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Steady State Thermoelastic Problem in an Infinite Elastic Layer Weakened by a Crack Lying in The Middle of the Layer

Rajesh Patra[1], Sakti Pada Barik[2], Pranay Kumar Chaudhuri[3]
[1]*(Department of Mathematics, Hooghly Engineering & Technology College, Hooghly-712103, West Bengal, India)*
[2]*(Department of Mathematics,Gobardanga Hindu College, 24-Parganas (N)-743273, West Bengal, India)*
[3]*(Retired Professor, Department of Applied Mathematics,University of Calcutta 92, A. P. C. Road, Kolkata - 700 009, India)*

Abstract: *The paper is concerned with a thermoelastic behavior in an infinite elastic layer of finite thickness with a crack in it lying in the middle of the layer and parallel to the faces of the layer. The faces of the layer are maintained at constant temperature of different magnitude. The problem is a steady state thermoelastic problem. The layer surfaces are supposed to be acted on by symmetrically applied concentrated forces of magnitude* $\frac{P}{2}$ *with respect to the centre of the crack. The applied concentrated force may be compressive or tensile in nature. The problem is solved by using integral transform technique .The solution of the problem has been reduced to the solution of a Cauchy type singular integral equation, which requires numerical treatment. The stress-intensity factor and the crack opening displacement are determined and thermal effects on various subjects of physical interest are shown graphically.*
Keywords: *Fourier integral transform, Singular integral equation, Stress-intensity factor.*

I. Introduction

Thermal loading on solids has significant effects in their after load behavior and so should be dealt with utmost care. As such, the study of thermoelastic problems has always been an important branch in solid mechanics (Nowacki [1] ; Nowinski [2]). In the design of a structure in engineering field, considerable attention on thermal stress is a natural task, because many structural components are subjected to severe thermal loading which might cause significant thermal stresses in the components, especially around any defect present in the solid. Thermal stresses along with the stresses due to mechanical loadings can give rise to stress concentration in an around the defects and can lead to considerable damage in the structure.

In literature, problems related to defects such as cracks in solids have been studied in detail for various kinds of solid medium. Cracks in a solid may be generated due to several reasons: such as uncertainties in the loading process, compositional defects in materials, inadequacies in the design, deficiencies in construction or maintenance of environmental conditions, and several others. Consequently, almost all structures contain cracks, either due to manufacturing defects or due to inappropriate thermal or mechanical loading. If proper attention to load condition is not paid, the size of the crack grows, leading to a catastrophically structure failure. A comprehensive list of work on crack problems by earlier investigators has been provided in Zhou and Hanson [3], Chaudhuri and Ray [4], Fabrikant [5], Dag et al.[6], Dag and Erdogan [7], Sherief and El-Mahraby [8], Chen et al. [9], Lee [10], Matbuly [11] etc. Among the recent works on crack problems in solids of above mentioned characteristics, notable are the works of Kanaun [12], Barik et al.[13], Birinci et al. [14], Rekik et al. [15], Wang and Han [16], Beom and Jang [17], Chen and Wang [18], Chen and Hu [19], Chang and Wang [20], Chudnovsky [21], Markov and Kanaun [22], Ding et al. [23], Wang et al. [24], Matysiak and Pauk [25] etc.

For a solid with a crack in it loaded mechanically or thermally, determination of stress-intensity factor (SIF) becomes a very important task in fracture mechanics. The SIF is a parameter that gives a measure of stress concentration around cracks and defects in a solid. SIF needs to be understood if we are to design fracture tolerant materials used in bridges, buildings, aircraft, or even bells. Polishing just won't do if we detect crack. For a thermoelastic crack problem thermal stress intensity factor is a very important subject of physical interest. Literature survey shows good number of papers dealing with thermal stress intensity factors. Among them mention may be made of the works of Lee and Park [26], Itou [27], Liu and Kardomateas [28], Nabavi and Shahani [29], Hu and Chen [30] etc.

The present investigation aims to find the elastostatic solution in an infinite layer with a crack in it and is under steady state thermal loading as well as mechanical loading. Following the integral transform technique the problem has been reduced to a problem of Cauchy type singular integral equation, which has been solved numerically. Finally, the stress-intensity factors and the crack opening displacements are determined for various thermal and mechanical loading conditions and the associated numerical results have been shown graphically.

II. Formulation of the Problem

We consider an infinitely long elastic layer of thickness 2h weakened by the presence of an internal crack of length 2b lying in the middle of the layer. The crack is opened by an uniform internal pressure p_0 along its surface. The strip is under the action of steady state thermal loading with its surfaces maintained at different constant temperatures. The layer is subjected to two different types of mechanical loadings on its surfaces in a direction perpendicular to its length (i) a symmetric pair of compressive concentrated normal loads $\frac{P}{2}$, (ii) a symmetric pair of tensile concentrated normal loads $\frac{P}{2}$ [Fig. 1(a) , (b)]. The gravitational force has not been taken into consideration. The problem is formulated in Cartesian co-ordinate system (x ,y) in which crack lies along x-axis with origin at the centre of the crack.

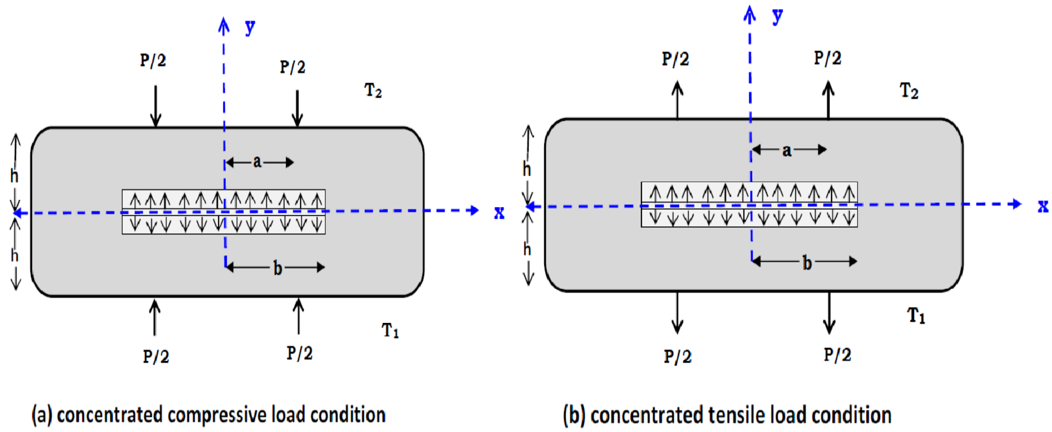

Fig. 1 Geometry of the Problem

The strain displacement relations, linear stress-strain relations and equations of equilibrium are, respectively, given by

$$\varepsilon_x = \frac{\partial u}{\partial x}, \qquad \varepsilon_y = \frac{\partial v}{\partial y}, \qquad \gamma_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right); \tag{1}$$

$$\begin{gathered}\sigma_x = \frac{\mu}{\kappa - 1}[(1+\kappa)\varepsilon_x + (3-\kappa)\varepsilon_y] \\ \sigma_y = \frac{\mu}{\kappa - 1}[(3-\kappa)\varepsilon_x + (1+\kappa)\varepsilon_y] \\ \tau_{xy} = 2\mu\gamma_{xy};\end{gathered} \tag{2}$$

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0 \tag{3}$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} = 0, \tag{4}$$

where $\kappa = 3 - 4\boldsymbol{\nu}$, $\boldsymbol{\nu}$ is the Poisson's ratio and μ is the elastic parameter. Before further proceeding it will be convenient to adopt non-dimensional variables by rescaling all length variables by the problem's length scale b and the temperature variable by the reference temperature scale T_0 :

$$u' = \frac{u}{b}, v' = \frac{v}{b}, x' = \frac{x}{b}, y' = \frac{y}{b}, h' = \frac{h}{b};$$

$$T' = \frac{T}{T_0}, T_1' = \frac{T_1}{T_0}, T_2' = \frac{T_2}{T_0}, \alpha' = \alpha T_0. \tag{5}$$

where α is the thermal expansion coefficient .

