e_3\}$, $A=\{e_2,e_3\}$ and $F_A = \{ (e_2, \{u_2, u_3\}), (e_3, \{u_1, u_2\}) \}$. Define $\tilde{\tau} = \{F_A, F_\emptyset, F_{A_5}, F_{A_8}, F_{A_{14}}\}$. Then $(F_A, \tilde{\tau})$ is a soft topological space. Since $\tilde{\tau}^\ell = \{F_\emptyset, F_A, F_{A_6}, F_{A_9}, F_{A_{11}}\}$. Here $F_{A_{16}}$ is a soft P-closed set and F_\emptyset is a soft closed set such that $F_\emptyset = (F_{A_{16}})^\circ \subseteq F_\emptyset \subseteq F_{A_{16}}$.

3.3. Remark. In a soft topological space $(F_E, \tilde{\tau})$ F_\emptyset and F_E are always soft P-closed sets.

3.4. Remark. Complement of soft P-closed set is soft P-open set. It means that, a soft set F_A is said to be soft pre-open set (soft P-open) if there exists a soft open set F_O such that $F_A \subseteq F_O \subseteq \overline{F_A}$. The set of all soft P-open set of F_E is denoted by $G_{sp}(F_E, \tilde{\tau})$ or $G_{sp}(F_E)$.

3.5. Remark. A soft set F_A is said to be soft P-clopen if it is both soft P-open and soft P-closed.

3.6. Example. Let us consider the soft topological space $(F_A, \tilde{\tau})$ that is given in Example 3.2. Then the classes of all soft P- closed sets are $F_{sp}(F_A) = \{F_\emptyset, F_A, F_{A_{10}}, F_{A_6}, F_{A_{11}}, F_{A_9}, F_{A_{12}}, F_{A_4}, F_{A_{16}}\}$.

3.7. Example. Let $U=\{u_1,u_2,u_3\}$, $E=\{e_1,e_2,e_3\}$, $A=\{e_1,e_2\}$ and $F_A = \{ (e_1, \{u_1\}), (e_2, \{u_1, u_2\}) \}$. The class of all soft sub sets over U of F_A is denoted by S (F_A). Then $F_{A_1}=\{(e_1, \{u_1\})\}$, $F_{A_2}=\{(e_1, \{u_1\}), (e_2, \{u_1\})\}$, $F_{A_3}=\{(e_1, \{u_1\}), (e_2, \{u_2\})\}$, $F_{A_4}=\{(e_2, \{u_1, u_2\})\}$, $F_{A_5}=\{(e_2, \{u_1\})\}$, $F_{A_6}=\{(e_2, \{u_2\})\}$, $F_{A_7}=F_A$, $F_{A_8}=F_\emptyset$. Define

$\tilde{\tau} = \{F_A, F_\emptyset, F_{A_1}, F_{A_2}, F_{A_3}, F_{A_4}, F_{A_5}\}$. Then $(F_A, \tilde{\tau})$ is a soft topological space. Since $\tilde{\tau}^\ell = \{F_\emptyset, F_A, F_{A_4}, F_{A_5}, F_{A_1}, F_{A_2}\}$.

Hence the class of all soft P-closed set is $F_{sp}(F_A) = \{F_\emptyset, F_A, F_{A_4}, F_{A_5}, F_{A_1}, F_{A_2}\}$.

3.8. Theorem.

- (i). Arbitrary soft intersection (union) of soft P-closed (P-open) set is soft P-closed (open) set.
- (ii) The soft union (intersection) of two soft P-closed (P-open) set need not be a soft P-closed (P-open) set.

3.9. Proposition.

Every soft closed (open) set is soft P-closed (P-open) set.

3.10. Definition. Let $(F_E, \tilde{\tau})$ be a soft topological space and $F_A \subseteq F_E$. Then the soft pre closure (soft P-closure)

of F_A denoted by $p(\overline{F_A})$ is defined as the soft intersection of all soft P-closed supersets of F_A .

