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Representing Tap-changer Transformers in Conic Relaxation Optimal Power Flows

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Abstract: *Conic optimization has been applied optimal power flow (OPF) problem recently. Present conic relaxation OPFs only consider for the continuous decision variables, and the transformer tap ratios have not been treated as optimization variables. Based on angle relaxed branch power flow, an extended branch power flow model with tap-changer transformers is proposed. The tap-changer transformers were modeled with bilinear functions. Then the McCormick envelopes were deployed to exactly reformulate the bilinear functions with linear constrains. The effectiveness of the proposed method is demonstrated by the simulation results obtained in the test systems.*
Keywords: *Conic optimization, discrete decision variables*

I. Introduction

The tap-changer transformers are widely used in power systems to realized voltage regulation. Traditionally the transformer tap ratios are set by automatic voltage regulators based on local voltage measurements or OPFs where the tap ratios are optimization variables to improve system efficiency. Since the OPFs could obtain certain overall system operational objectives, the tap ratios are included as discrete variables expanding the OPFs to mix-integer optimization (MIP) problems in this paper.

Due to the nonconvex nature of AC power flow equations, the MIP problems are always NP-hard to solve. By relaxing the relationship between voltages, the conic optimization was firstly deployed in [1] to solve the OPFs. Another conic relaxation method was proposed in [2], and the relationship between quadratic relaxations was also demonstrated.

While the above conic relaxations were all based on bus injection power flow, the authors in [3] proposed a novel conic relaxation by rewriting the power flow equations based on the branch power flow. And this method would always obtain much tighter results in radial networks [4].

However, all these conic relaxations had not taken the transformers' tap ratios as optimization variables. The traditional methods in OPFs to optimize the tap ratios were mostly based on bus injection power flow by fixing the admittance matrix. While according to our knowledge, in conic relaxed branch power flow, how to model the transformer tap ratios as decision variables still remains to be urgent and has not been well studied.

By modeling the tap-changer transformers in angle relaxed power flow, a convex envelope based relaxation for the transformers is proposed in this paper.

II. Angle Relaxed Power Flow Model

The angle relaxed branch power flow represented by linear equality functions and conic functions for radial power networks are shown as follows [3]:

$$\sum_{i \in \mathrm{G}_j} P_{G,i} - \sum_{i \in \mathrm{L}_j} P_{L,i} = P_{ij} - r_{ij} l_{ij} - \sum_{k:(j,k)\in \mathrm{E}} P_{jk}, \forall j \in \mathrm{B} \tag{1a}$$

$$\sum_{i \in \mathrm{G}_j} Q_{G,i} - \sum_{i \in \mathrm{L}_j} Q_{L,i} = Q_{ij} - x_{ij} l_{ij} - \sum_{k:(j,k)\in \mathrm{E}} Q_{jk}, \forall j \in \mathrm{B} \tag{1b}$$

$$v_j = v_i - 2(r_{ij} P_{ij} + x_{ij} Q_{ij}) + (r_{ij}^2 + x_{ij}^2) l_{ij}, \forall (i,j) \in \mathrm{E}/ \tag{1c}$$

$$l_{ij} \geq \frac{P_{ij}^2 + Q_{ij}^2}{v_i}, \forall (i,j) \in \mathrm{E} \tag{1d}$$

where $P_{G,i}$ and $Q_{G,i}$ are the active and reactive power output of the i-th generator, $P_{L,i}$ and $Q_{L,i}$ are the active and reactive power consumption of the i-th user, V_i is the voltage magnitude at bus i, $v_i := V_i^2$, r_{ij} and x_{ij} re the resistance and reactance on line (i,j), I_{ij} is the complex current flowing from bus i to j, $l_{ij} := |I_{ij}|^2$, P_{ij} and Q_{ij} are the active and reactive power from bus i to bus j, G_i and L_i are the set of generations and users connecting to the i-th bus, B is the set of buses, E is the set of branches.

III. Tap-Changer Transformer Models

3.1 Exact Model

Atypical tap-changer transformer could be represented as impedance $r_{ij} + jx_{ij}$ in series with an ideal transformerrepresenting the tap ratio $\sqrt{k_{ij}}:1$ as shown in Fig. 1 [1].The impedance $r_{ij} + jx_{ij}$ of transformer (i, j) is calculated with a specific tap ratio, i.e., k_{ij}=1.

