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DEA Model: A Key Technology for the Future

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Abstract: *Transportation is defined as the movement of passengers and freight from one place to another. Passenger is an important part of the overall development problem of the nation and it affects mostly all the aspects of mobility. The Transportation problem is one of the sub classes of LPP in which the objective is to transport various amount of a single homogeneous commodity, that are initially stored at various origins, to different destinations in such a way that the total transportation cost is minimum. Although the name of the problem is derived from transport to which it was first applied, the problem can also be used for machine allocation, plant location, product mix problem, and many others, so that the problem is not confined to transportation or distribution only. Data Envelopment Analysis (DEA) is a very powerful service management and benchmarking technique originally developed by Charnes et al (1) to evaluate nonprofit and public sector organizations. Linear programming problem (LPP) is the underlying methodology that makes DEA particularly powerful compared with alternative productivity management tools. A Transportation Problem can be solved very easily by different methods (NWC RULE, LCM & VAM) by recognizing and formulating into LPP. The IBFS obtained in Transportation Algorithm can be tested also by MODIFIED DISTRIBUTION METHOD.*
After studying this paper, we will be able to achieve the following objectives of Transportation System - a major problem of the metropolitan cities.
1- *Recognize and formulate the transportation problem as a linear programming problem.*
2- *Build a transportation table and describe its components.*
3- *Find an initial basic feasible solution of the transportation problem by using various methods.*
4- *Know in detail all the steps involved in solving a transportation problem by MODI problem.*
5- *Solve the unbalanced transportation problems by MODI method.*
6- *Identify the special situation in transportation problems; such as degeneracy and alternative optimal solution.*
7- *Resolve the special cases in transportation problems, where the objective may be of maximization or some transportation route may be prohibited.*
Keywords: *Linear programming problem, Data Environment Analysis, Transportation Problem, Modi distribution method, Initial Basic Feasible solution.*

I. Introduction

Linear programming is the underlying methodology that makes DEA [1] particularly powerful as compared with the other alternative productivity management tools. The most common methods among comparison or performance evaluation were regression analysis and stochastic frontier analysis. But, these measures are at times inadequate due to various reasons such as multiple inputs and outputs related to different resources, activities and various environmental related factors. DEA[2] enables to calculate the efficiency of an organization relatively to the identified group's best practice. The two major issues of Transportation problem are discussed with the help of a relevant example as follows:

a. Cost Maximization Problem

A dairy farm has three plants located throughout a state. Daily milk production at each plant is as follows:

Plant A……..5 million litres
Plant B……..2 million litres
Plant C……..8 million litres

Each day the firm must fulfill the needs of its four distribution centres. Minimum requirement at each centre is as follows:

Distribution Centre 1……6 million litres
Distribution Centre 2……4 million litres
Distribution Centre 3……2 million litres
Distribution Centre 4……3 million litres

Cost of shipping one million litres of milk from each plant to each distribution centre is given in the following table in hundreds of rupees:

Plants	Distribution Centres			
	1	2	3	4
A	2	3	9	5
B	0	1	5	2
C	5	7	12	7

The dairy farm wishes to decide as to how much should be the shipment from which plant to which distribution centre so that the cost of shipment may be minimum.

b. Profit Maximization Problem

A company manufacturing TV sets at two plants located at Hyderabad and Salem with a capacity of 300 units and 100 units respectively. The company supplies its TV sets to its four showrooms situated at Ernakulam, Delhi, Bangalore and Chennai which have a maximum demand of 85, 150, 150 and 55 units respectively. Due to the differences in the raw material cost and transportation cost the profit per unit in rupees differs which is shown below in the table.

City	Ernakulam	Delhi	Bangalore	Chennai
Hyderabad	80	60	190	175
Salem	70	85	50	65

Plan the production programme so as to maximize the profit. The company may have its production capacity at both the plants partly or wholly unused.