3.11. Remark. Since arbitrary soft intersection of soft P-closed sets is a soft P-closed set, then $p(\overline{F_A})$ is soft P-closed. Note that, $p(\overline{F_A})$ is the smallest soft P-closed set that containing F_A .

3.12. Definition. Let $(F_E, \tilde{\tau})$ be a soft topological space and $F_A \subseteq F_E$. Then the soft pre- interior (soft P-interior) of F_A denoted by $p(F_A)^\circ$ is defined as the soft union of all soft P-open subsets of F_A .

3.13. Theorem. Let $(F_E, \tilde{\tau})$ be a soft topological space and let F_A and F_B be a soft sets over U . Then

- (a) $F_A \subseteq p(\overline{F_A})$
- (b) F_A is soft P-closed iff $F_A = p(\overline{F_A})$
- (c) $F_A \subseteq F_B$, then $p(\overline{F_A}) \subseteq p(\overline{F_B})$
- (d) $p(\overline{F_\emptyset}) = F_\emptyset$ and $p(\overline{F_E}) = F_E$.
- (e) (i) $(p(\overline{F_A}))^\ell = p(F_A^\ell)^\circ$
 (ii) $(p(F_A)^\circ)^\ell = p(\overline{F_A^\ell})$
- (f) $p(\overline{F_A \cap F_B}) \subseteq p(\overline{F_A}) \cap p(\overline{F_B})$
- (g) $p(\overline{F_A \cup F_B}) = p(\overline{F_A}) \cup p(\overline{F_B})$
- (h) $p(p(\overline{F_A})) = p(\overline{F_A})$

IV. SOFT PRE-CONNECTED AND SOFT LOCALLY PRE-CONNECTED SPACE

4.1. Definition. Let $(F_E, \tilde{\tau})$ be a soft topological space. Two non-empty soft sub sets F_A and F_B of F_E are called soft pre- separated iff $p(\overline{F_A}) \cap F_B = F_A \cap p(\overline{F_B}) = F_\emptyset$. That means that a soft pre-separation of a soft topological space $(F_E, \tilde{\tau})$ is a pair F_A, F_B of soft pre -disjoint non- nullsoft pre-open sets whose union is F_E .

4.2. Theorem . Let $(F_E, \tilde{\tau})$ be a soft topological space. Then the following statements are equivalent:

- (1): F_\emptyset and F_E are the only soft pre-clopen sets in $(F_E, \tilde{\tau})$.

(2): $(F_E, \tilde{\tau})$ is not the soft union of two soft disjoint non-empty soft pre-open sets.

(3): $(F_E, \tilde{\tau})$ is not the soft union of two soft disjoint non-empty soft pre-closed sets.

(4): $(F_E, \tilde{\tau})$ is not the soft union of two non-empty soft pre-separated sets.

4.3. Definition. Let $(F_E, \tilde{\tau})$ be a soft topological space. If there doesn't exist a soft pre-separation of F_E , then it is said to be soft pre-connected (soft P-connected) otherwise it is soft pre-disconnected (soft P-disconnected).

4.4. Example. Let $(F_A, \tilde{\tau})$ be a soft topological space, where F_A and its soft subsets are considered as in Example

2.5. Let $\tilde{\tau} = \{F_A, F_\emptyset, F_{A_5}, F_{A_8}, F_{A_{14}}\}$ then $\tilde{\tau}^{\ell} = \{F_A, F_\emptyset, F_{A_6}, F_{A_9}, F_{A_{11}}\}$.

$G_{SP}(F_A) = \{F_A, F_\emptyset, F_{A_5}, F_{A_8}, F_{A_{14}}, F_{A_3}, F_{A_7}, F_{A_{13}}, F_{A_{15}}\}$. $F_{SP}(F_A) = \{F_\emptyset, F_A, F_{A_6}, F_{A_9}, F_{A_{11}}, F_{A_{10}}, F_{A_4}, F_{A_{12}}, F_{A_{16}}\}$. Here

we show that F_A is soft P-connected. We first choose $F_{A_4} = \{(e_2, \{u_2\})\}$ and $F_{A_7} = \{(e_2, \{u_3\}), (e_3, \{u_1, u_2\})\}$.