By introducing a virtual node x at the primary side of transformer, the KCL and KVL equationsfor node $x$and transformer (i, j) could be represented as following:

$$P_{ix} + jQ_{ij} = P_{xj} + jQ_{xj} \tag{2a}$$

$$v_x = v_i - 2(r_{ij}P_{ij} + x_{ij}Q_{ij}) + (r_{ij}^2 + x_{ij}^2)l_{ij}, \forall (i,j) \in \mathrm{K} \tag{2b}$$

$$v_x = k_{ij}v_j, \tag{2c}$$

whereK is the set of transformers. Since the transformer is ideal, there is no power loss on the transformer. When node x and j are integrated to one super node j, the KCL equations for node i and j stay the same with (1a) - (1b), while the KVL equation for transformer (i, j)should be modified as the following equation:

$$k_{ij}v_j = v_i - 2(r_{ij}P_{ij} + x_{ij}Q_{ij}) + (r_{ij}^2 + x_{ij}^2)l_{ij}, \forall (i,j) \in \mathrm{K} \tag{2d}$$

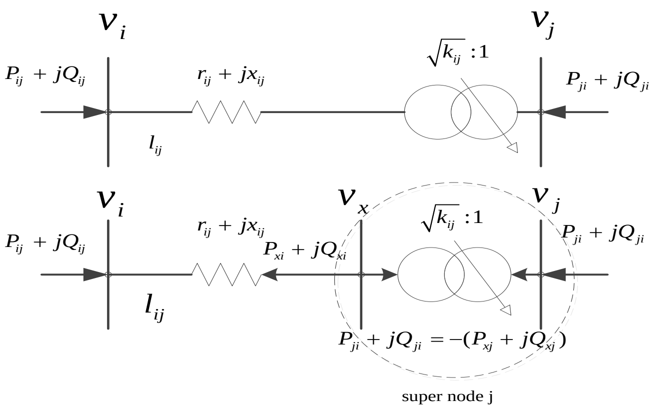

Fig.1 Tap-changing transformer model

3.2 Relaxed Model

The difficulty to reformulate the transformer model lies in the bilinear function$w$=kv, which resulting in the non-linear and non-convex of equation (2d). Since the tap ratios could not be adjusted continuously, the following constrains should hold for the tap ratios:

$$k = \sum_{i \in \mathrm{T}_i} a_i k_i^0 \tag{3a}$$

$$\sum_{i \in \mathrm{T}_i} a_i = 1 \tag{3b}$$

whereT_i is the set of states of transformer tap ratios,$\{k_1^0, k_2^0, \ldots\}$; a_i is a binary variable which represents whether$k$ is k_i^0 or not. And (3b) is the SOS-1 type constrain. Then the bilinear function w=kv could be represented by the following equation:

$$w = \sum_{i \in \mathrm{T}_i} w_i = \sum_{i \in \mathrm{T}_i} a_i k_i^0 v \tag{3c}$$

As shown in equation (3c), there still exist bilinear terms at each state. The traditional methods to transfer bilinear equations to linear constrains are linear reformulation technique, McCormick inequalities [5] and so on.In this paper, the McCormick inequalities are deployed to linearize the binary bilinear items in (3c) as following:

$$w_i \ge k_i^0 \underline{v} a_i \tag{3d}$$

$$w_i \ge k_i^0 (v + \overline{v} a_i - \overline{v}) \tag{3e}$$

$$w_i \le k_i^0 \overline{v} a_i \tag{3f}$$

$$w_i \le k_i^0 (v + \underline{v} a_i - \underline{v}) \tag{3g}$$

where $\underline{v}$ and $\bar{v}$ are the lower and upper boundary of v.

***Proposition* 1: Equations (3d)-(3g) are the exact linear relaxation of equation (3c).**
***Proof*:** When representingw_i by $a_i k_i^0 v$, it means the following disjunctive function:

$$w_i = \begin{cases} 0, a_i = 0 \\ k_i^0 v, a_i = 1 \end{cases} \tag{3h}$$

Whena_i=0, w_i inequations (3d)-(3g) equals 0. And when a_i=1, w_i in equations (3d)-(3g) equals$k_i^0 v$. Then there is no loss by representing (3c) with (3d)-(3g).
Then the bilinear function$w$=kvcould be exactly reformulated withlinear equations (3a) -(3b) and (3d) -(3g), which completes the proof.

IV. Example

The simulation is implemented on MATLAB version 8.2, using the YALMIP[6]with the commercialsolver Gurobi [7] on an Intel Corei5-3320M CPU at 2.6 GHz and 8 GB of RAM computer. A modified 69-bus test system was deployed to demonstrate the effectiveness of the method. The branch and load data could be obtained from [8], and branch 1, 9 and 59 are replaced with three tap-changer transformers with tapratios varyingfrom 0.9p.u. to 1.05 p.u. (0.01 p.u. each step).
The objective is to minimize the real power losses. Compared with the result (0.2111MW) obtained from the bus injection flow by the solver BONMIN [9], the optimal losses is 0.1983MW, reduced by 6.44%. And the solving time has significantly reduced from 85.1955s to 0.3054s. The maximum value of $v_i l_{ij}$-$P_{ij}^2 - Q_{ij}^2$ in (1d) is 1.0687e-06.
Future, three distributed generations are integrated to bus 3, 6 and 19 with the same active and reactive output range [0, 1] MW and [-0.3, 1] MVar, respectively. The real power losses would reduce to 0.1527MW, and the solving time remains almost the same in our method.While in the IEEE-33 bus test system, the solving time reduces to 0.1565s with three tap-changer transformers and three DGs integrated.

V. Conclusion

This paper presents a relaxation method for incorporating tap-changer transformers in a branch power flow based OPFs.

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