The places where the goods originate from (the plants or factories or godowns) are called the *sources* or the *origins*; and the places where they are to be supplied are known as the *destinations*.

In this terminology the problem of the manufacturer or distributer is to decide as to how many units to be transported from the different origins to different destinations so that the total transportation cost is minimum (or profit is maximum).

II. DEA and Basic Feasible Solution of A Transportation Problem

The number of constraints in a general transportation problem (no of cells in the transportation tableau) are (m+n) i.e. the number of rows + number of columns. The number of variables required for forming a basis is 1 less i.e. (m+n-1). This is so because there are only (m+n-1) independent variables in the solution basis. In other words with values of any (m+n-1) independent variables being given, the remaining would automatically be determined on the basis of those values. Also considering the conditions of feasibility and no negativity the number of basic variables representing transportation routes that are utilized equals (m+n-1) and all other variables be non basic or zero representing the unutilized routes. It means that a basic feasible solution of a transportation problem has exactly (m+n-1) positive components.

When a problem is solved some of the x_{ij}*'s* would assume positive values indicating utilized routes. The cells containing such values are called *occupied cells* and each of them represents the presence of the basic variable. For the remaining cells called the unoccupied cells x_{ij}*'s* would be zero. These are the routes that are not utilized by the transportation schedule and their corresponding variables x_{ij}*'s* are termed as non basic variables.

How to solve a Transportation Problem?

As a transportation problem, it can be put in a linear programming format. It can be solved by simplex method but the solution is going to be very lengthy because of the involvement of a large number of decisions as well as artificial variables. Hence, we modified the procedure of simplex method popularly known as Modified Simplex Method [3]. This method yields results faster and with least computational efforts. A significant point of difference between this method and simplex method is regarding the determination of the initial basic feasible solution. It involves the following major steps:

1. Formulate the given problem as the linear programming problem.
2. Setup the given LPP in the tabular form known as transportation problem.
3. Examine whether total supply equals total demand, if not introduce a dummy row/column having all the cost elements as zero and supply/demand as the positive difference of supply and demand.
4. Find an initial Basic Feasible Solution that must satisfy all the supply and demand conditions.
5. Examine the obtained solution for optimality, i.e. examine whether an improved transportation schedule with a lower cost is possible or not.
6. If the obtained solution is not found to be optimum, modify the shipping schedule by including that unoccupied cell whose inclusion may result in an improved solution.

7. Repeat the 4th step until no further improvement is possible.

We shall now discuss various methods available for finding an initial basic feasible solution and then attaining an optimum solution. There are several methods available to obtain an initial basic feasible solution of a transportation problem. These methods are mentioned below

a. North – West Corner Method (NWC Rule)
b. Least Cost Method
c. Vogel's Approximation Method

III. Linear Programming Problem Analysis

Data envelopment analysis is a linear programming problem that provides a means of calculating apparent efficiency levels within a group of organizations. The efficiency of an organization is calculated relative to the group's observed best practice.

IV. Optimality Test

The determination of an initial basic feasible solution to a transportation problem is followed by the next very critical step of how to arrive at the optimum solution. The basic technique is the same as in the simplex method namely:

1. Determining the net evaluations for non basic variables.
2. Choosing that opportunity cost which may improve the current basic feasible solution.
3. Determining the current occupied cells which leaves (i.e. becomes unoccupied) and repeating the steps 1 through 3 until an optimum solution is attained.

V. Mathematical Formulation and Solution of the Transportation Model

Let us consider a practical problem and then try to analyze and find a mathematical solution to the identified problem.

A company has three factories F_i (i=1, 2, 3) from which it transports the product to four warehouses W_j (j=1, 2, 3, 4). The unit costs of production at the three factories are Rs.4, Rs. 3, and Rs. 5 respectively. Given the following information on unit costs of transportation capacities at the three factories and of the requirement at the four warehouses, find the optimum allocation.