In the analysis below, for notational convenience, we shall use only dimensionless variables and shall ignore the dashes on the transformed non-dimensional variables. Mathematically, the problem under consideration is reduced to the solution of thermoelasticity equations:

(i) Equilibrium equations:

$$2(1-\nu)\frac{\partial^2 u}{\partial x^2}+(1-2\nu)\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 v}{\partial x\partial y}=2(1+\boldsymbol{\nu})\boldsymbol{\alpha}\frac{\partial \boldsymbol{T}}{\partial \boldsymbol{x}} \tag{6}$$

$$(1-2\nu)\frac{\partial^2 v}{\partial x^2}+2(1-\nu)\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 u}{\partial x\partial y}=2(1+\boldsymbol{\nu})\boldsymbol{\alpha}\frac{\partial \boldsymbol{T}}{\partial \boldsymbol{y}} \tag{7}$$

(ii) Steady state heat conduction equation:

$$\frac{\partial^2 \mathrm{T}}{\partial \mathrm{x}^2}+\frac{\partial^2 \mathrm{T}}{\partial \mathrm{y}^2}=0\,,\ (-\infty<x<\infty) \tag{8}$$

and

(iii) The boundary conditions:

(a) Thermal boundary conditions:

$$\mathrm{T}(\mathrm{x}\,,-\mathrm{h})=\mathrm{T}_1\,,\quad (|\mathrm{x}|<\infty) \tag{9}$$

$$\mathrm{T}(\mathrm{x}\,,\mathrm{h})=\mathrm{T}_2\,,\quad (|\mathrm{x}|<\infty) \tag{10}$$

$$\frac{\partial \mathrm{T}(\mathrm{x}\,,0^+)}{\partial \mathrm{y}}=\frac{\partial \mathrm{T}(\mathrm{x}\,,0^-)}{\partial \mathrm{y}}=\mathrm{H}[\mathrm{T}(\mathrm{x}\,,0^+)-\mathrm{T}(\mathrm{x}\,,0^-)],\quad (|\mathrm{x}|<1) \tag{11}$$

$$\mathrm{T}(\mathrm{x}\,,0^+)=\mathrm{T}(\mathrm{x}\,,0^-),\ (|\mathrm{x}|\geq 1) \tag{12}$$

$$\frac{\partial \mathrm{T}(\mathrm{x}\,,0^+)}{\partial \mathrm{y}}=\frac{\partial \mathrm{T}(\mathrm{x}\,,0^-)}{\partial \mathrm{y}}\ ,\ (|\mathrm{x}|\geq 1) \tag{13}$$

(b) Elastic boundary conditions:

$$\tau_{\mathrm{xy}}(\mathrm{x},0)=0,\quad (-\infty<x<\infty) \tag{14}$$

$$\tau_{\mathrm{xy}}(\mathrm{x},\mathrm{h})=0,\quad (-\infty<x<\infty) \tag{15}$$

$$\sigma_{\mathrm{y}}(\mathrm{x},\mathrm{h})=\mp\left[\frac{\mathrm{P}}{2}\delta(\mathrm{x}-\mathrm{a})+\frac{\mathrm{P}}{2}\delta(\mathrm{x}+\mathrm{a})\right],\ (-\infty<x<\infty) \tag{16}$$

$$\frac{\partial}{\partial x}[v(x,0)]=\begin{cases} f(x), |x|<1; \\ \quad 0, |x|>1. \end{cases} \tag{17}$$

$$\sigma_y(x,0)=-p_0,\ (-1\leq x\leq 1) \tag{18}$$

where u and v are the x and y components of the displacement vector; $\sigma_x, \sigma_y, \tau_{xy}$ are the normal and shearing stress components; the quantity H is the dimensionless thermal conductivity of the crack surface defined in Carslaw and Jaeger [31]; f(x) is an unknown function and $\delta(x)$ is the Dirac delta function. In equation (16) positive sign indicates tensile force while negative sign corresponds to compressive force.

III. Method Of Solution

(a) Thermal part:

To determine temperature field T(x,y) from equation (8) and boundary conditions (9) -(13) we assume

$$\mathrm{T}(\mathrm{x},\mathrm{y})=\mathrm{U}(\mathrm{x},\mathrm{y})+\mathrm{W}(\mathrm{y}) \tag{19}$$

where $\mathrm{U}(\mathrm{x},\mathrm{y})$ and $\mathrm{W}(\mathrm{y})$ are two unknown functions satisfying the conditions

$$\mathrm{U}(\mathrm{x}\,,-\mathrm{h})=\mathrm{U}(\mathrm{x}\,,\mathrm{h})=0\,, \tag{20}$$

and

$$W(-h) = T_1\,,\ W(h) = T_2\,. \tag{21}$$

Under these considerations we get

$$W(y) = \frac{(T_2 - T_1)}{2h}\,y + \frac{T_1 + T_2}{2} \tag{22}$$

and

$$U(x,y) = \begin{cases} (e^{\eta y} - e^{\eta(2h-y)})A_1, & y \geq 0; \\ (e^{\eta y} - e^{-\eta(2h+y)})\frac{1+e^{2\eta h}}{1+e^{-2\eta h}}A_1, & y \leq 0; \end{cases} \tag{23}$$

for certain constant A_1.

The appropriate temperature field satisfying the boundary conditions and regularity condition can be expressed as:

$$T(x,y) = \int_{-\infty}^{\infty}[e^{\eta y} - e^{\eta(2h-y)}]D(\eta)e^{-ix\eta}d\eta + W(y), \qquad y \geq 0 \tag{24}$$

$$T(x,y) = \int_{-\infty}^{\infty}[e^{\eta y} - e^{-\eta(2h+y)}]r_{ab}D(\eta)e^{-ix\eta}d\eta + W(y), \quad y \leq 0 \tag{25}$$

where $D(\eta)$ is an unknown function to be determined and

$$r_{ab} = \frac{1+e^{2\eta h}}{1+e^{-2\eta h}}\,. \tag{26}$$

Let us introduce the density function $\Theta(x)$, as

$$\Theta(x) = \frac{\partial T(x,0^+)}{\partial x} - \frac{\partial T(x,0^-)}{\partial x}\,. \tag{27}$$

It is clear from the boundary conditions (12) and (13) that

$$\int_{-1}^{1}\Theta(s)ds = 0 \tag{28}$$

and

$$\Theta(x) = 0, (|x| \geq 1). \tag{29}$$

Substituting (24) and (25) into (27) and using Fourier inverse transform, we have

$$D(\eta) = \frac{i(1+e^{-2\eta h})}{4\pi\eta(e^{-2\eta h} - e^{2\eta h})}\int_{-1}^{1}\Theta(s)e^{is\eta}ds, \quad (-1 < s < 1). \tag{30}$$

