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Dynamic Model of Windings Parameters for Electrical Machines with a Non-Symmetric Magnetic System

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Abstract: *Electrical Machines that are having clearly expressed poles in rotor or in stator possess non-symmetric magnetic systems. In this paper we study main winding inductances at the example of magnetic system with salient rotor.*
Keywords*: Dynamic Model, Winding Parameters, Electrical Machine, Non-Symmetric Magnetic System*

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I. INTRODUCTION

After the first representation of the alternating current (AC) electrical machine given by Nikola Tesla in the end of the XIXth century[1], the dynamic modeling of these machines became the corner stone in order to analyze and control them for a better yield and stability.

In this line, the Park theory – awardedin 2002 as the 2ndpublication that has had the most influence in power electronics theory –gives two-axis (d,q) transformation which is practical for the representation and modeling of the dynamic electromagnetic behavior of the AC electrical machine[2], [3].

Based on this theory, researchers and electrical engineers of the AIEE (American Institute of Electrical Engineers) further of the IEEE (Institute of Electrical and Electronics Engineers) developed a various dynamic model for the AC electrical machines with a particular interest on the impedance matrix [4], [5], [6].

Number of these models are convenient for the study of AC electrical machine with a symmetric magnetic system because we can consider a uniform variation of the magnetic field due to stator and rotor windings currents. But, in certain cases the stator and rotor windings are not uniformly distributed along the cores. The aim of this paper is then to study the magnetic distribution (conduction) along longitudinal and transversal axes of stator and rotor in order to give an inductance matrix representation that takes into account the non-symmetric magnetic system.

1- Main magnetic conduction along longitudinal axes of magnetic symmetry d [2], [3], [6], [7].

For convenient analysis conditions in the Machine section transversal plane, we represent symmetry axes d and q (figure 1).

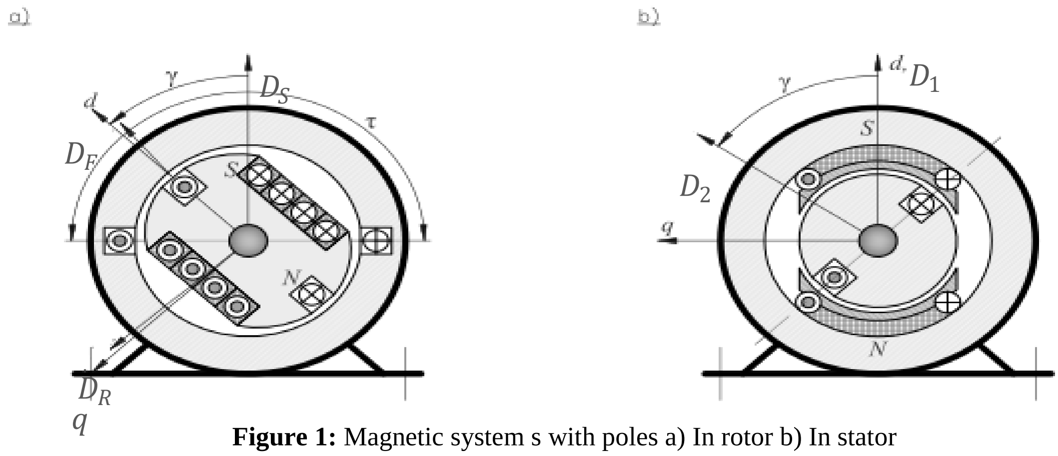

Figure 1: Magnetic system s with poles a) In rotor b) In stator

We suppose that, in the rotor poles, we have a winding with W_d −number of turns (figure 2). Magnetic axis of winding corresponds with longitudinal axissymmetry d:$\boldsymbol{D_d^T} = [1, 0]$.

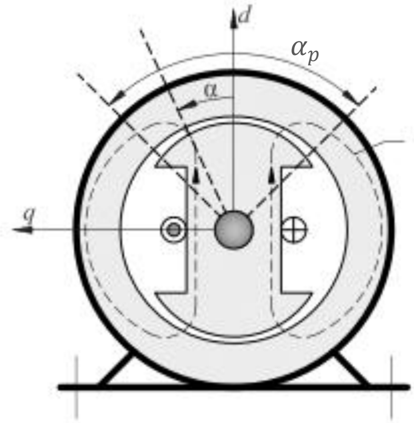

Figure 2 : Magnetic lines of fluxes along longitudinal axis d

Magnetomotrice force of winding is $F_d = W_d \cdot i_d$,where $i_d -$ winding current.