Then $p(\overline{F_{A_4}}) = \{(e_2, \{u_2\}), (e_3, \{u_1\})\}$ and so $p(\overline{F_{A_4}}) \tilde{\cap} F_{A_7} = \{(e_3, \{u_1\})\} = F_{A_{11}} \neq F_\emptyset$. We next choose

$F_{A_5} = \{(e_2, \{u_3\})\}$ and $F_{A_{10}} = \{(e_3, \{u_1, u_2\})\}$. Then $p(\overline{F_{A_5}}) = F_A$ and so

$p(\overline{F_{A_5}}) \tilde{\cap} F_{A_{10}} = \{(e_3, \{u_1, u_2\})\} = F_{A_{10}} \neq F_\emptyset$. We next choose $F_{A_3} = \{(e_2, \{u_2, u_3\})\}$ and

$F_{A_{10}} = \{(e_3, \{u_1, u_2\})\}$. Then $p(\overline{F_{A_3}}) = F_A$ and so $p(\overline{F_{A_3}}) \tilde{\cap} F_{A_{10}} = \{(e_3, \{u_1, u_2\})\} = F_{A_{10}} \neq F_\emptyset$. We next

choose $F_{A_{11}} = \{(e_3, \{u_1\})\}$ and $F_{A_{14}} = \{(e_2, \{u_2, u_3\}), (e_3, \{u_2\})\}$. Then

$p(\overline{F_{A_{11}}}) = \{(e_3, \{u_1\})\} = p(\overline{F_{A_{14}}}) = F_A$. But

$p(\overline{F_{A_{11}}}) \tilde{\cap} F_{A_{14}} = F_\emptyset$ and $F_{A_{11}} \tilde{\cap} p(\overline{F_{A_{14}}}) = \{(e_3, \{u_1\})\} = F_{A_{11}} \neq F_\emptyset$. Finally we choose $F_{A_{12}} = \{(e_3, \{u_2\})\}$

and $F_{A_{13}} = \{(e_2, \{u_2, u_3\}), (e_3, \{u_1\})\}$. Then $p(\overline{F_{A_{12}}}) = \{(e_2, \{u_2\}), (e_3, \{u_1, u_2\})\}$ and so

$p(\overline{F_{A_{12}}}) \tilde{\cap} F_{A_{13}} = \{(e_2, \{u_2\}), (e_3, \{u_1\})\} = F_{A_9} \neq F_\emptyset$. Thus we see that F_A can't be expressed as the soft union

of two soft P-separated sets and hence F_A is soft P-connected.

4.5. Proposition. The soft union F_A of any family $\{F_{A_i} : i \in I\}$ of soft P-connected sets having a non-empty soft intersection is a soft P-connected set.

4.6. Definition. Let $(F_E, \tilde{\tau})$ be a soft topological space and $(x_{f_E}) \in F_E$. The soft P-component of (x_{f_E}) is denoted by $(SP-C(x_{f_E}))$ and it is the soft union of all soft P-connected subsets of F_E containing (x_{f_E}) . The soft sets like $(SP-C(x_{f_E}))$ are called soft P-components of F_E . We can see from theorem 4.5 that the $(SP-C(x_{f_E}))$ is soft P-connected.

4.7. Theorem.

In a soft topological space $(F_E, \tilde{\tau})$

(i) each soft-pre component $(SP-C(x_{f_E}))$ is a maximal soft P-connected set in $(F_E, \tilde{\tau})$.

- (ii) The set of all distinct soft P-components of soft points of F_E form a partition of F_E and
- (iii) each SP-C (x_{f_E}) is soft P-closed in (F_E, τ) .

4.8. Definition. A soft topological space (F_E, τ) is called soft locally pre connected (soft locally P-connected) at $(x_{f_A}) \in F_E$ if and only if for every soft P-open set F_A containing (x_{f_A}) there exist a soft P-connected open set F_B such that $(x_{f_A}) \in F_B \subseteq F_A$. That means (F_E, τ) is called soft locally P-connected if and only if it is soft locally P-connected at every point of F_E .