Substituting (24) and (25) into (11) and applying the relation (30), we get the singular integral equation for $\Theta(s)$ as follows

$$\frac{1}{\pi}\int_{-1}^{1}\left[\frac{1}{s-x} + k_1(x,s)\right]\Theta(s)ds = \frac{T_1 - T_2}{h} \tag{31}$$

where

$$k_1(x,s) = \int_0^{\infty}\left[1 - \frac{2H}{\eta} + \frac{2+e^{2\eta h}+e^{-2\eta h}}{e^{-2\eta h} - e^{2\eta h}}\right]\sin\eta(x-s)d\eta \tag{32}$$

After determining $\Theta(s)$ from the singular integral equation (31) we have the temperature field along the axes as

$$T(x,0)=\begin{cases}\frac{1}{4}\int_{-1}^{1}\operatorname{sign}(x-s)\Theta(s)ds+\frac{T_1+T_2}{2}\ ,y\geq 0;\\ -\frac{1}{4}\int_{-1}^{1}\operatorname{sign}(x-s)\Theta(s)ds+\frac{T_1+T_2}{2}\ ,y\leq 0;\end{cases} \tag{33}$$

$$T(0,y)=\begin{cases}-\frac{1}{4\pi}\int_{0}^{\infty}\frac{1+e^{-2\eta h}}{e^{-4\eta h}-1}\left[e^{-\eta(2h-y)}-e^{-\eta y}\right]\frac{\sin(s\eta)}{\eta}\,d(\eta)\int_{-1}^{1}\Theta(s)\,ds+\frac{T_2-T_1}{2h}+\frac{T_1+T_2}{2}\ ,y\geq 0;\\ -\frac{1}{4\pi}\int_{0}^{\infty}\frac{1+e^{-2\eta h}}{e^{-4\eta h}-1}\left[e^{\eta y}-e^{-\eta(2h+y)}\right]\frac{\sin(s\eta)}{\eta}\,d(\eta)\int_{-1}^{1}\Theta(s)\,ds+\frac{T_2-T_1}{2h}+\frac{T_1+T_2}{2}\ ,y\leq 0;\end{cases} \tag{34}$$

(b) Elastic part:

First of all we observe that due to symmetry of the crack location with respect to the layer and of the applied load with respect to the crack, it is sufficient to consider solution of the problem in the region $0\leq x<\infty$. To solve the partial differential equations (6) and (7), Fourier transform is applied to the equations with respect to the variable x. The equations in the transformed domain can be written as

$$(1-2\nu)\frac{d^2\bar{u}}{dy^2}-2\xi^2(1-\nu)\bar{u}-i\xi\frac{d\bar{v}}{dy}=-2i\xi(1+\nu)\alpha\bar{T} \tag{35}$$

$$2(1-\nu)\frac{d^2\bar{v}}{dy^2}-(1-2\nu)\bar{v}-i\xi\frac{d\bar{u}}{dy}=2(1+\nu)\alpha\frac{d\bar{T}}{dy} \tag{36}$$

where $\bar{u}$ and $\bar{v}$ are the Fourier transforms of u and v, respectively with respect to x. Now, elimination of $\bar{u}$ from the equations (35) and (36) yields a differential equation in $\bar{v}$;

$$\frac{d^4\bar{v}}{dy^4}-2\xi^2\frac{d^2\bar{v}}{dy^2}+\xi^4\bar{v}=0. \tag{37}$$

Equation (37) is an ordinary differential equation in independent variable y, its general solution can be obtained as

$$\bar{v}(\xi,y)=(A+By)e^{-\xi y}+(C+Dy)e^{\xi y} \tag{38}$$

where A, B, C and D are the unknown constants. Use of (38) in equations (35) and (36) yields

$$\bar{u}(\xi,y)=\frac{1}{i\xi}\left[\{\xi(C+Dy)+D\kappa\}e^{\xi y}+\{-\xi(A+By)+B\kappa\}e^{-\xi y}\right]-\frac{2\alpha\bar{T}(7-\kappa)}{4i\xi} \tag{39}$$

where $i=\sqrt{-1}$. Application of Fourier inversion formula on the equations (38) and (39) we get

$$u(x,y)=\frac{i}{2\pi}\int_{-\infty}^{\infty}\left[\left\{A+\left(y-\frac{\kappa}{\xi}\right)B\right\}e^{-\xi y}-\left\{C+\left(y+\frac{\kappa}{\xi}\right)D\right\}e^{\xi y}\right]e^{-i\xi x}d\xi+\frac{i\alpha}{4\pi}(7-\kappa)\int_{-\infty}^{\infty}\frac{1}{\xi}\bar{T}\,e^{-i\xi x}d\xi \tag{40}$$

and $$v(x,y)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\left[(A+By)e^{-\xi y}+(C+Dy)e^{\xi y}\right]e^{-i\xi x}d\xi\,. \tag{41}$$

Substituting (40) and (41) into the transformed equivalent forms of equations (1) and (2) in dimensionless variables, we obtain

$$\frac{1}{2\mu}\sigma_x(x,y)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\left[\xi e^{-\xi y}A+\left\{\xi y+\frac{\kappa^2+2\kappa-3}{2(1-\kappa)}\right\}e^{-\xi y}B-\xi e^{\xi y}C+\left\{-\xi y+\frac{\kappa^2+2\kappa-3}{2(1-\kappa)}\right\}e^{\xi y}D\right]e^{-i\xi x}d\xi$$

$$-\frac{\alpha(1+\kappa)(7-\kappa)}{4(1-\kappa)}T(x,y) \tag{42}$$

$$\frac{1}{2\mu}\sigma_y(x,y)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\left[-\xi e^{-\xi y}A+\left\{-\xi y-\frac{\kappa^2-2\kappa+1}{2(1-\kappa)}\right\}e^{-\xi y}B+\xi e^{\xi y}C+\left\{\xi y-\frac{\kappa^2-2\kappa+1}{2(1-\kappa)}\right\}e^{\xi y}D\right]e^{-i\xi x}d\xi$$

$$-\frac{\alpha(3-\kappa)(7-\kappa)}{4(1-\kappa)}T(x,y) \quad (43)$$

$$\frac{1}{2\mu}\tau_{xy}(x,y)=\frac{i}{2\pi}\int_{-\infty}^{\infty}\left[-\xi e^{-\xi y}A+\left\{-\xi y+\frac{\kappa+1}{2}\right\}e^{-\xi y}B-\xi e^{\xi y}C+\left\{-\xi y-\frac{\kappa+1}{2}\right\}e^{\xi y}D\right]e^{-i\xi x}d\xi$$

$$+\frac{i\alpha(7-\kappa)}{8\pi}\int_{-\infty}^{\infty}\frac{1}{\xi}\frac{\partial\overline{T}}{\partial y}\,e^{-i\xi x}d\xi \quad (44)$$