Magnetomotrice force creates magnetic field.

If we assume that magnetic induction in rotor and stator cores is equal to zero, then total current law can be written as follows:

$$F_d = W_d \cdot i_d = 2 \cdot H_d \cdot \delta(\alpha)$$

Where $H_d -$ induction of magnetic field in the air gap,
α −Angle coordinate of air gap (central angle, measured related to longitudinal axis of magnetic symmetry d)

δ in general is a function of angle α:

$$\delta(\alpha) = \delta_0 . f(\alpha)$$

Where δ_0 – expression of air gap along longitudinal axis of magnetic symmetry d:
$f(\alpha)$ – function, whose values are symmetric relatively to axis d, with $f(0) = 1$

We assume that magnetic induction in air gap is concentrated under poles, and out of poles it is zero. The distribution of induction is expressed as follows:

$$H_d(\alpha) = \begin{cases} \frac{H_{d0}}{f(\alpha)} if \ \alpha \epsilon \left[-\frac{\alpha_p}{2}, \frac{\alpha_p}{2}\right] \\ -\frac{H_{d0}}{f(\alpha)} \ if \ \alpha \ \epsilon \left[\pi - \frac{\alpha_p}{2}, \pi + \frac{\alpha_p}{2}\right] \end{cases} \quad (1)$$

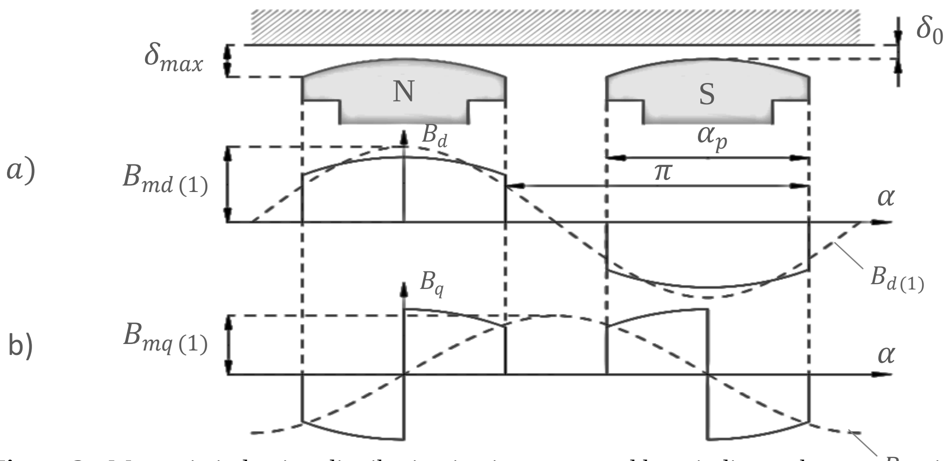

Figure 3 : Magnetic induction distribution in air gap created by windings whose magnetic axes are directed along magnetic symmetry a) d b) q

Where $H_{d_0} = W_d . \frac{i_d}{2\delta_0} -$ magnetic induction in the center of pole on axis d, corresponding to current i_d.

$\alpha_p -$ angle of pole fermeture.

Magnetic field $B_d(\alpha) = \mu_0 . H_d(\alpha)$ has a distribution along air gap analog to magnetic field (figure 3.a)

From approximation with main harmonic:

$$B_{d(1)} = B_{md\,(1)} cos\alpha$$

$$B_{md\,(1)} = \frac{1}{\pi}\int_{-\pi}^{\pi} B_d(\alpha)\cos(\alpha)d\alpha = \frac{4}{\pi}B_o K_d$$

Where $K_d = \int_0^{\frac{\alpha_p}{2}} \frac{\cos\alpha}{f(\alpha)} d\alpha$ −form coefficient.

The main magnetic flux along air gap is therefore:

$$\phi_{dd} = \int_{dS\epsilon S_0} B_{d\,(1)}.dS = \lambda_0.K_d.F_d$$

Where $dS = R.l_o.d\alpha;\ \alpha\ \epsilon\ [-\frac{\pi}{2},\frac{\pi}{2}]$.

$S_0 = \pi R l_0;$

λ_0 – magnetic conduction of machine with symmetric magnetic system.

The main magnetic conduction of first harmonic along longitudinal axis of magnetic symmetry d is:

$$\lambda_d = \frac{\phi_{dd}}{F_d} = \lambda_0.K_d$$

2- The Main magnetic conduction along transversal axis of magnetic symmetry q[2], [3], [6], [7].