4.9. Example. Let (F_A, τ) be a soft topological space, where F_A and its soft subsets are consider as in Example 3.7.

$$\text{Consider } \tau = \{F_A, F_\emptyset, F_{A1}, F_{A4}, F_{A3}, F_{A6}\}$$

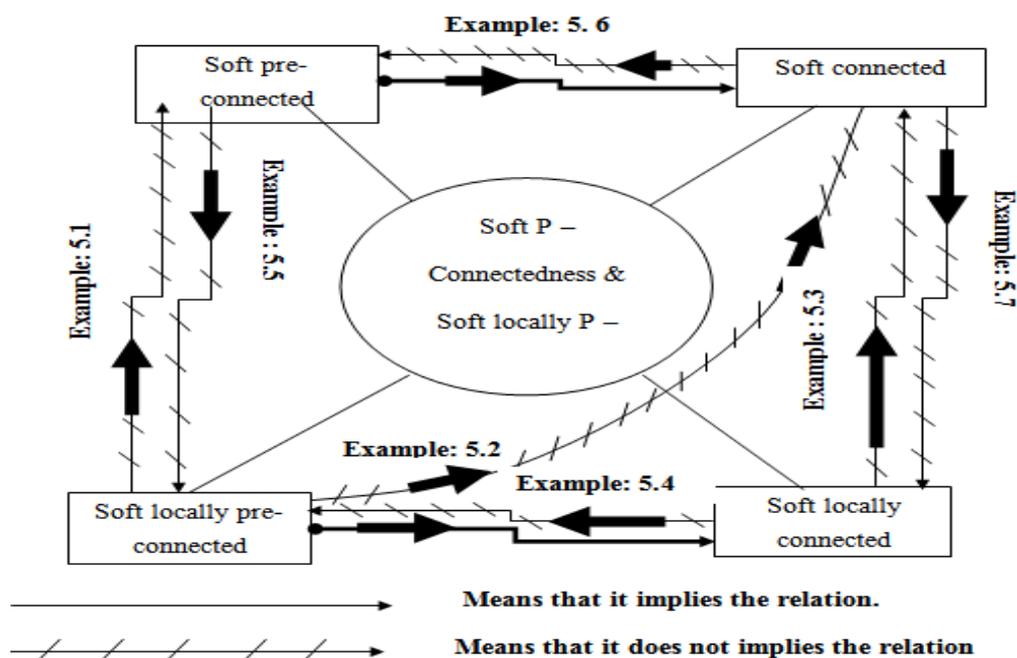
$$\text{then } \tau^c = \{F_\emptyset, F_A, F_{A4}, F_{A1}, F_{A5}, F_{A2}\}, G_{sp}(F_A) = \{F_A, F_\emptyset, F_{A1}, F_{A4}, F_{A3}, F_{A6}, F_{A2}\}.$$

$F_{sp}(F_A) = \{F_\emptyset, F_A, F_{A4}, F_{A1}, F_{A5}, F_{A2}, F_{A6}\}$. The soft P-open sets containing the soft set F_{A1} is a soft P-connected and open. Therefore F_A is softlocally P-connected at $(e_1, \{u_1\})$. The soft P-open sets containing $(e_2, \{u_1, u_2\})$ are F_A and F_{A4} . Clearly F_{A4} is soft P-connected and open. Therefore F_A is soft locally P-connected at $(e_2, \{u_1, u_2\})$. Therefore F_A is soft locally P-connected.

4.10. Theorem. A soft topological space (F_E, τ) is soft locally P-connected if and only if the soft P-Components of soft P-open sets are soft open set.

V. ANALYSIS OF SOFT LOCALLY PRE-CONNECTEDSPACE WITH SOFT PRE-CONNECTED SPACE.

In this section we find out the relations between soft locally pre- connected space, soft pre-connected space, soft locally connected space, soft connected space by the way of some examples which is given in the below diagram.