Utilizing of the boundary conditions(14) -(17), the unknown constants A,B,C and D can be found out and substitution of these values into the equation (18) will lead to the following singular integral equation:

$$\frac{1}{\pi}\int_{-1}^{1}f(t)\left[\frac{1}{t-x}+k_2(t,x)\right]dt=\frac{\kappa+1}{2}\left[-\frac{p_0}{2\mu}\pm\frac{P}{4\pi\mu}k_3(x)\right]$$

$$+\left[-\frac{\alpha(7-\kappa)(\kappa+1)}{64\pi^2}k_4(t,x)+\frac{\alpha(3-\kappa)(7-\kappa)(\kappa+1)}{8(1-\kappa)}T(x,0)\right]$$

$$-\frac{\alpha(7-\kappa)(\kappa+1)}{64\pi}\left[-(\kappa-1)(5-\kappa)(T_1-T_2)+\frac{4(3-\kappa)T^*}{1-\kappa}\right],(-1\le x\le 1), \quad (45)$$

where $$T^*=\begin{cases}T_2\,, & y\ \ge 0;\\ T_1\,, & y\ \le 0\,;\end{cases} \quad (46)$$

$$k_2(t,x)=-2\int_0^{\infty}\frac{(1+2\xi h+2\xi^2h^2)e^{-2\xi h}-e^{-4\xi h}}{1+4\xi he^{-2\xi h}-e^{-4\xi h}}\sin\xi(x-t)d\xi \quad (47)$$

$$k_3(x)=\int_0^{\infty}\frac{2e^{-\xi h}[1+\xi h+(-1+\xi h)e^{-2\xi h}]}{1+4\xi he^{-2\xi h}-e^{-4\xi h}}[\cos\xi(x-a)+\cos\xi(x+a)]\,d\xi \quad (48)$$

and

$$k_4(t,x)=\int_0^{\infty}\left[\frac{(\kappa-1)(1+e^{-2\eta h})^2\{1+2(1-\kappa+4\eta^2h^2)e^{-2\eta h}+e^{-4\eta h}\}}{\eta(\kappa+1)(e^{-4\eta h}-1)(1+4\eta he^{-2\eta h}-e^{-4\eta h})}+\frac{8\eta he^{-\eta h}(e^{-3\eta h}+e^{-5\eta h})(e^{-2\eta h}-1)}{(e^{-4\eta h}-1)(1-e^{-4\eta h}+4\eta he^{-2\eta h})}\right]\sin\eta(t-x)d\eta \quad (49)$$

The kernels $k_2(t,x), k_3(x)$ and $k_4(t,x)$ are bounded and continuous in the closed interval $-1\le x\le 1$. The integral equations must be solved under the following single-valuedness condition

$$\int_{-1}^{1}f(t)dt=0. \quad (50)$$

IV. Solution of the Integral Equations

(a) Thermal part:

The singular integral equation (31) is a Cauchy-type singular integral equation for an unknown function $\Theta(s)$. For the evaluation of thermal stress it is necessary to solve the integral equation (31).For this purpose we write

$$\Theta(s)=\frac{\Upsilon(s)}{\sqrt{1-s^2}}\,,\ (-1<s<1) \quad (51)$$

where $\Upsilon(s)$ is a regular and bounded unknown function. Substituting (51) into equation (31) and using Gauss-Chebyshev formula (Erdogan and Gupta [32]), we obtain

$$\frac{1}{N}\left[\sum_{k=1}^{N}\left\{\frac{1}{s_k-x_i}+k_1(x_i,s_k)\right\}\Upsilon(s_k)\right]=\frac{T_1-T_2}{h}\ ,i=1,2,\dots\dots,N-1 \tag{52}$$

and

$$\frac{\pi}{N}\sum_{k=1}^{N}\Upsilon(s_k)=0 \tag{53}$$

where s_k and x_i are given by

$$s_k=\cos\left(\frac{2k-1}{2N}\pi\right),\ (k=1,2,3,\dots\dots\dots.,N) \tag{54}$$

$$x_i=\cos\left(\frac{\pi i}{N}\right),\ (i=1,2,3,\dots\dots..,N-1). \tag{55}$$

We observe that corresponding to $(N-1)$ collocation points $x_i=\cos\left(\frac{\pi i}{2(N+1)}\right)$, $i=1,2,\dots\dots,(N-1)$ we have a set of N linear equations in N unknowns $\Upsilon(s_1),\Upsilon(s_2),\dots\dots.,\Upsilon(s_N)$.
This linear algebraic system of equations are solved numerically by utilizing Gaussian elimination method.

(b) Elastic part:
The singular integral equation (45) is a Cauchy-type singular integral equations for an unknown function f(t). Expressing now the solution of equation (45) in the form

$$f(t)=\frac{\psi(t)}{\sqrt{1-t^2}}\ ,\ (-1<t<1) \tag{56}$$

where $\psi(t)$ is a regular and bounded unknown function and using Gauss-Chebyshev formula (Erdogan and Gupta [32]) to evaluate the integral equation (45), we obtain, for $y\geq 0$

$$\begin{aligned}\frac{1}{N}\left[\sum_{k=1}^{N}\left[\frac{1}{t_k-x_r}+k_2(t_k,x_r)\right]\psi(t_k)\right]&=\frac{\kappa+1}{2}\left[-\frac{p_0}{2\mu}\pm\frac{P}{4\pi\mu}k_3(x_r)\right]\\&+\sum_{k=1}^{N}\left[-\frac{\alpha(7-\kappa)(\kappa+1)}{64\pi^2}k_4(t_k,x_r)+\frac{\alpha(3-\kappa)(7-\kappa)(\kappa+1)}{32(1-\kappa)}\mathrm{sign}(x_r-t_k)\right]\psi(t_k)\\&+\frac{\alpha(3-\kappa)(7-\kappa)(\kappa+1)}{16(1-\kappa)}(T_1+T_2)\\&-\frac{\alpha(7-\kappa)(\kappa+1)}{64\pi}\left[-(\kappa-1)(5-\kappa)(T_1-T_2)+\frac{4(3-\kappa)T_2}{1-\kappa}\right],r=1,2,\dots\dots..,N-1\end{aligned} \tag{57}$$

$$\frac{1}{N}\sum_{k=1}^{N}\psi(t_k)=0 \tag{58}$$

and for $y\leq 0$

$$\begin{aligned}\frac{1}{N}\left[\sum_{k=1}^{N}\left[\frac{1}{t_k^*-x_r}+k_2(t_k^*,x_r)\right]\psi(t_k)\right]&=\frac{\kappa+1}{2}\left[-\frac{p_0}{2\mu}\pm\frac{P}{4\pi\mu}k_3(x_r)\right]\\&+\sum_{k=1}^{N}\left[-\frac{\alpha(7-\kappa)(\kappa+1)}{64\pi^2}k_4(t_k,x_r)+\frac{\alpha(3-\kappa)(7-\kappa)(\kappa+1)}{32(1-\kappa)}\mathrm{sign}(x_r-t_k)\right]\psi(t_k)\\&+\frac{\alpha(3-\kappa)(7-\kappa)(\kappa+1)}{16(1-\kappa)}(T_1+T_2)\end{aligned}$$

$$-\frac{\alpha(7-\kappa)(\kappa+1)}{64\pi}\left[-(\kappa-1)(5-\kappa)(T_1-T_2)+\frac{4(3-\kappa)T_1}{1-\kappa}\right], r = 1,2,\ldots\ldots\ldots, N-1 \qquad (59)$$

$$\frac{1}{N}\sum_{k=1}^{N}\psi(t_k) = 0 \qquad (60)$$

where t_k and x_r are given by

$$t_k = \cos\left(\frac{2k-1}{2N}\pi\right), \ (k = 1,2,3,\ldots\ldots\ldots, N) \qquad (61)$$

$$x_r = \cos\left(\frac{r\pi}{N}\right), \ (r = 1,2,3,\ldots\ldots\ldots, N-1). \qquad (62)$$

We observe that corresponding to $(N-1)$ collocation points $x_r = \cos\left(\frac{r\pi}{2(N+1)}\right)$, $r = 1,2,\ldots\ldots,(N-1)$ the equations (58) ,(59) or equations (60) ,(61) represent a set of N linear equations in N unknowns $\psi(t_1), \psi(t_2), \ldots\ldots, \psi(t_N)$. This linear algebraic system of equations are solved numerically by utilizing Gaussian elimination method.