Factory	Unit cost of Production (*Rs./unit*)	Transportation cost (*Rs./unit*)				Capacity
		W_1	W_2	W_3	W_4	
F_1	4	5	7	3	8	300
F_2	3	4	6	9	5	500
F_3	5	2	6	4	5	200
Requirements		200	300	400	100	1000

Solution: Using *Vogel's Approximation Method*, an initial basic feasible solution is displayed in the following transportation table:

					Row Difference		
	5	7	300 / 3	8	2	-	-
	100 / 4	300 / 6	9	100 / 5	1	1	1
	100 / 2	6	100 / 4	5	2	2	3
Column Difference	2	0	1	0			
	2	0	5	0			
	2	0	-	0			

The number of occupied cells are 3+4-1 i.e. 6. This indicates that the basic feasible solution obtained above is non – degenerate. Using MODI method, an optimum solution is obtained as follows:

Initial Iteration: Compute the numbers u_i (i=1, 2, 3) and v_j (j=1, 2, 3, 4) for the occupied cells and then compute the opportunity costs d_{ij} for the non occupied cells. These are shown in the table given below wherein we have set u_2 =0.

					u_i
	-4 5	-4 7	300 3	-6 8	-3
	100 4	300 6	-3 9	100 5	0
	100 2	-2 6	100 4	-2 5	-2
v_j	4	6	6	5	

Since all the opportunity costs d_{ij} are negative, an optimum solution is obtained.
The optimum solution is:
$x_{13} = 300$, $x_{21} = 100$, $x_{22} = 300$, x24 = 100, $x_{31} = 100$ and $x_{33} = 100$.
The total minimum cost of production as well as transportation is:
Rs. 4 *300 *3 + Rs.3 * (100 *4 + 300 * 6 + 100 * 5) + Rs.5 * (100 *2 + 100 * 4)
i.e. Rs. 3600 + Rs.8100 + Rs.3000 = Rs. 14700.
Unbalanced Degeneracy Alternative

VI. Merits and Demerits

The major advantage of using DEA is that it includes multiple input and output to evaluate the technical efficiency. The competitive organizations that are not efficient enough are identified and for such organizations it plays very potential role models, which can be further adopted by these organizations.

DEA [4] is quite a similar to any of the empirical techniques that is majorly based on a number of simplifying assumptions which has to be acknowledged while interpreting the results of DEA.

The major limitation of DEA is that it is a much more deterministic technique rather than statistical. It generally generates the results which are more sensitive to the measurement errors.

Since it is capable of only measuring the efficiency relative to a particular sample, it is not able to handle the comparative scores between 2 different studies. DEA technique is very sensitive to the input and output specifications and the sample size. Despite of these limitations it is a very useful tool to examine the efficiency of various government service providers.

VII. Conclusion

This paper has introduced the concept of efficiency measurement of decision making units and the DEA technology for the measurement. In India, Operations Research came into existence in 1949 with the opening of an O.R. unit at the Regional Research Laboratory at Hyderabad. At the same time, another group was set up in the Defence Science Laboratory which devoted itself to the problems of stores, purchase and planning. In 1953, an O.R. unit was established in the Indian Statistical Institute, Calcutta for the application of O.R methods in national planning and survey. O.R. society of India was formed in 1957. It became a member of the International Federation of O.R. Societies in 1959. Towards the applications of O.R. in India, Prof. Mahalonobis made the first important application. He formulated the second Five – Year Plan with the help of O.R. techniques to forecast the trends of demand, availability of resources and for scheduling the complex schemes necessary for developing our country's economy. The Transportation Problem is one of the sub – classes of LPP in which the objective is to transport the various amounts of a single homogeneous commodity, that are initially stored at various origins, to different destinations in such a way that the total transportation cost is minimum. Although the name of the problem is derived from transport to which it was first applied, the problem can also be used for machine allocation, plant location, product mix problem, and many others, so that the problem is not confined to transportation or distribution only.

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