5.1. Example. Soft locally P-connectedness does not implies Soft P-connected.

Let $(F_A, \tilde{\tau})$ be a soft locally P-connected then it is not soft P- connected. Where F_A and its soft subsets are

consider as in Example 3.7. Consider $\tilde{\tau} = \{F_A, F_\emptyset, F_{A_3}, F_{A_6}, F_{A_5}, F_{A_8}\}$

Here $\tilde{\tau}^c = \{F_A, F_\emptyset, F_{A_6}, F_{A_3}, F_{A_7}, F_{A_4}\}$. $G_{sp}(F_A) = \{F_A, F_\emptyset, F_{A_3}, F_{A_6}, F_{A_5}, F_{A_8}, F_{A_4}\}$

$F_{sp}(F_A) = \{F_A, F_\emptyset, F_{A_6}, F_3, F_7, F_{A_4}, F_{A_8}\}$. The soft P-open sets containing $(e_1, \{u_1\})$ are F_A, F_{A_3}, F_{A_5} and F_{A_4} . Clearly

the soft set F_{A_3} is soft P-connected and open. Therefore F_A is soft locally P-connected at $(e_1, \{u_1\})$. Next the soft

P-open sets containing $(e_2, \{u_1, u_2\})$ are F_A and F_{A_6} . Clearly F_{A_6} is soft P-connected and open. Therefore F_A is

soft locally P-connected at $(e_2, \{u_1, u_2\})$. Therefore F_A is soft locally P-connected .Now we show that F_A is not

soft P-connected. Let $F_A = F_{A_3} \cup F_{A_6}$ where, $F_{A_3} = \{(e_1, \{u_1\})\}$ and $F_{A_6} = \{(e_2, \{u_1, u_2\})\}$.Then

$p(\overline{F_{A_3}}) = \{(e_1, \{u_1\})\} = F_{A_3}$ and $p(\overline{F_{A_6}}) = \{(e_2, \{u_1, u_2\})\} = F_{A_6}$ and so $p(\overline{F_{A_3}}) \cap F_{A_6} = F_\emptyset$ and

$F_{A_3} \cap p(\overline{F_{A_6}}) = F_\emptyset$ and so \square_{\square_3} and \square_{\square_6} are two soft P-separated sets. Hence F_A can be expressed as the soft

union of two soft P-separated sets and so F_A is not soft P-connected.

5.2. Example. Soft locally P-connectedness does not implies soft connectedness.

In Example 5.1, it is verified that $(\square_\square, \tilde{\sigma})$ is soft locally P-connected, but we show that it is not soft connected.

Let $\square_\square = \square_{\square_3} \cup \square_{\square_6}$. Then $\overline{(\square_{\square_3})} = \{(\square_j, \{\square_j\})\} = \square_{\square_3}$ and $\overline{(\square_{\square_6})} = \square_{\square_6}$ and so $\square_{\square_3} \cap \overline{(\square_{\square_6})} = \square_\emptyset$,

$\overline{(\square_{\square_3})} \cap \square_{\square_6} = \square_\emptyset$. Therefore F_A can be expressed as the soft union soft separated sets of F_A . Hence it is not

soft connected.

5.3. Example. Soft locally connectedness does not implies soft connectedness. Let $(\square_\square, \tilde{\sigma})$ be the soft

topological space, considered as in Example 5.1. Here, the soft open sets containing $(e_1, \{u_1\})$ are $\square_{\square_3}, \square_{\square_5}$ and

F_A . Clearly \square_{\square_3} is soft connected and open. Therefore F_A is locally connected at $(e_1, \{u_1\})$. The soft opensets

containing $(e_2, \{u_1, u_2\})$ are \square_{\square_6} and F_A . Clearly \square_{\square_6} is soft connected and open. Therefore F_A is soft locally

connected at $(e_2, \{u_1, u_2\})$. Therefore F_A is soft locally connected, but we show that it is not soft connected.

5.4. Example. If a soft topological space $(\square_\square, \tilde{\sigma})$ is soft locally P-connected then it is soft locally connected

but the converse is not true as shown by the following example.