V. Determination of Stress Intensity Factor

Presence of a crack in a solid significantly affects the stress distribution compared to the state when there is no crack. While the stress distribution in a solid with a crack in the region far away from the crack is not much disturbed, the stresses in the neighbourhood of the crack tip assumes a very high magnitude. In order to predict whether the crack has a tendency to expand further, the stress intensity factor (**SIF**), a quantity of physical interest, has been defined in fracture mechanics. The load at which failure occurs is referred to as the fracture strength. The stress intensity factor is defined as

$$k(b) = \lim_{x^*\to 1}\sqrt{2b(x^*-1)}\,\sigma_y^*(x^*,0) \qquad (63)$$

Use of the equations (18), (45) and having some workings, the expression for k(b) is obtained as

$$\frac{\kappa+1}{4\mu\sqrt{b}}k(b) = k'(b)\ (\text{say}) = -\psi(1) \qquad (64)$$

where $\psi(1)$ can be found out from $\psi(t_k), (k = 1,2,3,\ldots, N)$ using the interpolation formulas given by Krenk [33].

Following the method as in Gupta and Erdogan [34] we obtain the crack surface displacement in the form

$$v'(x,0) = \int_{-1}^{x}\frac{\psi(t)}{\sqrt{1-t^2}}dt\,, \ (-1 < x < 1) \qquad (65)$$

where
$$v'(x,0) = \frac{v(x,0)}{b} \qquad (66)$$

which can be obtained numerically, using say, Simpson's $\frac{1}{3}$ integration formula and appropriate interpolation formula.

VI. Numerical Results And Discussions

The present study is related to the study of an internal crack problem in an infinite elastic layer with thermal effect. The main objective of the present discussion is to study the effects of temperature as well as of applied loads on stress intensity factor and crack opening displacement. Solution of the problem can be obtained using numerical methods. Following the standard numerical method described in section 4, the normal displacement component and the stress intensity factor are computed and shown graphically.

Fig. 2(a) shows temperature distribution on the crack faces for various values of $\frac{b}{h}$. As expected, the result shows that temperature distribution increases with the decrease of layer thickness. Fig. 2(b) shows temperature distribution along x = 0, taking $T_1 > T_2$ .Temperature decreases linearly from the lower to the upper surface of the layer. There is one point to note here that the temperature at a particular point on x = 0 below the line of crack varies inversely with the layer thickness, whereas the behavior is opposite for a point on x = 0 above the line of crack.

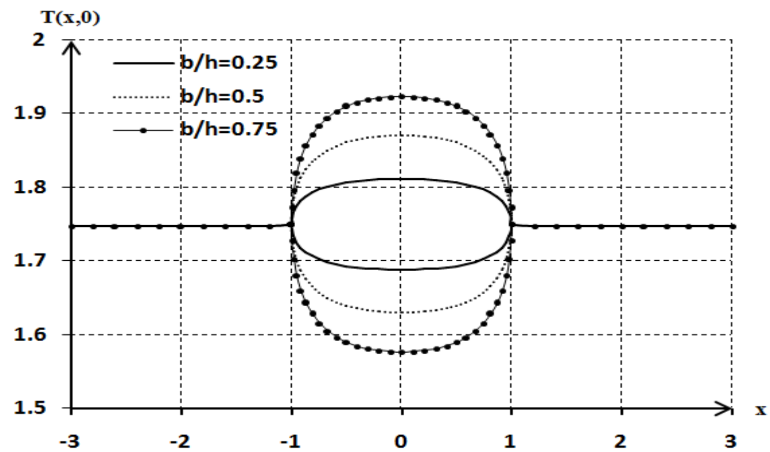

Fig. 2(a) Temperature distribution on crack face and extension line for diffrerent b/h when K=1.8, H=1.0, α=1.5, T_1=2.5, T_2=1.0

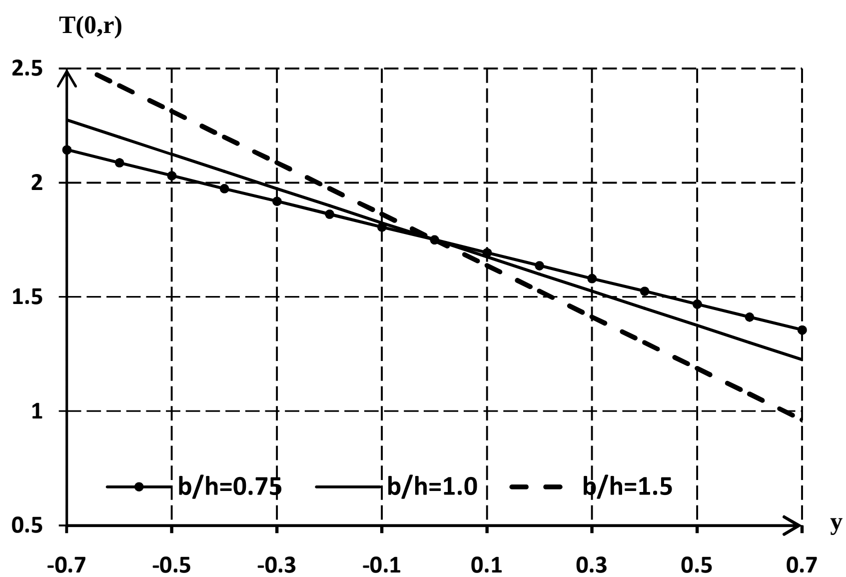

Fig. 2(b)Temperature on crack faces at different b/h (K=1.8, H=1.0, α=1.5, T_1=2.5, T_2=1.0)

The variation of normalized stress intensity factor with $\frac{b}{h}$ are shown in Figs. 3(upper layer), 4(lower layer) for both the cases of two symmetric pair of compressive and tensile concentrated forces. It is observed from Figs. 3(a), 4(a) that for compressive concentrated forces the normalized stress-intensity factor $k'(b)$ decreases with the increase of the load ratio Q, and the increase of $k'(b)$ is quite significant for smaller values of Q. It is also observed from Figs. 3(a), 4(a) that the load ratio Q has not of much effect on the stress intensity factor $k'(b)$ when the crack length is sufficiently small. Contrary to this, in Figs. 3(b), 4(b) where the force is of tensile nature, $k'(b)$ increases with Q. For small crack length, the behavior of $k'(b)$ is similar to the case of compressive concentrated load. Figs 5 and 6 show the variations in the stress intensity factor with the distance of the point of application of compressive / tensile load for fixed load ratio Q. It is observed that in both layers for compressive load conditions normalized stress-intensity factor increases with $\frac{a}{b}$ while the effect is opposite for tensile load conditions.