We suppose that in notching for rotor poles there is a non-salient winding with W_q turns (figure 4).

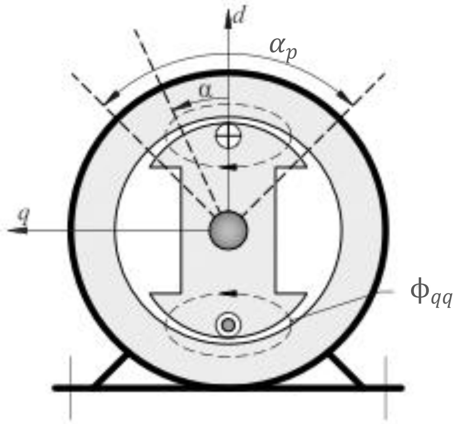

Figure 4 : Magnetic lines of fluxes along transversal axis q

The magnetic axis of winding corresponds with axis of transversal symmetry q:

$\boldsymbol{D_q^T} = [0,1]$. Magnetomotrice force of winding $F_q = W_q.i_q$, where$i_q -$ winding current. It creates magnetic field.

If we assume that magnetic induction H_q in stator and rotor cores is equal to zero, H_q is concentrated in the air gap:

$$F_q = W_q.i_q = 2.H_q.\delta(\alpha) = 2H_q\delta_0.f(\alpha)$$

We assume that magnetic induction H_q in air gap is concentrated in pole and out of pole is zero. Thus

$$H_q(\alpha) = \begin{cases} \frac{H_{q_0}}{f(\alpha)}\ if\ \alpha\epsilon\left[0,\frac{\alpha_p}{2}\right] U\ [\pi-\frac{\alpha_p}{2},\pi] \\ -\frac{H_{q_0}}{f(\alpha)}\ if\ \alpha\epsilon\left[\pi,\pi+\frac{\alpha_p}{2}\right] U\ [-\frac{\alpha_p}{2},0] \end{cases} \quad (2)$$

Where $H_{q_0} = W_q.\frac{i_q}{2\delta_0}$ – magnetic induction in center of pole in axis d, corresponding withi_q.

Magnetic field $B_q(\alpha) = \mu_0.H_q(\alpha)$ has similar distribution along air gap with magnetic induction. (See figure 3.b)

The distribution of magnetic field along pole division angle can be approximated with first harmonic $B_{q\,(1)} = B_{mq\,(1)}sin\alpha$

The amplitude of first harmonic of magnetic field

$$B_{mq\,(1)} = \frac{1}{\pi}\int_{-\pi}^{\pi} B_q(\alpha)\sin(\alpha)d\alpha = \frac{4}{\pi}B_0 K_q$$

Where $K_q = \int_0^{\frac{\alpha_p}{2}} \frac{sin\alpha}{f(\alpha)} d\alpha$ is form coefficient.

The main flux along air gap is created by main harmonic of magnetic field:

$$\phi_{qq} = \int_{dS\epsilon S_0} B_{q\,(1)}.dS = \lambda_0.K_q.F_q$$

Where $dS = R.l_o.d\alpha$;$\alpha\epsilon\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$; $S_0 = \pi R l_0;$

$\lambda_0 -$ magnetic conduction of machine with symmetric magnetic system.
The main conduction along transversal axis of magnetic symmetry q:

$$\lambda_q = \frac{\phi_{qq}}{F_q} = \lambda_0 . K_q$$

3- Matrices of winding inductances in salient polesmachine [2], [3], [8], [9]

We suppose that in machine poles there are two windings, whose axes are directed along axes d and q (figure 5).

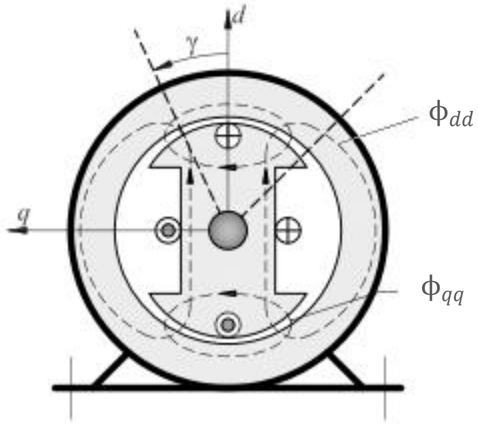

Figure 5 : Machine with windings whose magnetic axes are directed along axes d and q

The main inductances of windings are expressed as follows:

$$\mathbf{L_{dd}} = W_d . \lambda_d . W_d = W_d . \boldsymbol{D_d^T} . \lambda_d . \boldsymbol{D_d} . W_d$$
$$\mathbf{L_{qq}} = W_q . \lambda_q . W_q = W_q . \boldsymbol{D_q^T} . \lambda_q . \boldsymbol{D_q} . W_q$$

Magnetic conduction λ_q is less than λ_d.