Let $(\square_\square, \tilde{\sigma})$ be the soft topological space, which is given in Example 3.6. Consider

$\tilde{\sigma} = \{\square_\square, \square_\emptyset, \square_{\square_2}, \square_{\square_8}, \square_{\square_{14}}\}$. Here $\tilde{\sigma}^c = \{\square_\emptyset, \square_\square, \square_{\square_6}, \square_{\square_7}, \square_{\square_{11}}\}$,

$\square_{\square_\square}(F_A) = \{\square_\square, \square_\emptyset, \square_{\square_2}, \square_{\square_8}, \square_{\square_{14}}, \square_{\square_3}, \square_{\square_7}, \square_{\square_{13}}, \square_{\square_{15}}\}$.

$\square_{\square_\square}(F_A) = \{\square_\emptyset, \square_\square, \square_{\square_6}, \square_{\square_7}, \square_{\square_{11}}, \square_{\square_{10}}, \square_{\square_4}, \square_{\square_{12}}, \square_{\square_{16}}\}$.

It is verified that $(\square_{\square}, \tilde{\square})$ is soft locally connected. But we show that it is not soft locally P-connected. Here \square_{\square_3} is a P-open set containing $(e_2, \{u_2, u_3\})$, but there is no soft open subset of \square_{\square_3} containing $(e_2, \{u_2, u_3\})$ and so $(\square_{\square}, \tilde{\square})$ is not soft locally P-connected at $(e_2, \{u_2, u_3\})$. Therefore $(\square_{\square}, \tilde{\square})$ is not soft locally P-connected.

5.5. Example. Soft P-connectedness does not implies soft locally P-connectedness as shown by the following example .We consider the soft topological space in Example 5. 4. It is clear that F_A is soft P-connected, We first choose $\square_{\square_4} = \{(\square_2, \{\square_2\})\}$ and $\square_{\square_7} = \{(\square_2, \{\square_3\}), (\square_3, \{\square_1, \square_2\})\}$. Then

$$\square(\overline{\square_{\square_4}}) = \{(\square_2, \{\square_2\})\} = \square_{\square_4} \quad \square(\overline{\square_{\square_7}}) = \square_{\square} \quad \text{Here}$$

$$\square(\overline{\square_{\square_4}}) \tilde{\cap} \square_{\square_7} = \square_{\emptyset} \quad \square_{\square_4} \tilde{\cap} \square(\overline{\square_{\square_7}}) = \square_{\square_4} \neq \square_{\emptyset}.$$

We next choose $\square_{\square_5} = \{(\square_2, \{\square_3\})\}$ and

$$\square_{\square_{10}} = \{(\square_3, \{\square_1, \square_2\})\}.$$

Then $\square(\overline{\square_{\square_5}}) = \square_{\square}$ and so $\square(\overline{\square_{\square_5}}) \tilde{\cap} \square_{\square_{10}} = \{(\square_3, \{\square_1, \square_2\})\} = \square_{\square_{10}} \neq \square_{\emptyset}$.we

next choose, $\square_{\square_{11}} = \{(\square_2, \{\square_2, \square_3\})\}$ and $\square_{\square_{10}} = \{(\square_3, \{\square_1, \square_2\})\}$. Then $\square(\overline{\square_{\square_5}}) = \square_{\square}$ and so

$$\square(\overline{\square_{\square_5}}) \tilde{\cap} \square_{\square_{10}} = \{(\square_3, \{\square_1, \square_2\})\} = \square_{\square_{10}} \neq \square_{\emptyset}.$$

We next choose $\square_{\square_{11}} = \{(\square_3, \{\square_1\})\}$ and

$$\square_{\square_{14}} = \{(\square_2, \{\square_2, \square_3\}), (\square_3, \{\square_2\})\}.$$

Then $\square(\overline{\square_{\square_{11}}}) = \{(\square_3, \{\square_1\})\} = \square_{\square_{11}}$, $\square(\overline{\square_{\square_{14}}}) = \square_{\square}$. But

$$\square(\overline{\square_{\square_{11}}}) \tilde{\cap} \square_{\square_{14}} = \square_{\emptyset} \quad \square_{\square_{11}} \tilde{\cap} \square(\overline{\square_{\square_{14}}}) = \{(\square_3, \{\square_1\})\} = \square_{\square_{11}} \neq \square_{\emptyset}.$$