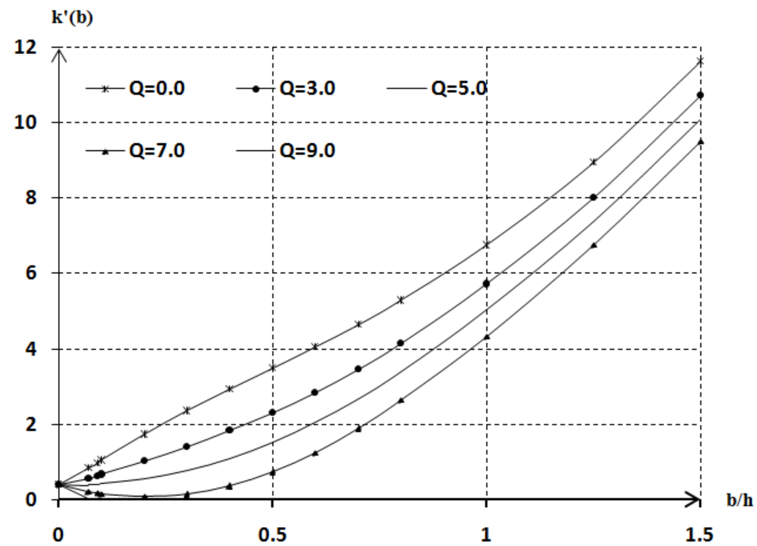

Fig. 3 (a) Variation of normalized stress-intensity factor k'(b) with b/h for different loads Q in the case of compressive forces for the upper layer when a/b=0.0, K=1.8 , H=1.0, α=1.5, T_1=2.5, T_2=1.0

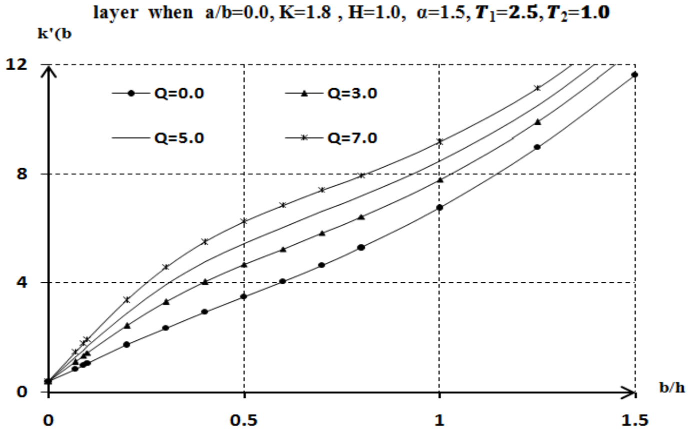

Fig. 3(b) Variation of normalized stress-intensity factor k'(b) with b/h for different loads Q in the case of tensile forces for upper layer when a/b=0.0, K=1.8, H=1.0, α=1.5, T_1=2.5, T_2=1.0

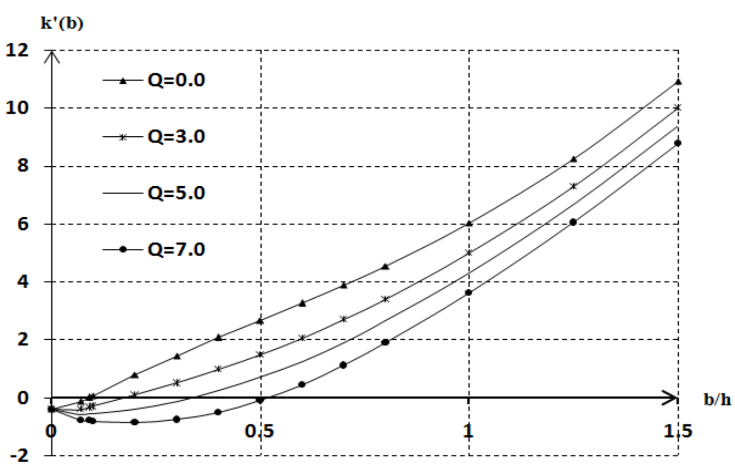

Fig. 4 (a) Variation of normalized stress-intensity factor k'(b) with b/h for different loads Q in the case of compressive forces for the lower layer when a/b=0.0,K=1.8 , H=1.0, α=1.5, T_1=2.5, T_2=1.0

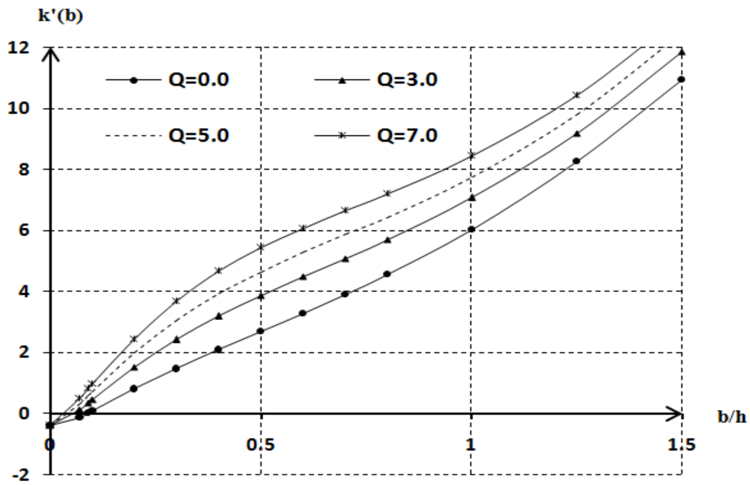

Fig. 4(b) Variation of normalized stress-intensity factor k'(b) with b/h for different loads Q in the case of tensile forces for lower layer when a/b=0.0, K=1.8, H=1.0, α=1.5, T_1=2.5, T_2=1.0

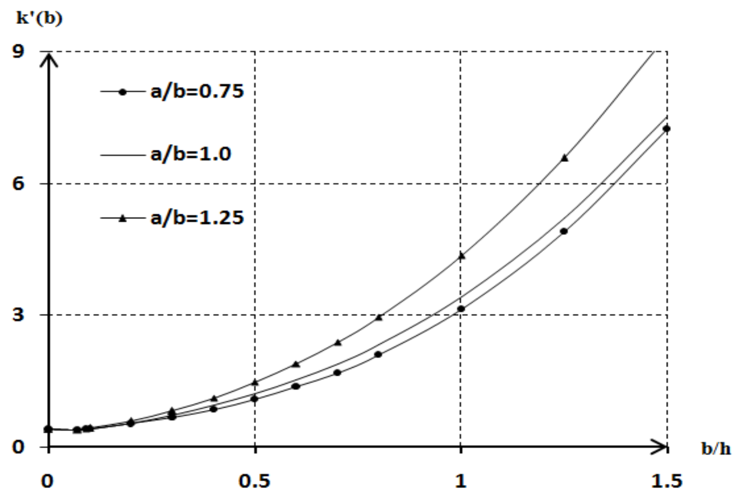

Fig. 5(a) Variation of normalized stress intensity factor k'(b) for different values of a/b in the case of compressive forces for upper layer when Q=5.0, K=1.8, H=1.0, α=1.5, T_1=2.5, T_2=1.0

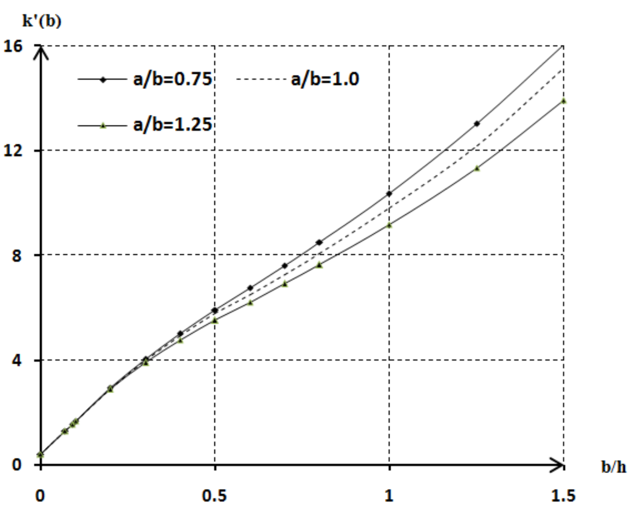

Fig. 5(b) Variation of normalized stress intensity factor k'(b) for different values of a/b in the case of tensile forces (Q=5.0, K=1.8, H=1.0, α=1.5, T_1=2.5, T_2=1.0)