The main magnetic conductions of windings along axes d and q are characterized by diagonal matrices:

$$\boldsymbol{\Delta_{dq}} = diag(\lambda_d, \lambda_q) \quad (3)$$

The matrix of inductances of rotor windings in figures (1, 3) can be represented as follows:

$$\mathbf{L} = \boldsymbol{W} . \boldsymbol{D^T} . \boldsymbol{\Delta_{dq}} . \boldsymbol{D} . \boldsymbol{W} \quad (4)$$

Where $\boldsymbol{W} = diag(W_d, W_q)$ – diagonal matrix of rotor windings turns.
$\boldsymbol{D} = [D_d, D_q] = 1$ is phase matrix rotor windings

In notching for rotor poles we can enroll n-phase winding. (figure 6)

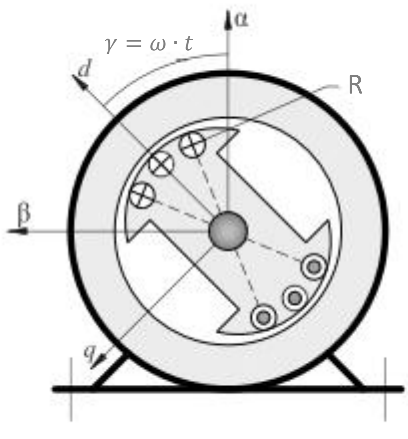

Figure 6 : Electrical machine with three-phase non-salient winding in rotor

Space orientation of magnetic axes n-phase symmetric non salient rotor winding is characterized by phase matrix:

$$D_R = \begin{pmatrix} \cos(\alpha_1) \cos(\alpha_1 + \rho_R) \dots \dots \dots \dots \dots . \cos(\alpha_1 + (n-1)\rho_R) \\ \sin(\alpha_1) \sin(\alpha_1 + \rho_R) \dots \dots \dots \dots \dots . \sin(\alpha_1 + (n-1)\rho_R) \end{pmatrix}$$

Where $\alpha_1 = \frac{[\rho_R(1-n)-\pi]}{2}$ – deviation angle of magnetic axis of first winding phase relatively to magnetic symmetry d.

ρ_R – deviation angle of magnetic axes of phase winding each other related.

Phase rotor salient windings are uniformly distributed along pole fermeture angle α_p that is why we can assume $\alpha_p = n\rho_R$.

The matrix of inductances of n-phase non-salient rotor winding looks as follows:

$$\mathbf{L_{RR}} = W_R.\mathbf{D}_{\boldsymbol{R}}^{\mathbf{T}}.\Delta_{dq}.\mathbf{D_R}.W_R \quad (5)$$

Where $W_R = diag(w_1, \ldots., w_n)$ is the diagonal matrix of rotor phase number of turns.

4- Matrix of winding inductances in non-salient machine with non-symmetric magnetic system [7], [8], [9], [10]

Let us consider magnetic processes in the electrical machine created by non-salient stator winding A (figure 7). We assume that in stator we have a fixed coordinate system (α, β), and in rotor, axes of magnetic symmetric (d, q).