Finally we choose

$$\square_{\square_{12}} = \{(\square_3, \{\square_2\})\}$$

and $\square_{\square_{13}} = \{(\square_2, \{\square_2, \square_3\}), (\square_3, \{\square_1\})\}$. Then $\square(\overline{\square_{\square_{12}}}) = \{(\square_3, \{\square_2\})\} = \square_{\square_{12}}$ and

$$(\overline{\square_{\square_{13}}}) = \square_{\square} .$$

Therefore we have $\square(\overline{\square_{\square_{12}}}) \tilde{\cap} \square_{\square_{13}} = \square_{\emptyset}$, and $\square_{\square_{12}} \tilde{\cap} \square(\overline{\square_{\square_{13}}}) = \square_{\square_{12}} \neq \square_{\emptyset}$. Thus we see

that F_A can't be expressed as the soft union of two P-separated sets and hence F_A is soft P-connected. But in Example 5. 4, we have F_A is not soft locally P-connected.

5.6. Example. Soft connectedness does not imply soft P-connectedness.

Let $(\square_{\square}, \tilde{\square})$ be the soft topological space that is consider as in Example 3.6. Consider

$$= \{\square_{\square}, \square_{\emptyset}, \square_{\square_3}, \square_{\square_{11}}, \square_{\square_{13}}\} .$$

Here $\tilde{\square}^{\tilde{\square}} = \{\square_{\emptyset}, \square_{\square}, \square_{\square_{10}}, \square_{\square_{14}}, \square_{\square_{12}}\}$.

$$\square_{\square}(\tilde{F}_A) = \{\square_{\square}, \square_{\emptyset}, \square_{\square_3}, \square_{\square_{11}}, \square_{\square_{13}}, \square_{\square_4}, \square_{\square_5}, \square_{\square_6}, \square_{\square_7}, \square_{\square_8}, \square_{\square_{12}}, \square_{\square_{13}}\} .$$

$$\square_{\square}(\tilde{F}_A) = \{\square_{\emptyset}, \square_{\square}, \square_{\square_{10}}, \square_{\square_{14}}, \square_{\square_7}, \square_{\square_8}, \square_{\square_5}, \square_{\square_4}, \square_{\square_6}, \square_{\square_{12}}, \square_{\square_{16}}\} .$$

Since it is clear that $(\square_{\square}, \tilde{\square})$ is soft connected. Since the only soft clopen sets are F_A and \square_{\emptyset} . But we show that

it is not soft P-connected. Let $\square_{\square} = \square_{\square_4} \cup \square_{\square_7}$, then $\square(\overline{\square_{\square_4}}) = \{(\square_2, \{\square_2\})\} = \square_{\square_4}$,

$$\square(\overline{\square_{\square_7}}) = \{(\square_2, \{\square_3\}), (\square_3, \{\square_1, \square_2\})\} = \square_{\square_7} .$$

We have $\square(\overline{\square_{\square_4}}) \tilde{\cap} \square_{\square_7} = \square_{\emptyset}$ and $\square_{\square_4} \tilde{\cap} \square(\overline{\square_{\square_7}}) = \square_{\emptyset}$.

Hence F_A can be expressed as a soft union of two soft P-separated sets \square_{\square_4} and \square_{\square_7} . Hence F_A is not soft P-connected.

5.7. Example. Soft connectedness does not implies soft locally connectedness.

We consider the soft topological space $(\square_{\square}, \tilde{\square})$ that is given in Example 5. 4. Here it is verified that $(\square_{\square}, \tilde{\square})$

can't be expressed as a soft union of two soft separated sets. Hence it is soft connected. But the soft point

$(\square_2, \{\square_2, \square_3\}) \notin \square_\square$ then there doesn't exist a soft connected open set containing $(\square_2, \{\square_2, \square_3\})$ in $(\square_\square, \square)$.

Hence it is not soft locally connected at $(\square_2, \{\square_2, \square_3\})$. Therefore $(\square_\square, \square)$ is not soft locally connected.

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