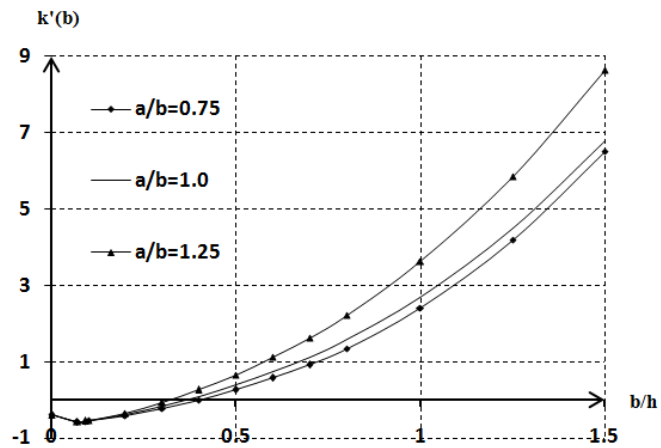

Fig. 6(a) Variation of normalized stress intensity factor k'(b) for different values of a/b in the case of compressive forces for lower layer when Q=5.0, K=1.8, H=1.0, α=1.5, T_1=2.5, T_2=1.0

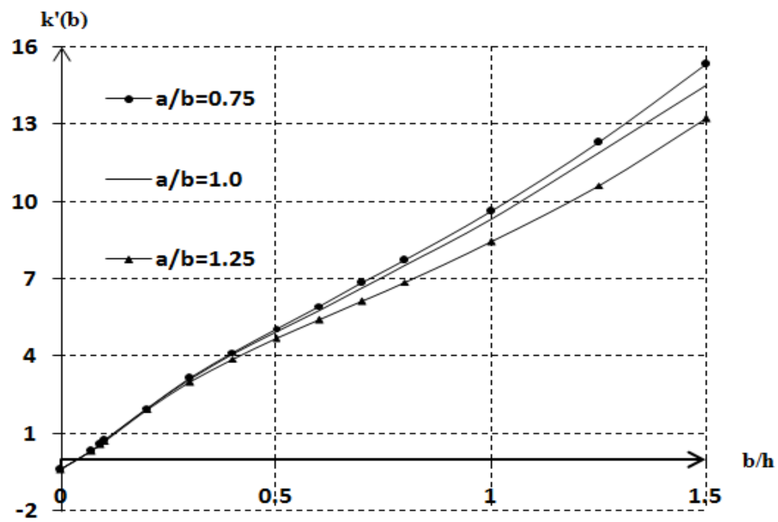

Fig. 6(b) Variation of normalized stress intensity factor k'(b) for different values of a/b in the case of tensile forces for lower layer when Q=5.0, K=1.8, H=1.0, α=1.5, T_1=2.5, T_2=1.0

Figs. 7 and 8 depict the variation of normalized crack surface displacement $v'(x,0)$ with x for different values of load ratio Q. It is clear from Figs. 7(a), 8(a), that for compressive nature of forces the normalized crack surface displacement $v'(x,0)$ decreases as load ratio Q increases, but for tensile load conditions it increases as Q increases. For both the cases of compressive and tensile concentrated forces the graphs show that the normalized crack surface displacement is symmetrical with respect to the origin. We have one more natural observation here. The crack surface displacement has almost a peak value near the centre of the crack for tensile load conditions, whereas for compressive load conditions it has the local minimum value. The effect of $\frac{a}{b}$ on normalized crack surface displacement $v'(x,0)$ is observed in Figs. 9 and 10 for both the cases of compressive and tensile concentrated forces. It is observed in Figs. 9(a), 10(a), that for compressive concentrated loading the normalized crack surface displacement increases with the increased values of $\frac{a}{b}$ but behavior is just opposite (Figs. 9(b),10(b)) for tensile concentrated loading.

An important observation may be available from Fig. 3(a), under the compressive load condition: we find that with the increase of Q, there is a critical crack length for which the stress intensity factor has zero value. From Fig. 3(a), if Q = 9.0, the critical crack length is approximately 0.08.

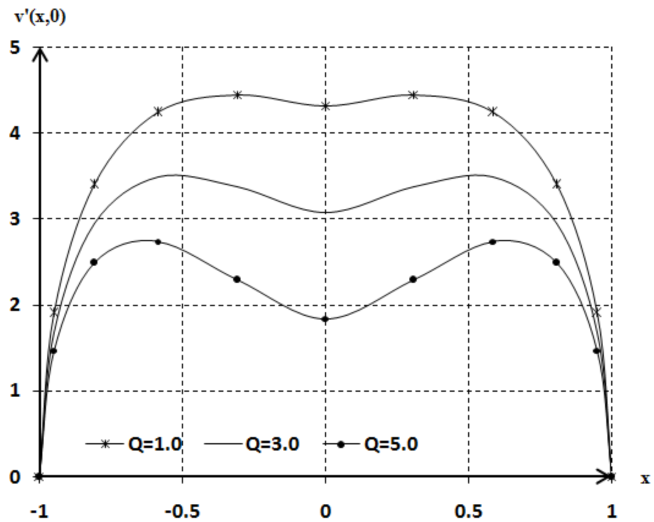

Fig. 7(a) Variation of normalized crack surface displacement v'(x,0) for different loads Q in the case of compressive forces for upper layer when a/b=0.0, b/h=1.0, K=1.8 , H=1.0, α=1.5, T_1=2.5, T_2=1.0

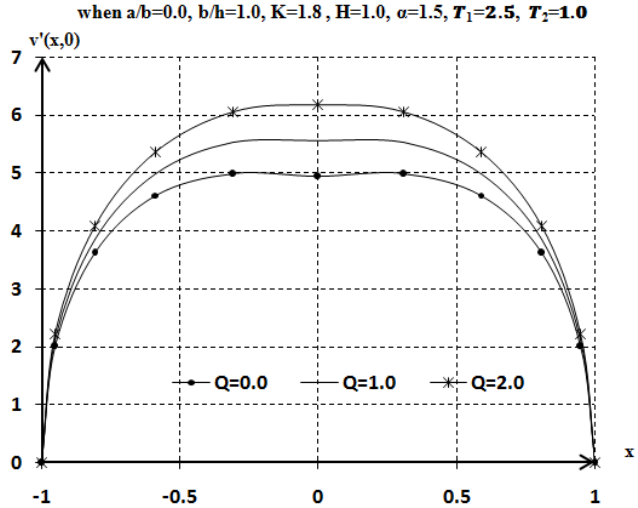

Fig. 7(b) Variation of normalized crack surface displacement v'(x,0) for different loads Q in the case of tensile forces for upper layer when a/b=0.0, b/h=1.0, K=1.8, H=1.0, α=1.5, T_1=2.5, T_2=1.0