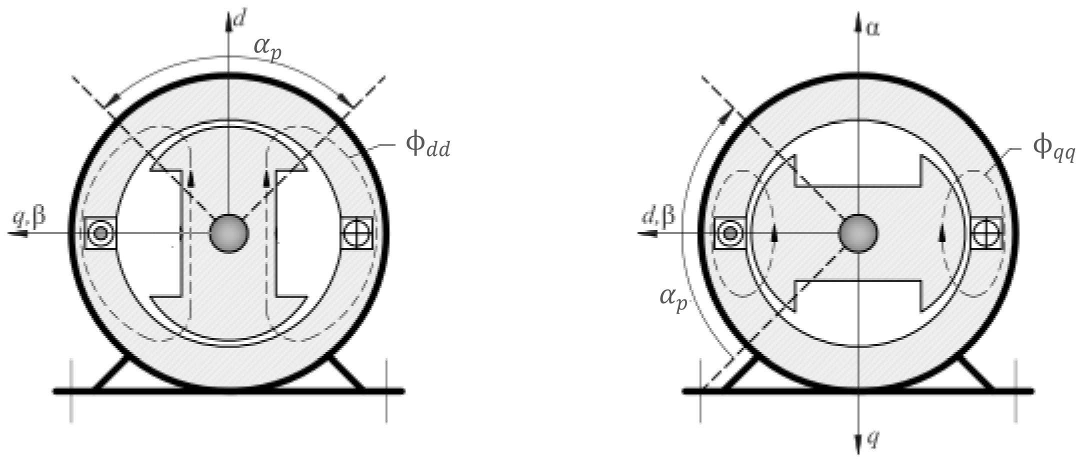

Figure 7 : Magnetic lines of fluxes along d and q, created by stator winding

The position of coordinate axes (α, β) can be chosen arbitralrily. Thus the magnetic axis of stator winding in (α, β) system is characterized by vector$\mathrm{D}_S^T = [\cos(\alpha_A) \sin(\alpha_A)]$. Where α_A is the angle between the coordinate axis α and the magnetic axis of winding A.

When the latter corresponds with longitudinal axis rotor magnetic system d, the magnetic field created by A will be analog to the magnetic field created by a longitudinal winding in the rotor.

If magnetic axis of winding corresponds with axis of rotor transversal magnetic symmetry q, then the magnetic field in machine will be analog to magnetic field created by transversal winding situated in notching of rotor poles (figure 3). Therefore, inductances of winding when rotor rotation will change from maximal value $L_{dd} = w_A^2.\lambda.d$ to minimal value $L_{qq} = w_A^2\lambda.q$.

Winding magnetic axis $\mathrm{D}_S$can be expressed in axes of magnetic symmetry d and q that are deviated relatively to α, β by angle γ:

$$\mathbf{D}_{\boldsymbol{S}\to\boldsymbol{R}} = \nabla(\gamma)^T.\mathrm{D}_S$$

Where, $\boldsymbol{\nabla}(\boldsymbol{\gamma})$is the rotation matrix.

We can therefore express the inductance of winding in the case of non-symmetric magnetic system:

$$L_{AA} = W_A.\mathrm{D}_{S\to R}^T.\Delta_{\mathrm{dq}}.\mathrm{D}_{S\to R}.W_A$$

$$\mathbf{L_{AA}} = \boldsymbol{W_A}.\mathbf{D_S^T}.\boldsymbol{\nabla}(\boldsymbol{\gamma}).\boldsymbol{\Delta}_{\mathbf{dq}}.\boldsymbol{\nabla}(\boldsymbol{\gamma})^{\mathbf{T}}.\mathbf{D_S}.\mathbf{W_A} \ (6)$$

Where Δ_{dq} is defined in (3).

In the developed form

$$L_{AA} = L_{dd}\cos(\gamma_A)^2 + L_{qq}\sin(\gamma_A)^2 = L_0 + L_m\cos(2\gamma_A)$$

Where$\gamma_A = \gamma - \alpha_A$;$L_0 = \frac{L_{dd} + L_{qq}}{2}$; $L_m = \frac{L_{dd} - L_{qq}}{2}$;

The formula (6) can be generalized in the case of arbitrary number of windings in stator. In this case we use phase matrix windings:

$$\mathbf{L_{SS}} = \boldsymbol{W_S}.\boldsymbol{\Delta}_{\boldsymbol{S}}^{\boldsymbol{T}}.\boldsymbol{\nabla}(\boldsymbol{\gamma}).\boldsymbol{\Delta}_{\boldsymbol{dq}}.\boldsymbol{\nabla}(\boldsymbol{\gamma})^{\boldsymbol{T}}.\boldsymbol{\Delta}_{\boldsymbol{S}}.\boldsymbol{W_S} \ (7)$$

II. CONCLUSION

Magnetic systems with clearly expressed poles have magnetic asymmetry. For formalism of magnetic processes in the machine, we introduce axes longitudinal d and transversal q of magnetic symmetry. The main magnetic conductions of windings along axes of magnetic symmetry are various and are characterized by matrix (3). The most convenient form of representation for inductances of stator and rotor multiphase winding is presented in (7) with the use of matrix of main magnetic conductions, phase matrix and rotation matrix.

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