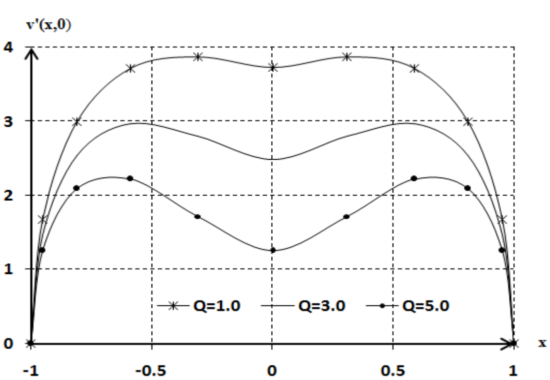

Fig. 8(a) Variation of normalized crack surface displacement v'(x,0) for different loads Q in the case of compressive forces for lower layer when a/b=0.0, b/h=1.0, K=1.8 , H=1.0, α=1.5, T_1=2.5, T_2=1.0

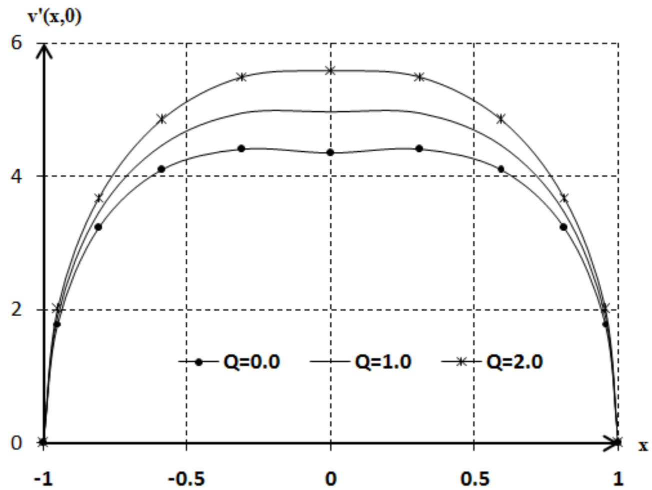

Fig. 8(b) Variation of normalized crack surface displacement v'(x,0) for different loads Q in the case of tensile forces for lower layer when a/b=0.0, b/h=1.0,K=1.8 , H=1.0, α=1.5, T_1=2.5, T_2=1.0

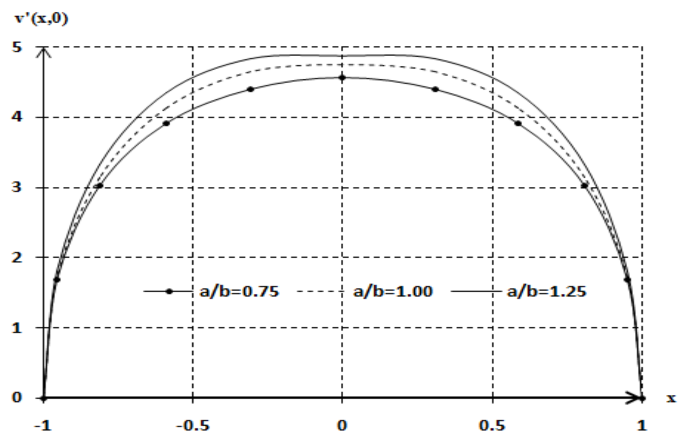

Fig. 9(a) Variation of normalized crack surface displacement v'(x,0) for various values of a/b in the case of compressive forces for upper layer when Q=1.5,b/h=1.0, K=1.8 , H=1.0, α=1.5, T_1=2.5, T_2=1.0

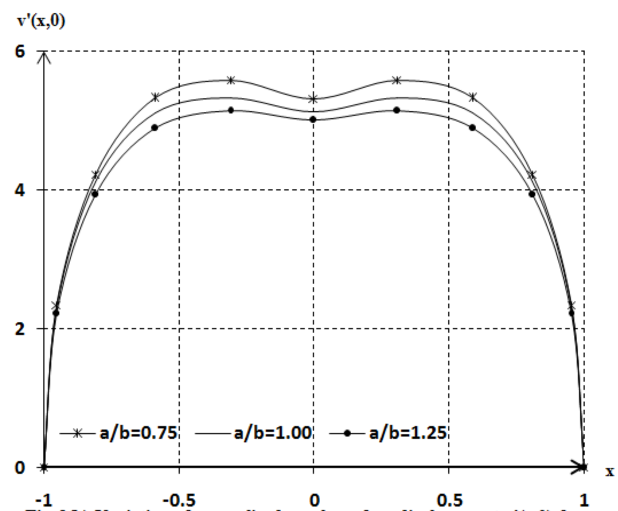

Fig. 9(b) Variation of normalized crack surface displacement v'(x,0) for various values of a/b in the case of tensile forces for upper layer when Q=1.5, b/h=1.0, K=1.8, H=1.0, α=1.5, T_1=2.5, T_2=1.0

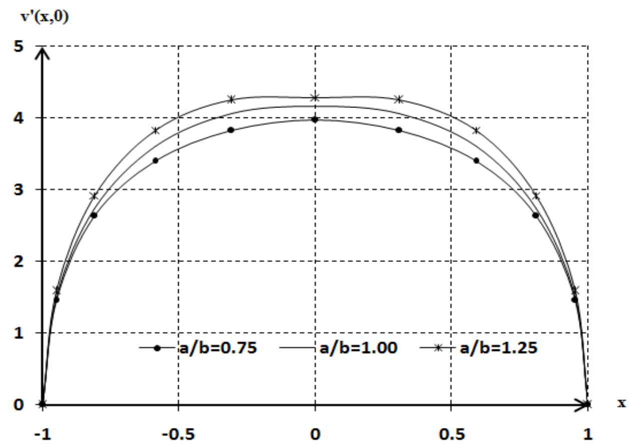

Fig. 10(a) Variation of normalized crack surface displacement v'(x,0) for various values of a/b in the case of compressive forces for lower layer when Q=1.5, b/h=1.0, K=1.8 , H=1.0, α=1.5, T_1=2.5, T_2=1.0

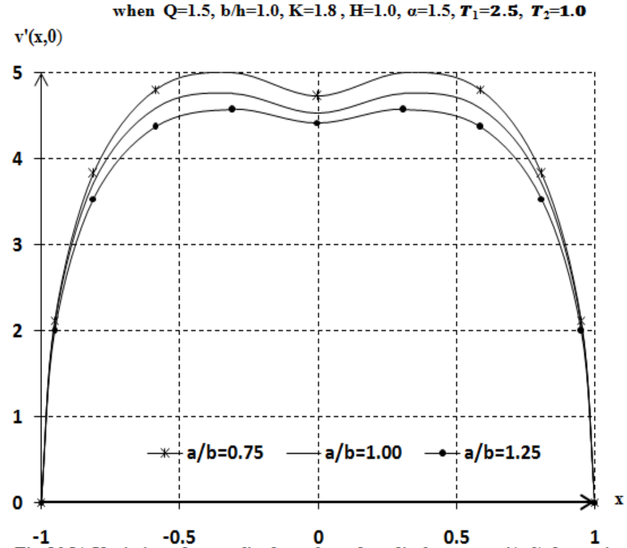

Fig. 10(b) Variation of normalized crack surface displacement v'(x,0) for various values of a/b in the case of tensile forces for lower layer when Q=1.5, b/h=1.0, K=1.8, H=1.0, α=1.5, T_1=2.5, T_2=1.0

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